IV. CONCLUSIONS

As a numerical technique, the least square error minimization is near in style to point matching and Fourier matching. All three methods are versatile in being able to join up a patchwork of regions, each with a complete expansion that exactly satisfies the differential equations of the problem. In each method, some different criterion is put forward to approximately satisfy the remaining boundary conditions.

Point matching is undoubtedly the easiest scheme to implement, as it replaces integration by sampling of fields at discrete points. In its simple form, however, it is known that it can fail to converge or give useful answers [7],[13]. Fourier matching and least squares both involve integration of the boundary residuals (boundary errors). Inner products are needed of all boundary residuals formed with either a set of test functions (Fourier) or the same boundary residuals (least squares).

In the earlier sections, it has been demonstrated that the least squares method, as used for scattering problems, can equally be applied to eigenvalue problems. In Section III, examples are given of two such applications, the microstrip example being taken as a test case to compare with Fourier matching. For this example, the field analysis leading to the formulation of the matrix elements is very similar for the two techniques; in each case, four integrals are required along a boundary interface. The two matrix orders are the same for the same choice of expansion sets. The least squares approach requires the (lowest eigenvalue) solution of a real, symmetric, positive-definite matrix whereas Fourier matching requires the evaluation of the determinant of a real but nonsymmetric and nondefinite matrix. Least squares therefore needs half the storage for the matrix elements. In this work, the least squares matrix was solved by Householder tridiagonalization [12] followed by Sturm sequence and bisection—the Fourier matrix by Gaussian elimination with partial pivoting [14].

For the least squares, solution can be via inverse iteration [15], with Choleski decomposition [14]. Whether using the Fourier or least squares approach, computing time is likely to be comparable for the evaluation of the matrix elements; similarly, for the solution of the matrix. There is therefore little between the methods, in terms of computing time, for a given matrix order.

Overall, the least squares approach would seem to have two potentially important advantages. Firstly, by the empirical choice of optimum weighting factors with low-order matrices these factors can then be used to advantage for higher matrix orders, and considerable acceleration of convergence has been obtained compared with Fourier matching. Best weighting factors were found to occur when arranged (as is easily done) for equal contribution to the error norm from the different boundary residuals. Secondly, in contrast to point matching (and as described in [1],[17], and [18]) it is rigorously convergent.

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The Microstrip Double-Ring Resonator

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***Abstract*—The resonance frequencies and the fields of a microstrip double-ring resonator are discussed. It is shown that no pure even or odd mode can be excited on the resonator. Therefore it is concluded that the microstrip double-ring resonator principally cannot be used to measure the phase velocities of the even and the odd modes on a coupled microstrip line.**

I. INTRODUCTION

Gould and Talboys [1] described a method for measuring the wavelengths on coupled microstrip lines, using a double-ring resonator. The described method has been used by Getsinger [2] too, to prove a theory for calculating the even- and odd-mode wavelengths. Gould and Talboys [1] assumed that an even and an odd mode can be excited on the double-ring resonator despite the fact that the two coupled rings are of different lengths. Furthermore they described that they measured an additional splitting of the resonance frequencies in the case of loosely coupled rings, using a field probe to detect the different resonances.

II. THE STRAIGHT MICROSTRIP DOUBLE-LINE RESONATOR OF DIFFERENT LINE LENGTHS

To get a first insight into the field distribution of a double-ring resonator, the resonator is unrolled and a resonator of two straight coupled microstrip lines, as shown in Fig. 1, is considered. This

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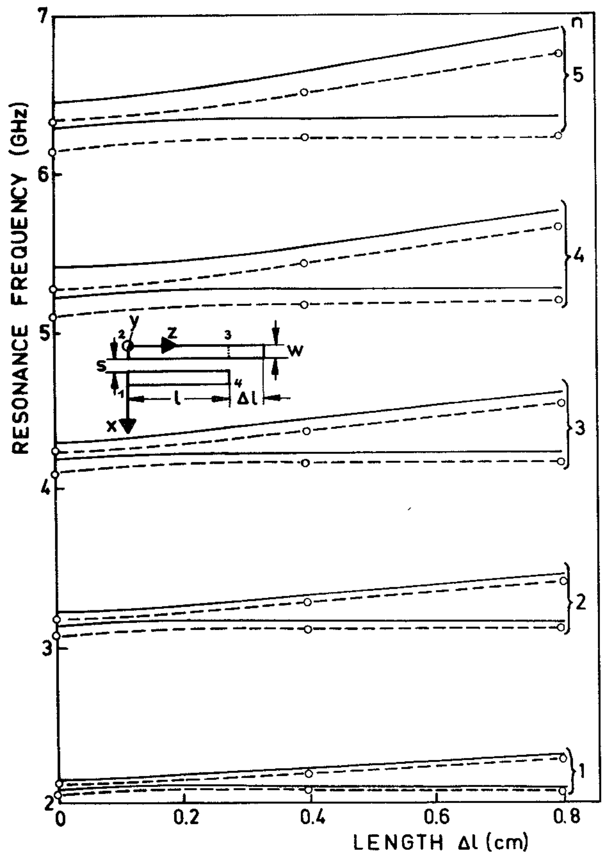

Fig. 1. Resonance frequencies and structure of the straight double-line resonator of constant length $l + \Delta l$. Resonance frequencies in dependence on the difference Δl of the line lengths. Substrate material Polyguide ($\epsilon_r = 2.32$, $h = 0.156$ cm). Resonator length $l + \Delta l = 10.01$ cm, $w = 0.44$ cm, $s = 0.5$ cm. ○ Experimental results.

resonator can be used to give a first approximation for the fields of the double-ring resonator, despite the fact that the conditions for the electromagnetic fields on both resonators are not really identical (the line of length Δl (Fig. 1) is not coupled to the other line, the linear pair has one asymmetrical termination, whereas in the case of the ring, the difference of the line lengths is uniformly distributed around the ring). Nevertheless the behavior of both resonators should be very similar.

To calculate the resonance frequencies and the field distributions of the straight double-line resonator as shown in Fig. 1, a coupled microstrip line of length l, which is connected to a short piece of an open-ended microstrip line of length Δl at one port and which is open ended at all other ports, is considered. From the equations for the voltages and the currents of the even and the odd mode on that line the following homogeneous set of equations for the voltages at ports U_1–U_4 can be derived:

$$\begin{aligned} Y_{11}U_1 + Y_{12}U_2 + Y_{13}U_3 + Y_{14}U_4 &= 0\\ Y_{12}U_1 + Y_{11}U_2 + Y_{14}U_3 + Y_{13}U_4 &= 0\\ Y_{13}U_1 + Y_{14}U_2 + (Y_{11} + 1/Z)U_3 + Y_{12}U_4 &= 0\\ Y_{14}U_1 + Y_{13}U_2 + Y_{12}U_3 + Y_{11}U_4 &= 0. \end{aligned} \qquad (1)$$

Y_{1j} ($j = 1,2,3,4$) are the elements of the admittance matrix of a pair of coupled lines

$$Y_{11} = -\tfrac{1}{2}j\left(\frac{\cot\theta_e}{Z_{0e}} + \frac{\cot\theta_o}{Z_{0o}}\right)$$

$$Y_{12} = -\tfrac{1}{2}j\left(\frac{\cot\theta_e}{Z_{0e}} - \frac{\cot\theta_o}{Z_{0o}}\right)$$

$$Y_{13} = -\tfrac{1}{2}j\left(\frac{-1}{Z_{0e}\sin\theta_e} + \frac{1}{Z_{0o}\sin\theta_o}\right)$$

$$Y_{14} = -\tfrac{1}{2}j\left(\frac{-1}{Z_{0e}\sin\theta_e} - \frac{1}{Z_{0o}\sin\theta_o}\right) \qquad (2)$$

considering the symmetry and the reciprocity of the structure. θ_e and θ_o are the electrical lengths of the line of geometrical length l for the even and the odd mode; Z_{0e} and Z_{0o} are the characteristic impedances of the even and the odd modes, respectively. Z in (1) is the characteristic impedance of the uncoupled line of length Δl.

Equation (1) only has a solution if the determinant of the system is zero, meaning

$$((Y_{11} + Y_{12})^2 - (Y_{13} + Y_{14})^2)((Y_{11} - Y_{12})^2 - (Y_{13} - Y_{14})^2) + (Y_{11}/Z)\cdot[Y_{11}^2 - Y_{12}^2 - Y_{13}^2 - Y_{14}^2 + (2Y_{12}Y_{13}Y_{14}/Y_{11})] = 0. \qquad (3)$$

The zeros of (3) in dependence on the frequency can be calculated numerically, yielding the resonance frequencies of the resonator.

Fig. 1 shows the resonance frequencies of a resonator of constant length $l + \Delta l = 10.01$ cm on Polyguide substrate material ($\epsilon_r = 2.32$) for different values of Δl. The measured resonance frequencies are somewhat smaller than the theoretical ones due to the neglected end effects of the resonators. This is true especially for the higher order resonances. Nevertheless the agreement between theory and experiment is good.

The calculated voltages and currents of the resonator show that the field distribution of the resonator is not symmetrical because of the unsymmetrical loading at the four ports. These calculations are in very good agreement with experimental results of resonators, which are excited highly symmetrical. Fig. 2(a) and (b) shows the

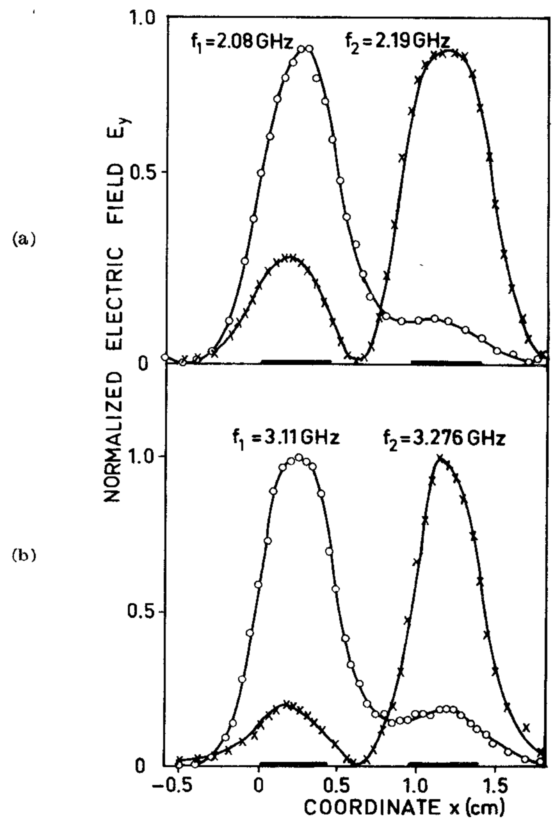

Fig. 2. Field component E_y of four resonances of the straight double-line resonator with lines of different lengths as a function of the coordinate x. Resonator dimensions as in Fig. 1, $\Delta l = 0.4$ cm.

measured field component of the electric field perpendicular to the substrate surface, measured with a field probe at different resonance frequencies. The field distribution is very unsymmetrical, though it still has the characteristics of the odd mode (zero between the conductors [3]: Fig. 2(a), $f_2 = 2.19$ GHz and Fig. 2(b), $f_2 = 3.276$ GHz) and of the even mode (no zero between the conductors: Fig. 2(a), $f_1 = 2.08$ GHz and Fig. 2 (b), $f_1 = 3.11$ GHz). Both field distributions can be interpreted as a superposition of the fields of an even and an odd mode. The results of the measurements are (neglecting the sinus dependence of the amplitudes on the z-coordinate) nearly independent of the length coordinate z of the resonator, excluding the ends of the resonators. It can be excluded that the unsymmetrical field distribution results from the influence of dissipation, for in the case of $\Delta l = 0$ a symmetrical field distribution can be measured if the resonator is excited symmetrically [3].

III. THE DOUBLE-RING RESONATOR

Gould and Talboys [1] described that they measured a splitting of the even and the odd modes of the double-ring resonator. They interpreted this splitting as the resonances of the two single rings of different lengths. The authors of this paper tried intensively to measure this splitting, but they never succeeded. From the notice given in [1] that the splitting only has been measured using field probes, it is suggested that the splitting of the resonance frequencies is due to the disturbance of the symmetry of the resonator as described in [4].

As in the case of the straight double-line resonator the field distribution of the double-ring resonator has been measured. Fig. 3 shows the measured field distributions of two adjoint resonances of a resonator 1) with large distance s between the lines and 2) with small distance s between the lines. The field distributions have been measured at $\varphi = 180°$ with the excitation at $\varphi = 0°$. The resonances are principally, as Fig. 3 shows, resonances of the single rings; the field distribution is very unsymmetrical, though the characteristics of the even and the odd modes of a symmetrical straight line resonator still can be detected (e.g., zero of the "odd mode" between the lines). Nevertheless the field distributions clearly show that no pure even or odd mode can be excited.

To find a method to calculate the resonance frequencies of the double-ring resonator approximately, three different methods have been examined. Firstly, a mean radius $r_m = (r_{mo} + r_{mi})/2$ (r_{mo} is the mean radius of the outer ring, and r_{mi} is the mean radius of the inner ring) can be defined, to calculate the resonance frequencies by assuming that an even and an odd mode are excited on the ring resonator (mean circumference $l_m = 2\pi r_m$). This leads to the resonance frequencies

$$f_1 = \frac{n \cdot c_0}{l_m(\epsilon_{effe})^{1/2}} \quad f_2 = \frac{m \cdot c_0}{l_m(\epsilon_{effo})^{1/2}}, \qquad m,n = 1,2,\cdots \tag{4}$$

where ϵ_{effe} and ϵ_{effo} are the effective dielectric constants of the even and the odd modes on a straight coupled microstrip line; c_0 is the phase velocity of light in vacuum.

As Fig. 4 shows, the so calculated resonance frequencies are in quite good agreement with experimental results, as long as the distance between the lines is small ($s \leq 0.1$ cm for a resonator on Polyguide material, $h = 0.156$ cm). For larger values of s the agreement between theory and experiment becomes worse. The theoretical resonance frequencies of the "even" and the "odd mode" become identical for large values of s, whereas the differences between the experimental resonance frequencies of the "even" and the "odd mode" become larger with growing s.

A second approximation for the resonance frequencies can be

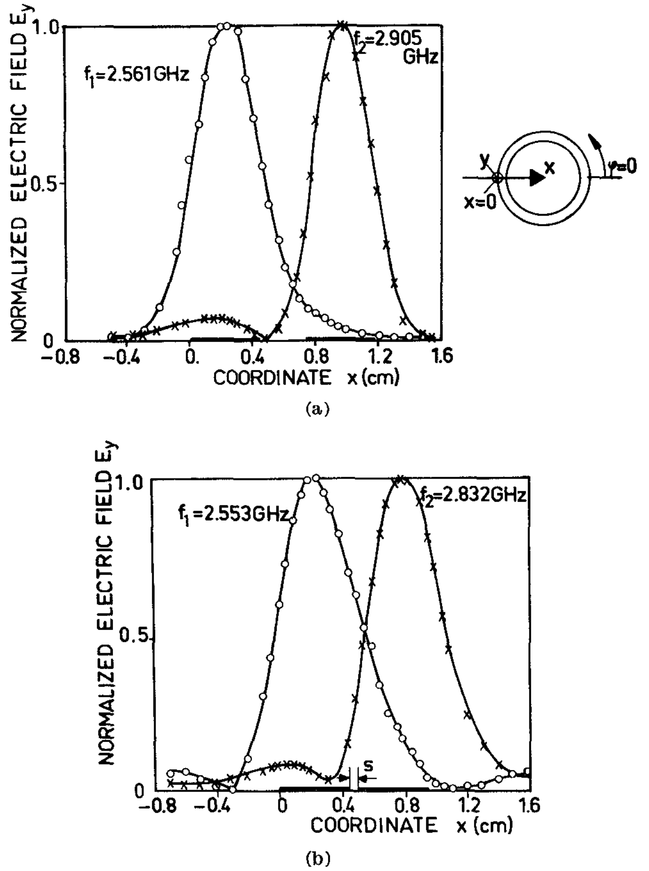

Fig. 3. Field component E_y of four resonances of the double-ring resonator. (a) Resonator dimensions: $r_4 = 6.84$ cm, $w = 0.44$ cm, $s = 0.30$ cm, $h = 0.156$ cm. Substrate material Polyguide (r_4 is the outer radius of the outer ring). (b) Resonator dimensions: $r_4 = 6.84$ cm, $w = 0.44$ cm, $s = 0.05$ cm, $h = 0.156$ cm. Substrate material Polyguide.

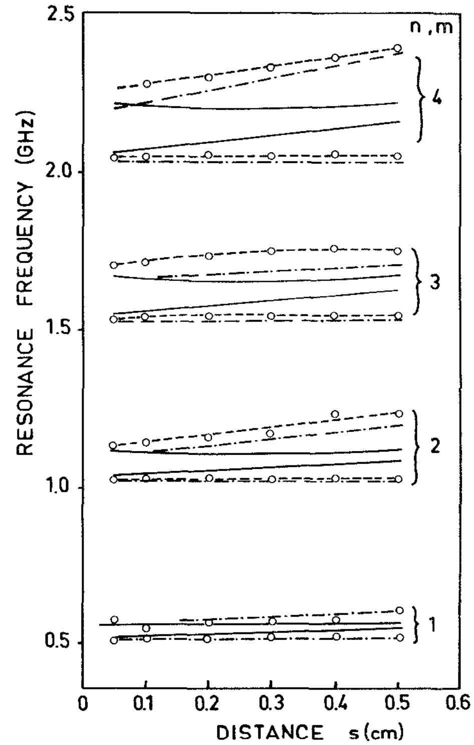

Fig. 4. Resonance frequencies of the double-ring resonator as a function of the distance s between the rings. ——— calculated by (4), —·—·— calculated by (5), —○—○— experimental results. Resonator dimensions: $r_4 = 6.84$ cm = const, $w = 0.44$ cm, $h = 0.156$ cm. Substrate material Polyguide (r_4 is the outer radius of the outer ring).

derived by assuming that the resonances are those of the single rings. This leads to the resonance frequencies

$$f_1 = \frac{n \cdot c_0}{2\pi r_{mo}(\epsilon_{eff})^{1/2}} \quad f_2 = \frac{m \cdot c_0}{2\pi r_{mi}(\epsilon_{eff})^{1/2}}, \qquad m,n = 1,2,\cdots \quad (5)$$

where ϵ_{eff} is the effective dielectric constant of the single microstrip line of the same width w as the resonator has. As the field distributions in Fig. 3 show, (5) should be at least a first approximation for the resonance frequencies, because the resonances are mainly those of the single rings. Fig. 4 shows that (5) is quite a good approximation, especially the growing difference between f_1 and f_2; with growing distance, s is described quite well. Up to the fifth higher order resonance ($n = 5$, $m = 5$), the accuracy of (5) is better than 3 percent for all resonators which have been examined (0.05 cm $\leq s \leq$ 0.5 cm, Polyguide material, $\epsilon_r = 2.32$, $h = 0.156$ cm), whereas the agreement between (4) and the experimental results is not so good (accuracy of about 5 percent for f_1 and about 9 percent for f_2, $m = 4$, $n = 4$, and $s = 0.5$ cm).

As has been shown in Section II, the unrolled double-ring resonator is the straight double-line resonator of different line lengths as shown in Fig. 1. So the $n \cdot \lambda_g$-resonance frequencies of the straight double-line resonator should be an approximation for the resonance frequencies of the double-ring resonator. Fig. 5 shows the comparison between the $n \cdot \lambda_g$-resonance frequencies of the straight double-line resonator and the measured resonance frequencies of the double-ring resonators. The agreement between theory and experiment is excellent for all values of s, as long as the mode numbers n,m are small ($n,m \leq 3$). The deviation between theory and experiment increases with the increasing value of n,m. This is due to the fact that with larger n,m (meaning with increasing resonance frequencies) the difference Δl of the circumferences of the two rings becomes of the order $\lambda_g/2$, which leads to a bad approximation of the double-ring resonator by the straight double-line resonator.

In conclusion, as far as we think, the double-ring resonator principally is not a good arrangement to measure the phase velocities of the even and the odd modes of a coupled microstrip line. Only in the case of very closely coupled lines can it be used to measure v_{phe} and v_{pho}, for in this case all three described theories are good approximations for the resonance frequencies and, e.g., (4) can be used to measure ϵ_{effe} and ϵ_{effo}. Furthermore the mean circumference of the resonator in this case should be larger than $5\lambda_g$ to avoid the influence of the curvature of the lines on the resonance frequencies (see, e.g., [5]).

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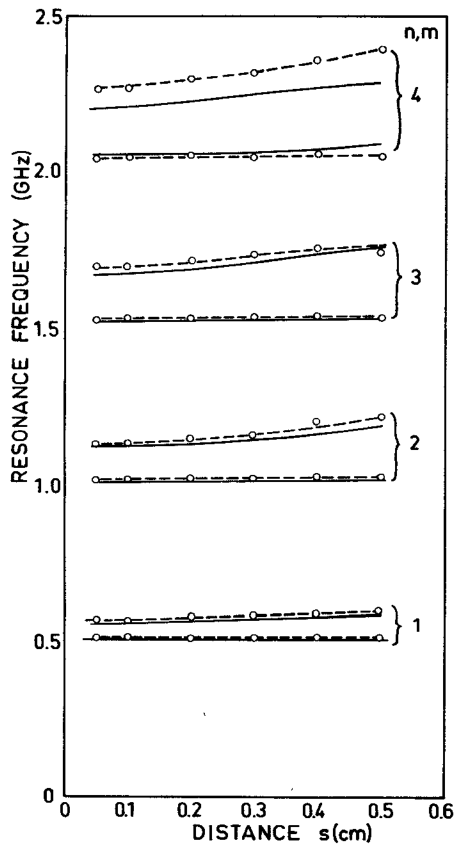

Fig. 5. Resonance frequencies of the double-ring resonator as a function of the distance s between the rings. ——— calculated by (3), ○ experimental results. Resonator dimensions as in Fig. 4.

A Coupled-Line Model for Dispersion in Parallel-Coupled Microstrips

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***Abstract*—A new circuit model is derived for parallel-coupled microstrip consisting of two separate pairs of coupled lines. Each pair consists of a homogenoeus TEM line coupled to a homogeneous TE line. One pair represents the hybrid even mode, the other represents the odd mode. Data calculated from the model are compared with experimental dispersion data for various parallel-coupled microstrip geometries. Agreement is excellent.**

The procedure for deriving the equivalent circuit is an example of a general technique for using coupled lines to model longitudinally uniform but transversely inhomogeneous lossless waveguide.

The representation of fields in longitudinally uniform but transversely inhomogeneous metallic-bound waveguides by the use of an infinite number of coupled TE and TM transmission lines was first introduced by Schelkunoff [1]. More recently, it was shown that by appropriately truncating the Schelkunoff representation, one can obtain practical models consisting of a finite number of coupled lines, from which the propagation functions of the structure can be approximated [2]. Moreover, even in cases in which the Schelkunoff parameters cannot be easily calculated, a practical coupled-line model can

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