Product Design and Capital Efficiency in Participating Life Insurance under Risk Based Solvency Frameworks

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Overview of Research Papers

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1 Research Context and Summary of Research Papers

1.1 Field of Research

This cumulative thesis contributes to the field of product design as well as profit and risk analysis for participating life insurance contracts with interest rate guarantees.

The professional discussions on life insurance during the last years have strongly been shaped by the concepts of market consistent valuation and risk based solvency frameworks (such as Solvency II in the European Union or the Swiss Solvency Test in Switzerland).\(^1\) On the one hand, these concepts have required the development of new methods, models and measures for valuing and analyzing insurance contracts and portfolios which has proved to be a challenging topic in research and practice. On the other hand, these concepts in combination with the unprecedented market environment of currently low interest rates have increased the pressure for innovative product designs in life insurance that are able to reduce the insurer’s risk effectively. And as a consequence of these changes, it appears reasonable to develop adequate new management and asset strategies for insurance portfolios. The research papers of the thesis deal with new questions that arise from this context.

While the market consistent valuation of participating life insurance business is already well addressed in literature (e.g. Sheldon and Smith [2004], Wuethrich et al. [2010]), the valuation under solvency frameworks has hardly been explored. In addition, portfolio analyses which relate profitability to solvency capital requirement are an important aspect that appears tempting to be examined. Therefore, the objective of this thesis is to approach and focus on the latter topics.

\(^1\)See for example Becker et al. [2014] for the discussion of the market consistent valuation in the actuarial context, and Eling et al. [2007] for an overview of the solvency regulation development.
While there is well-established literature that analyzes interest rate guarantees and surplus participation schemes in participating contracts,\(^2\) the systematic decomposition of the different functions that interest rates have in a contract and analyses of their impact on the required yield to be earned on the assets are new approaches in this thesis.

Furthermore, questions of policyholders’ interests in this market environment as well as optimized asset allocation and new business strategies on the background of solvency requirements lead to essentially unexplored research fields, and are addressed in this thesis.

### 1.2 Motivation and Objectives

Participating life insurance products have been an important pillar throughout the last decades to provide reliable and sustainable old-age provision in many European countries, and particularly in Germany.\(^3\) With these products, the policyholder obtains a guaranteed benefit based on prudent actuarial calculation and additionally participates in investment and other surpluses (where minimum levels are usually defined by regulation\(^4\)). The paid premiums are accumulated in a collective insurance portfolio during the lifetime of the contract providing an effective vehicle for balancing risk within the collective and over time.

However, participating life insurance has faced serious trouble within the last years: Persistently low interest rates in the capital markets make it difficult to yield the re-

\(^2\)For example, see Briys and de Varenne [1997] for a risk-neutral approach. See Barbarin and Devolder [2005] and Kling et al. [2007] for approaches including actuarial aspects in terms of imposed risk to the insurer in a real-world probability framework. See Gatzert et al. [2012] for a policyholders’ view. More detailed references are given in the papers.

\(^3\)According to statistical publications of the German Insurance Association (GDV) more than 70% of the old-age provision contracts in force by the end of 2014 in the German market were participating contracts.

\(^4\)See, for example, the "Mindestzuführungsverordnung" (MindZV) for the regulation in the German market.
turns required for the guaranteed benefits of older contracts, and as a consequence high capital reserves under risk based solvency frameworks like Solvency II are required. Strong interest rate guarantees in the contracts make the products expensive if valued market consistently, while the low level of interest rates make new business contracts appear less attractive for policyholders. Hence, more and more insurers currently question if it is reasonable to continue offering participating life insurance contracts.

There is an ongoing discussion how market consistent valuation and solvency frameworks can be adapted such that they adequately consider the special (long-term) construction of participating life insurance products. However, it is even more important to think about new and modified product designs in participating life insurance that really reduce the financial risk of the insurer and lead to a better profit and risk perspective in these frameworks.

Therefore, this thesis aims to thoroughly analyze the interest rate guarantees embedded in participating contracts and the financial risk they imply. Based on this analysis, the objective is to derive alternative product designs with modified guarantees that provide benefits for the insurer in terms of profitability, reduced risk and financial relief, but can also lead to advantages for the policyholder in terms of attractive payout profiles. To compare the effects of the different products in an insurance portfolio, comprehensive analyses of representative portfolios under profit and risk measures ought to be performed, particularly with respect to the new solvency frameworks. In

5Contracts sold in Germany in the late 1990s, for example, have a technical interest rate (used for the calculation of premiums and actuarial reserves) of 4.0%.

6Irrespective of solvency capital requirements, German life insurers have to set up the so-called ‘Zinszusatzreserve’ under local accounting rules, which is an additional (book value) reserve for contracts with a technical interest rate above a certain threshold. Note, however, that this country specific regulation is not considered in the models of this thesis.

7For example, Zurich Deutscher Herold Lebensversicherung stopped new business in participating life insurance in 2013. Generali Deutschland announced in a press release in May 2015 their target to discontinue participating life insurance and focus on offering unit-linked insurance. The Talanx group announced in July 2015 to sell new business only without guaranteed interest rate from the end of 2016 on.
order to investigate possible benefits for the policyholder, it appears reasonable to analyze possible adjustments to management rules with respect to surplus sharing and asset allocation. Finally, in order to draw conclusions for forward-looking strategies in portfolio management, not only portfolios with in-force business should be considered, but also representative portfolios that comprise new business.

Altogether, the following questions are addressed in this thesis:

1. What are the functions of interest rates and interest rate guarantees in traditional participating life insurance contracts, and how do they affect the required yield that has to be earned every year on the assets? How can products with modified guarantees be designed to reduce the risk for the insurer?

2. How can the insurer assess insurance contracts considering profitability as well as risk? What is a reasonable measure for relating both?

3. How can policyholders benefit from alternative product designs as well? How can the risk reducing potential of alternative guarantees be used to provide attractive return profiles for the policyholder?

4. What are the effects and interactions of products with alternative guarantees if brought into an existing portfolio of traditional products? What conclusions can be drawn for new business strategies with participating life insurance, and what is the development of risk exposure in the future depending on these strategies?

1.3 Summary of Research Papers and Thematic Relationship

Research Paper 1 – Participating Life Insurance Contracts under Risk Based Solvency Frameworks: How to Increase Capital Efficiency by Product Design

In the first paper we start with systematically analyzing the functions of interest rates and guarantees embedded in participating life insurance contracts. Based on
the findings of these analyses we develop product designs with modified guarantees. Furthermore, the concept of capital efficiency which relates profitability and risk is introduced. With the help of an exemplary insurance portfolio and by calculating several profit and risk measures, the different product designs are analyzed in terms of capital efficiency.

There is some literature that has analyzed interest rate guarantees in participating life insurance contracts and the financial risk for the insurer that results from them, e.g. Briys and de Varenne [1997] and several others. However, we focus on a new approach by identifying and systematically analyzing the different functions that interest rates have in a contract. We observe that in a traditional participating contract the technical interest rate has three different roles: the role as pricing rate essentially to determine the relation of premiums to guaranteed benefit, the role as reserving rate for the calculation of the actuarial reserves, and finally the role as year-to-year minimum guaranteed interest rate implying a minimum return on the contract’s account value. In a traditional participating contract, all three roles are unified in one rate, i.e. they are set at the same level. In Germany, this rate has usually been homogeneous in the market, and aligned with the maximum rate allowed for reserving (given by the "Deckungsrückstellungsverordnung" (DeckRV), §2). This design of traditional contracts with 'maximal guarantees’ – implying a required yield on the assets backing the liabilities on that same level for every year until maturity – has become a severe financial burden for insurers in the context of difficult market conditions and risk based solvency frameworks. Therefore, the paper defines product designs with alternative guarantees by detaching the roles of interest rates explained above from each other, i.e. by using three different interest rates. We mainly focus on two alternative contract designs: While both designs have the same pricing and reserving rate as the traditional product, the first alternative has a 0% year-to-year guarantee, i.e. the account value may not decrease, and the second alternative has no year-to-year guarantee. For

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8For an overview, see the literature references in the first section of the paper.
a better understanding of the resulting effects of these alternative guarantees on the annual required yield, we first analyze and present some exemplary scenarios of a single contract: In times when asset returns in the market are larger than the contract’s technical rate, a financial buffer can be built up in the case of alternative guarantees that strongly reduces the required yield for the following years. In adverse scenarios with temporarily low asset returns, this reduced required yield can help to prevent a shortfall.

Then, we compare insurance portfolios consisting of the different contract types, and analyze their profitability and financial risk for the insurer. We project the balance sheet of the portfolios in a set of stochastic risk-neutral scenarios until maturity of the contracts including typical accounting and management rules for the assets and liabilities. We assume that the surplus participation of the contracts is based on book value returns as it is in line with German regulation. For the analyses, we consider the present value of future profits as a profitability measure, as well as different risk measures like the time value of options and guarantees and solvency capital requirement\textsuperscript{9}. We also introduce the concept of capital efficiency, that relates profitability and risk, and analyze the different designs in that context. In general, we can see that products with alternative guarantees are able to reduce financial risks and increase capital efficiency. Under basic assumptions (for the asset allocation, for the level of interest rates in the market, and for the initial reserves in the portfolio) the profitability increases by about 17% compared to the traditional product. The time value of options and guarantees – indicating asymmetry of the shareholders’ cash flows – decreases by more than 90%. The alternative products also exhibit a significantly lower solvency capital requirement. It is particularly interesting to compare the impacts on profitability by either changing the type or the level of guarantee: Starting from the traditional product where the guaranteed maturity benefit is based on an interest rate

\textsuperscript{9}In this paper, we consider the solvency capital requirement for interest rate risk following the Solvency II standard formula (cf. the Delegated Regulation (EU) 2015/35 in EIOPA [2015]).
of 1.75%, changing the type of guarantee to the proposed alternatives (but not the level) has the same impact as reducing the level of guarantee (but not the type) to an interest rate of 0.9%.

Furthermore, we also perform our analyses under different sensitivities by varying the interest rate level of the capital market, asset allocations and initial reserves on the contracts. The results of the sensitivity analyses in general confirm the conclusions from the base case. Here, it is remarkable that the reduction of solvency capital requirement from the traditional to the alternative contract design is very robust throughout the base case and all sensitivities, and always decreases by slightly above one percentage point: for example, from 2.73% to 1.66% in the base case (measured in relation to the present value of future premium income from the portfolio).

In summary, the first paper answers the first two research questions.

This paper is a joint work with Andreas Reuß and Jochen Ruß, and has been published in the book Innovations in Quantitative Risk Management in the series "Springer Proceedings in Mathematics & Statistics" (Vol. 99) in January 2015.

The paper has been presented at the following conferences:

- 17th Annual Conference of the Asia-Pacific Risk and Insurance Association (APRIA, 2013) in New York City, USA,
- 48th Actuarial Research Conference (ARC, 2013) in Philadelphia, USA,
- 30th International Congress of Actuaries (ICA, 2014) in Washington D.C., USA.

Research Paper 2 – Participating Life Insurance Products with Alternative Guarantees: Reconciling Policyholders’ and Insurers’ Interests

Based on the findings of the first paper, in the second paper we look at the traditional and alternative products from both, the insurer’s and the policyholder’s view. Only providing participating products with alternative (weaker) guarantees will not
convince potential clients without any compensation for them. The idea is that alternative guarantees allow, due to their risk reducing potential, for product designs with an improved asset allocation and profit participation such that advantages for both, insurers and policyholders, can be created.

There is some literature analyzing participating life insurance contracts from the insurer’s and policyholder’s view (e.g. Gatzert et al. [2012]). The new approach of this paper is the explicit search for specific product designs with alternative guarantees that reconcile the interests of both, policyholders and insurers. Furthermore, the analyses are performed in consideration of new risk based solvency frameworks like Solvency II. We assume that the objective for the insurer is to maintain profitability and reduce solvency capital requirement, and for the policyholder to obtain an attractive risk-return-profile.

The stochastic projection model with risk-neutral scenarios is based on the first paper. However, the risk-return-profiles and the distribution of the maturity benefit for the policyholder are analyzed under a suitable real-world measure.

First, we select specific contract designs of the traditional and alternative product types by varying the asset allocation and the policyholder’s profit participation rate such that they are equally profitable for the insurer and equally fair for the policyholder under a risk-neutral measure. We call such a sample of contracts ‘iso-profit’. Under these premises, the alternative contracts generally allow for a higher participation rate and/or a larger equity ratio in the asset allocation due to the lower financial risk of their guarantees. For example, starting from a traditional contract with a participation rate of 90% and an equity ratio of 5%, for the alternative contracts either the participation rate can be increased to about 91.5% or the equity ratio can be increased to roughly 10 to 13% without changing the insurer’s profitability and the risk-neutral fair value from the policyholder’s perspective. For these base case results, we act on certain best
estimate assumptions about the capital market level and management rules stated in the paper.

In a second step, we analyze the solvency capital requirement for market risk (following the Solvency II standard formula) for selected "iso-profit" contracts. We find that products with alternative guarantees reduce the solvency capital requirement significantly, compared to the respective traditional "iso-profit" contract with the same equity ratio.

These characteristics will give the insurer some room to design alternative contracts that provide attractive risk-return-profiles for the policyholder, but still reduce the required solvency capital.

For example, as stated above, if we assume 'iso-profit' and the same asset allocation, the alternative product can come with a participation rate of about 91.5% while the traditional product has 90%; at the same time the solvency capital requirement is reduced from 3.4% to 1.65% with the alternative product. If we also allow for a higher equity ratio (and in consequence a higher expected return for the policyholder), products can be designed where the insurer’s risk lies anywhere between this reduced risk (1.65%) and the risk of the traditional product (3.4%) – still without changing the insurer’s profitability or the fair value. However, products at the two extremes of this range do not appear to be particularly attractive for either the insurer or the policyholder. An unchanged asset allocation and "maximally" reduced risk for the insurer does not provide an attractive risk-return-profile for the policyholder. An unchanged solvency risk level and higher equity ratio provides a higher expected return for the policyholder, but does not fulfill the insurer’s aim of de-risking. Therefore, we focus on product designs that are between these two cases: In the example above, if we select a product design that 'gives back' 50% of the risk reduction to the policyholder (i.e. a product design with a capital requirement of about 2.5%), the policyholder’s expected return amounts to about 2.6%, compared to the traditional product with
2.49%. Even though, due to the weaker guarantees, in adverse scenarios the return of the alternative products is below the traditional product’s return, this downside risk for the client is limited due to the guaranteed maturity benefit in all products.

To sum up, it can be concluded that well-balanced product designs with alternative guarantees are able to provide a reasonable solvency risk reduction as well as a substantial increase in expected returns, and thus an interesting risk-return-profile if a somewhat higher variance (lower payouts in adverse scenarios, higher payouts in better scenarios) is accepted.

Moreover, the paper includes sensitivity analyses assuming different asset allocations (in the initial traditional product used as starting point) as well as different levels of asset returns in the capital market. The results of the sensitivity analyses are in line with the base case.

In summary, the second paper answers the third research question listed above.

This paper is a joint work with Andreas Reuß and Jochen Ruß, and has been published in the open access risk management journal *Risks. Special Issue "Life Insurance and Pensions"* 4(2) in May 2016.

The paper has been presented at the following conferences:

- 18th International Congress on Insurance: Mathematics & Economics (IME, 2014) in Shanghai, China,
- 2nd European Actuarial Journal (EAJ) Conference (2014) in Vienna, Austria,
- World Risk and Insurance Economics Congress (WRIEC, 2015) in Munich, Germany,

Based on the product designs and findings of the first paper, the objective of the third paper is to perform profit and risk analyses of a typical insurance portfolio built up in the German market environment of the past decades to which contracts with traditional and alternative guarantees are added. It particularly investigates the effects and interactions of contracts with alternative guarantees in the light of the current low interest rate environment if the contracts are brought into a traditional portfolio and covered by the same pool of assets. Such effects and interactions cannot be observed with stand-alone analyses of the different products. Beyond that, the paper analyzes new business strategies for this typical participating portfolio and corresponding developments of the profit and risk perspective in the future.

Some papers have investigated the effects of the current low interest rate environment on representative insurance portfolios, investment decisions and surplus participation in the German market (e.g. Berdin and Gründl [2015], Hieber et al. [2015]). The main contributions of this paper however are to analyze profitability and financial risk of a representative portfolio of participating contracts with the different types of traditional and alternative guarantees introduced in the first paper, to study the interactions of these products in a single portfolio, and to examine new business strategies based on that.

For deriving the representative portfolio, we assume a constant new business volume since 1988 until the start of the projection. We model contracts with different technical interest rates according to the regulation of the past decades. We consider the changing market conditions observed in the past, and for the asset modeling we derive the coupon rates and equity returns from historical data for the German market. The stochastic projection model is based on the first paper.
In addition to the case where the insurer has sold only traditional products until the start of the projection, we also consider the hypothetical case that the insurer has sold alternative products from 1988, and some scenarios where the insurer changed the product strategy to selling alternative products at some point in time between 1988 and the start of the projection.

The results show that profitability (measured by the present value of future profits) would be significantly higher – about 80% – compared to the traditional portfolio if the insurer had sold alternative products from the beginning. Furthermore, the required yield of the traditional portfolio at the start of the projection is relatively high (about 3.5%) compared to current asset returns in the market. This creates substantial financial risk for the insurer. In the case where alternative contracts had been sold in the past, it would be close to zero. Similarly, the solvency capital requirement would be significantly lower. Hence, in line with the findings of the first paper, the results of this paper show that the alternative products would also have been more capital efficient, i.e. increased profitability and reduced capital requirement, in a setting that reflects the market environment of the past decades.

In a next step, we analyze the impacts of contracts with alternative guarantees that are brought into an existing portfolio of traditional contracts from a certain year on. We see a significant effect for all profit and risk measures, but with a different speed and magnitude. Particularly for reducing solvency capital requirement, alternative contracts have to be in the portfolio for some years before the effects become significant.

We then answer the question whether new business in the current situation is advisable, and analyze possible new business strategies. For this purpose, we first perform a present-value analysis of stochastic scenarios where several new business options are compared to the runoff case; and in a second part, we consider explicit projections of the in-force and new business portfolio under planning scenarios and analyze the resulting profit and risk situation in the future. The latter one is also a focus of the so-
called Own risk and solvency assessment (ORSA) in the risk management requirements set out in Pillar 2 of the Solvency II framework.

In the present-value analysis we can see that if the insurer considers profitability and capital requirement, new business with any product type is better than stopping new business altogether in the model, since the technical interest rates (and therefore also the guarantees) are reduced on average with new contracts. However, new business margins of alternative contracts are significantly larger than for the traditional product since the alternative guarantees of the new contracts progressively reduce the required yield of the aggregate portfolio. However, it needs to be noted that the model does not consider certain factors (like fixed costs etc.) that can affect the profitability of runoff portfolios and new business in reality.

Finally, the ORSA analysis of the profit and risk situation in the future proves to be in line with the previous results. It can be shown that alternative guarantees in the portfolio lead to a substantial movement towards more capital efficiency in the future, however the effect on solvency capital requirement needs some time to emerge. In a more adverse scenario where there is no profit for any product strategy, alternative guarantees are at least able to mitigate the insurer’s risks.

In summary, the third paper answers the last research question listed above.

This paper has been submitted to the European Actuarial Journal for review. It has been presented at the 19th International Congress on Insurance: Mathematics & Economics (IME, 2015) in Liverpool, UK.

References


2 Research Papers

The following sections provide full reproductions of the research papers submitted with this dissertation. In Section 2.1 the first paper “Participating Life Insurance Contracts under Risk Based Solvency Frameworks: How to Increase Capital Efficiency by Product Design” is presented, followed by an appendix in Section 2.2 containing further analyses to this topic which are not part of the publication of the first paper in the book “Innovations in Quantitative Risk Management”. The paper “Participating Life Insurance Products with Alternative Guarantees: Reconciling Policyholders’ and Insurers’ Interests” is printed in Section 2.3. Lastly, Section 2.4 contains the paper “Runoff or Redesign? Alternative Guarantees and New Business Strategies for Participating Life Insurance”.
2.1 Participating Life Insurance Contracts under Risk Based Solvency Frameworks: How to Increase Capital Efficiency by Product Design

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Participating Life Insurance Contracts under Risk Based Solvency Frameworks: How to Increase Capital Efficiency by Product Design

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Abstract Traditional participating life insurance contracts with year-to-year (cliquet-style) guarantees have come under pressure in the current situation of low interest rates and volatile capital markets, in particular when priced in a market consistent valuation framework. In addition, such guarantees lead to rather high capital requirements under risk-based solvency frameworks such as Solvency II or the Swiss Solvency Test (SST). We introduce several alternative product designs and analyze their impact on the insurer’s financial situation. We also introduce a measure for Capital Efficiency that considers both, profits and capital requirements, and compare the results of the innovative products to the traditional product design with respect to Capital Efficiency in a market consistent valuation model.

Keywords Capital efficiency · Participating life insurance · Embedded options · Interest rate guarantees · Market consistent valuation · Risk based capital requirements · Solvency II · SST

1 Introduction

Traditional participating life insurance products play a major role in old-age provision in Continental Europe and in many other countries. These products typically come with a guaranteed benefit at maturity, which is calculated using some guaranteed minimum interest rate. Furthermore, the policyholders receive an annual surplus participation that depends on the performance of the insurer’s assets. With the so-
called cliquet-style guarantees, once such surplus has been assigned to the policy at
the end of the year, it increases the guaranteed benefit based on the same guaranteed
minimum interest rate. This product design can create significant financial risk.

Briys and de Varenne [8] were among the first to analyze the impact of interest rate
guarantees on the insurer’s risk exposure. However, they considered a simple point-
to-point guarantee where surplus (if any) is credited at maturity only. The financial
risks of cliquet-style guarantee products have later been investigated, e.g., by Grosen
and Jorgensen [17]. They introduce the “average interest principle”, where the insurer
aims to smooth future bonus distributions by using a bonus reserve as an additional
buffer besides the policy reserve (the client’s account). Besides valuing the contract
they also calculate default probabilities (however, under the risk-neutral probability
measure $Q$). Grosen et al. [19] extend the model of Grosen and Jorgensen [17], and
introduce mortality risk. Grosen and Jorgensen [18] modify the model used by Briys
and de Varenne [8] by incorporating a regulatory constraint for the insurer’s assets
and analyzing the consequences for the insurer’s risk policy. Mittersen and Persson
[23] analyze a different cliquet-style guarantee framework with the so-called terminal
bonuses, whereas Bauer et al. [4] specifically investigate the valuation of participating
contracts under the German regulatory framework.

While all this work focuses on the risk-neutral valuation of life insurance contracts
(sometimes referred to as “financial approach”), Kling et al. [20, 21] concentrate
on the risk a contract imposes on the insurer (sometimes referred to as “actuarial
approach”) by means of shortfall probabilities under the real-world probability
measure $P$.

Barbarin and Devolder [3] introduce a methodology that allows for combining
the financial and actuarial approach. They consider a contract similar to Briys
and de Varenne [8] with a point-to-point guarantee and terminal surplus participation.
To integrate both approaches, they use a two-step method of pricing life insurance
contracts: First, they determine a guaranteed interest rate such that certain regulatory
requirements are satisfied, using value at risk and expected shortfall risk measures.
Second, to obtain fair contracts, they use risk-neutral valuation and adjust the par-
ticipation in terminal surplus accordingly. Based on this methodology, Gatzert and
Kling [14] investigate parameter combinations that yield fair contracts and analyze
the risk implied by fair contracts for various contract designs. Gatzert [13] extends
this approach by introducing the concept of “risk pricing” using the “fair value of
default” to determine contracts with the same risk exposure. Graf et al. [16] (also
building on Barbarin and Devolder [3]) derive the risk minimizing asset allocation
for fair contracts using different risk measures like the shortfall probability or the
relative expected shortfall.

Under risk-based solvency frameworks such as Solvency II or the Swiss Solvency
Test (SST), the risk analysis of interest rate guarantees becomes even more impor-
tant. Under these frameworks, capital requirement is derived from a market consistent
valuation considering the insurer’s risk. This risk is particularly high for long term
contracts with a year-to-year guarantee based on a fixed (i.e., not path dependent)
guaranteed interest rate. Measuring and analyzing the financial risk in relation to the
required capital, and analyzing new risk figures such as the Time Value of Options
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and Guarantees (TVOG) is a relatively new aspect, which gains importance with new solvability frameworks, e.g., the largest German insurance company (Allianz) announced in a press conference on June 25, 2013 the introduction of a new participating life insurance product that (among other features) fundamentally modifies the type of interest rate guarantee (similar to what we propose in the remainder of this paper). It was stressed that the TVOG is significantly reduced for the new product. Also, it was mentioned that the increase of the TVOG resulting from an interest rate shock (i.e., the solvency capital requirement for interest rate risk) is reduced by 80% when compared to the previous product. This is consistent with the findings of this paper.

The aim of this paper is a comprehensive risk analysis of different contract designs for participating life insurance products. Currently, there is an ongoing discussion, whether and how models assessing the insurer’s risk should be modified to reduce the capital requirements (e.g., by applying an “ultimate forward rate” set by the regulator). We will in contrast analyze how (for a given model) the insurer’s risk, and hence capital requirement can be influenced by product design. Since traditional cliquet-style participating life insurance products lead to very high capital requirements, we will introduce alternative contract designs with modified types of guarantees, which reduce the insurer’s risk and profit volatility, and therefore also the capital requirements under risk-based solvency frameworks. In order to compare different product designs from an insurer’s perspective, we develop and discuss the concept of Capital Efficiency, which relates profit to capital requirements. We identify the key drivers of Capital Efficiency, which are then used in our analyses to assess different product designs.

The remainder of this paper is structured as follows:

In Sect. 2, we present three considered contract designs that all come with the same level of guaranteed maturity benefit but with different types of guarantee:

- Traditional product: a traditional contract with a cliquet-style guarantee based on a guaranteed interest rate $> 0$.
- Alternative product 1: a contract with the same guaranteed maturity benefit, which is, however, valid only at maturity; additionally, there is a 0% year-to-year guarantee on the account value meaning that the account value cannot decrease from one year to the next.
- Alternative product 2: a contract with the same guaranteed maturity benefit that is, however, valid only at maturity; there is no year-to-year guarantee on the account value meaning that the account value may decrease in some years.

1 Cf. [1], particularly slide D24.

2 Of course, there already exist other well-established measures linking profit to required capital, such as the return on risk-adjusted capital (RORAC). However, they may not be suitable to assess products with long-term guarantees since they consider the required capital on a one-year basis only. To the best of our knowledge there is no common measure similar to what we define as Capital Efficiency that relates the profitability of an insurance contract to the risk it generates, and hence capital it requires over the whole contract term.
On top of the different types of guarantees, all three products include a surplus participation depending on the insurer’s return on assets. Our model is based on the surplus participation requirements given by German regulation. That means in particular that each year at least 90% of the (book value) investment return has to be distributed to the policyholders.

To illustrate the mechanics, we will first analyze the different products under different deterministic scenarios. This shows the differences in product design and how they affect the insurer’s risk.

In Sect. 3, we introduce our stochastic model, which is based on a standard financial market model: The stock return and short rate processes are modeled using a correlated Black-Scholes and Vasicek model. We then describe how the evolution of the insurance portfolio and the insurer’s balance sheet are simulated in our asset-liability-model. The considered asset allocation consists of bonds with different maturities and stocks. The model also incorporates management rules as well as typical intertemporal risk sharing mechanisms (e.g., building and dissolving unrealized gains and losses), which are an integral part of participating contracts in many countries and should therefore not be neglected.

Furthermore, we introduce a measure for Capital Efficiency based on currently discussed solvency regulations such as the Solvency II framework. We also propose a more tractable measure for an assessment of the key drivers of Capital Efficiency.

In Sect. 4, we present the numerical results. We show that the alternative products are significantly more capital efficient: financial risk, and therefore also capital requirement is significantly reduced, although in most scenarios all products provide the same maturity benefit to the policyholder. We observe that the typical “asymmetry”, i.e., particularly the heavy left tail of the insurer’s profit distribution is reduced by the modified products. This leads to a significant reduction of both, the TVOG and the solvency capital requirement for interest rate risk.

Section 5 concludes and provides an outlook for further research.

## 2 Considered Products

In this section, we describe the three different considered contract designs. Note that for the sake of simplicity, we assume that in case of death in year $t$, always only the current account value $AV_t$ (defined below) is paid at the end of year $t$. This allows us to ignore mortality for the calculation of premiums and actuarial reserves.

---

3 The correlated Black-Scholes and Vasicek model is applied in Zaglauer and Bauer [29] and Bauer et al. [5] in a similar way.

4 Note: In scenarios where the products’ maturity benefits do differ, the difference is limited since the guaranteed maturity benefit (which is the same for all three products) is a lower bound for the maturity benefit.


2.1 The Traditional Product

First, we consider a traditional participating life insurance contract with a cliquet-style guarantee. It provides a guaranteed benefit $G$ at maturity $T$ based on annual premium payments $P$. The pricing is based on a constant guaranteed interest rate $i$ and reflects annual charges $c_t$. The actuarial principle of equivalence\(^5\) yields

$$
\sum_{t=0}^{T-1} (P - c_t) \cdot (1 + i)^{T-t} = G. 
$$

(1)

During the lifetime of the contract, the insurer has to build up sufficient (prospective) actuarial reserves $AR_t$ for the guaranteed benefit based on the same constant interest rate $i$:

$$
AR_t = G \cdot \left( \frac{1}{1+i} \right)^{T-t} - \sum_{k=t}^{T-1} (P - c_k) \cdot \left( \frac{1}{1+i} \right)^{k-t}. 
$$

(2)

The development of the actuarial reserves is then given by:

$$
AR_t = (AR_{t-1} + P - c_{t-1}) \cdot (1 + i).
$$

Traditional participating life insurance contracts typically include an annual surplus participation that depends on the performance of the insurer’s assets. For example, German regulation requires that at least a “minimum participation” of $p = 90\%$ of the (local GAAP book value) earnings on the insurer’s assets has to be credited to the policyholders’ accounts. For the traditional product, any surplus assigned to a contract immediately increases the guaranteed benefit based on the same interest rate $i$. More precisely, the surplus $s_t$ is credited to a bonus reserve account $BR_t$ (where $BR_0 = 0$) and the interest rate $i$ will also apply each year on the bonus reserve:

$$
BR_t = BR_{t-1} \cdot (1 + i) + s_t.
$$

The client’s account value $AV_t$ consists of the sum of the actuarial reserve $AR_t$ and the bonus reserve $BR_t$; the maturity benefit is equal to $AV_T$.

As a consequence, each year at least the rate $i$ has to be credited to the contracts. The resulting optionality is often referred to as asymmetry: If the asset return is above $i$, a large part (e.g., $p = 90\%$) of the return is credited to the client as a surplus and the shareholders receive only a small portion (e.g., $1 - p = 10\%$) of the return. If, on the other hand, the asset returns are below $i$, then 100\% of the shortfall has to be compensated by the shareholder. Additionally, if the insurer distributes a high surplus, this increases the insurer’s future risk since the rate $i$ has to be credited also to this surplus amount in subsequent years. Such products constitute a significant

\(^5\) For the equivalence principle, see e.g., Saxer [25], Wolthuis [28].
financial risk to the insurance company, in particular in a framework of low interest rates and volatile capital markets.\(^6\)

The mechanics of this year-to-year guarantee are illustrated in Fig. 1 for two illustrative deterministic scenarios. We consider a traditional policy with term to maturity \(T = 20\) years and a guaranteed benefit of \(G = \€20,000\). Following the current situation in Germany, we let \(i = 1.75\%\) and assume a surplus participation rate of \(p = 90\%\) on the asset returns.

The first scenario is not critical for the insurer. The asset return (which is here arbitrarily assumed for illustrative purposes) starts at 3\%, then over time drops to 2\% and increases back to 3\% where the x axis shows the policy year. The chart shows this asset return, the “client’s yield” (i.e., the interest credited to the client’s account including surplus), the “required yield” (which is defined as the minimum rate that has to be credited to the client’s account), and the insurer’s yield (which is the portion of the surplus that goes to the shareholder). Obviously, in this simple example, the client’s yield always amounts to 90\% of the asset return and the insurer’s yield always amounts to 10\% of the asset return. By definition, for this contract design, the required yield is constant and always coincides with \(i = 1.75\%\).

In the second scenario, we let the asset return drop all the way down to 1\%. Whenever 90\% of the asset return would be less than the required yield, the insurer has to credit the required yield to the account value. This happens at the shareholder’s expense, i.e., the insurer’s yield is reduced and even becomes negative. This means that a shortfall occurs and the insurer has to provide additional funds.

It is worthwhile noting that in this traditional product design, the interest rate \(i\) plays three different roles:

- pricing interest rate \(i_p\) used for determining the ratio between the premium and the guaranteed maturity benefit,
- reserving interest rate \(i_r\), i.e., technical interest rate used for the calculation of the prospective actuarial reserves,
- year-to-year minimum guaranteed interest rate \(i_g\), i.e., a minimum return on the account value.

\(^6\) This was also a key result of the QIS5 final report preparing for Solvency II, cf. [2, 11].
2.2 Alternative Products

We will now introduce two alternative product designs, which are based on the idea to allow different values for the pricing rate, the reserving rate and the year-to-year minimum guaranteed interest rate on the account value. So Formulas 1 and 2 translate to the following formulae for the relation between the annual premium, the guaranteed benefit and the actuarial reserves:

\[
\sum_{t=0}^{T-1} (P - c_t) \cdot (1 + i_p)^{T-t} = G
\]

\[
AR_t = G \cdot \left( \frac{1}{1 + i_r} \right)^{T-t} - \sum_{k=t}^{T-1} (P - c_k) \cdot \left( \frac{1}{1 + i_r} \right)^{k-t}.
\]

Note, that in the first years of the contract, negative values for \(AR_t\) are possible in case of \(i_p < i_r\), which implies a “financial buffer” at the beginning of the contract. The year-to-year minimum guaranteed interest rate \(i_g\) is not relevant for the formulae above, but it is simply a restriction for the development of the client’s account, i.e.,

\[
AV_t \geq (AV_{t-1} + P - c_{t-1}) \cdot (1 + i_g).
\]

where \(AV_0 = \max \{AR_0, 0\}\) is the initial account value of the contract.

The crucial difference between such new participating products and traditional participating products is that the guaranteed maturity benefit is not explicitly increased during the lifetime of the contract (but, of course, an increase in the account value combined with the year-to-year minimum guaranteed interest rate can implicitly increase the maturity guarantee).

In this setting, the prospective reserve \(AR_t\) is only a minimum reserve for the guaranteed maturity benefit: The insurer has to make sure that the account value does not fall below this minimum reserve. This results in a “required yield” explained below. Under “normal” circumstances the account value (which is also the surrender value) exceeds the minimum reserve. Therefore, the technical reserve (under local GAAP), which may not be below the surrender value, coincides with the account value.

The required yield on the account value in year \(t\) is equal to

\[
z_t = \max \left\{ \max \{AR_t, 0\}, \frac{AV_{t-1} + P - c_{t-1}}{AV_{t-1} + P - c_{t-1}} - 1, i_g \right\}.
\]

The left part of (3) assures that the account value is nonnegative and never lower than the actuarial reserve. The required yield decreases if the bonus reserve (which is included in \(AV_{t-1}\)) increases.
The surplus participation rules remain unchanged: the policyholder’s share $p$ (e.g., 90%) of the asset return is credited to the policyholders (but not less than $z_t$). Hence, as long as the policyholder’s share is always above the technical interest rate used in the traditional product, there is no difference between the traditional and the alternative product designs.

Obviously, only combinations fulfilling $i_g \leq i_p \leq i_r$ result in suitable products: If the first inequality is violated, then the year-to-year minimum guaranteed interest rate results in a higher (implicitly) guaranteed maturity benefit than the (explicit) guarantee resulting from the pricing rate. If the second inequality is violated then at $t = 0$, additional reserves (exceeding the first premium) are required.

In what follows, we will consider two concrete alternative contract designs. Obviously, the choice of $i_g$ fundamentally changes the mechanics of the guarantee embedded in the product (or the “type” of guarantee), whereas the choice of $i_p$ changes the level of the guarantee. Since the focus of this paper is on the effect of the different guarantee mechanisms, we use a pricing rate that coincides with the technical rate of the traditional product. Hence, the guaranteed maturity benefit remains unchanged. Since the legally prescribed maximum value for the reserving rate also coincides with the technical rate of the traditional product, we get $i_p = i_r = 1.75\%$ for both considered alternative designs.

In our alternative product 1, we set $i_g = 0\%$ (0% year-to-year guarantee) and for alternative 2 we set $i_g = -100\%$ (no year-to-year guarantee). In order to illustrate the mechanics of the alternative products, Figs. 2 and 3 show the two scenarios from Fig. 1 for both alternative contract designs. In the first scenario (shown on the left), the required yield $z_t$ on the account value gradually decreases for both alternative contract designs since the bonus reserve acts as some kind of buffer (as described above). For alternative 1, the required yield can of course not fall below $i_g = 0\%$, while for the alternative 2 it even becomes negative after some years.

The adverse scenario on the right shows that the required yield rises again after years with low asset returns since the buffer is reduced. However, contrary to the traditional product, the asset return stays above the required level and no shortfall occurs.

Fig. 2 Two illustrative deterministic scenarios for alternative 1 product: asset returns and yield distribution
From a policyholder’s perspective, both alternative contract designs provide the same maturity benefit as the traditional contract design in the first scenario since the client’s yield is always above 1.75%. In the second scenario, however, the maturity benefit is slightly lower for both alternative contract designs since (part of) the buffer built up in years 1 to 8 can be used to avoid a shortfall. In this scenario, the two alternative products coincide, since the client’s yield is always positive.

Even if scenarios where the products differ appear (or are) unlikely, the modification has a significant impact on the insurer’s solvency requirements since the financial risks particularly in adverse scenarios are a key driver for the solvency capital requirement. This will be considered in a stochastic framework in the following sections.

3 Stochastic Modeling and Analyzed Key Figures

Since surplus participation is typically based on local GAAP book values (in particular in Continental Europe), we use a stochastic balance sheet and cash flow projection model for the analysis of the product designs presented in the previous section. The model includes management rules concerning asset allocation, reinvestment strategy, handling of unrealized gains and losses and surplus distribution. Since the focus of the paper is on the valuation of future profits and capital requirements we will introduce the model under a risk-neutral measure. Similar models have been used (also in a real-world framework) in Kling et al. [20, 21] and Graf et al. [16].

3.1 The Financial Market Model

We assume that the insurer’s assets are invested in coupon bonds and stocks. We treat both assets as risky assets in a risk-neutral, frictionless and continuous financial
market. Additionally, cash flows during the year are invested in a riskless bank account (until assets are reallocated). We let the short rate process \( r_t \) follow a Vasicek\(^7\) model, and the stock price \( S_t \) follow a geometric Brownian motion:

\[
\begin{align*}
    \text{dr}_t &= \kappa (\theta - r_t) \, dt + \sigma_r \, dW_t^{(1)} \\
    \frac{dS_t}{S_t} &= r_t \, dt + \rho \sigma_S \, dW_t^{(1)} + \sqrt{1 - \rho^2} \sigma_S \, dW_t^{(2)},
\end{align*}
\]

where \( W_t^{(1)} \) and \( W_t^{(2)} \) each denote a Wiener process on some probability space \( (\Omega, \mathcal{F}, \mathbb{P}, \mathbb{Q}) \) with a risk-neutral measure \( \mathbb{Q} \) and the natural filtration \( \mathcal{F}_t = \sigma \left( \left( W_s^{(1)}, W_s^{(2)} \right), s < t \right) \). The parameters \( \kappa, \theta, \sigma_r, \sigma_S \) and \( \rho \) are deterministic and constant. For the purpose of performing Monte Carlo simulations, the stochastic differential equations can be solved to

\[
\begin{align*}
    S_t &= S_{t-1} \cdot \exp \left( \int_{t-1}^t r_u \, du - \frac{\sigma_S^2}{2} t + \int_{t-1}^t \rho \sigma_S \, dW_u^{(1)} + \int_{t-1}^t \sqrt{1 - \rho^2} \sigma_S \, dW_u^{(2)} \right) \quad \text{and} \\
    r_t &= e^{-\kappa} \cdot r_{t-1} + \theta \left( 1 - e^{-\kappa} \right) + \int_{t-1}^t \sigma_r \cdot e^{-\kappa(t-u)} \, dW_u^{(1)},
\end{align*}
\]

where \( S_0 = 1 \) and the initial short rate \( r_0 \) is a deterministic parameter. Then, the bank account is given by \( B_t = \exp \left( \int_0^t r_u \, du \right) \). It can be shown that the four (stochastic) integrals in the formulae above follow a joint normal distribution.\(^8\) Monte Carlo paths are calculated using random realizations of this multidimensional distribution. The discretely compounded yield curve at time \( t \) is then given by\(^9\)

\[
\begin{align*}
    r_t(s) &= \\
    &\exp \left[ \frac{1}{s} \left( \frac{1 - e^{-\kappa s}}{\kappa} r_t + \left( s - \frac{1 - e^{-\kappa s}}{\kappa} \right) \cdot \left( \theta - \frac{\sigma_r^2}{2\kappa^2} \right) + \left( \frac{1 - e^{-\kappa s}}{\kappa} \right)^2 \frac{\sigma_r^2}{4\kappa} \right) \right] - 1
\end{align*}
\]

for any time \( t \) and term \( s > 0 \). Based on the yield curves, we calculate par yields that determine the coupon rates of the considered coupon bonds.

---

\(^7\) Cf. [27].

\(^8\) Cf. Zaglauer and Bauer [29]. A comprehensive explanation of this property is included in Bergmann [6].

\(^9\) See Seyboth [26] as well as Branger and Schlag [7].
Table 1  Balance sheet at time $t$

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$BV_t^S$</td>
<td>$X_t$</td>
</tr>
<tr>
<td>$BV_t^B$</td>
<td>$AV_t$</td>
</tr>
</tbody>
</table>

3.2 The Asset-Liability Model

The insurer’s simplified balance sheet at time $t$ is given by Table 1. Since our analysis is performed for a specific portfolio of insurance contracts on a stand-alone basis, there is no explicit allowance for shareholders’ equity or other reserves on the liability side. Rather, $X_t$ denotes the shareholders’ profit or loss in year $t$, with corresponding cash flow at the beginning of the next year. Together with $AV_t$ as defined in Sect. 2, this constitutes the liability side of our balance sheet.

In our projection of the assets and insurance contracts, incoming cash flows (premium payments at the beginning of the year, coupon payments and repayment of nominal at the end of the year) and outgoing cash flows (expenses at the beginning of the year and benefit payments at the end of the year) occur. In each simulation path, cash flows occurring at the beginning of the year are invested in a bank account. At the end of the year, the market values of the stocks and coupon bonds are derived and the asset allocation is readjusted according to a rebalancing strategy with a constant stock ratio $q$ based on market values. Conversely, $(1 - q)$ is invested in bonds and any money on the bank account is withdrawn and invested in the portfolio consisting of stocks and bonds.

If additional bonds need to be bought in the process of rebalancing, the corresponding amount is invested in coupon bonds yielding at par with term $M$. However, toward the end of the projection, when the insurance contracts’ remaining term is less than $M$ years, we invest in bonds with a term that coincides with the longest remaining term of the insurance contracts. If bonds need to be sold, they are sold proportionally to the market values of the different bonds in the existing portfolio.

With respect to accounting, we use book-value accounting rules following German GAAP, which may result in unrealized gains or losses (UGL): Coupon bonds are considered as held to maturity and their book value $BV_t^B$ is always given by their nominal amounts (irrespective if the market value is higher or lower). In contrast, for the book value of the stocks $BV_t^S$, the insurer has some discretion.

Of course, interest rate movements as well as the rebalancing will cause fluctuations with respect to the UGL of bonds. Also, the rebalancing may lead to the realization of UGL of stocks. In addition, we assume an additional management rule with respect to UGL of stocks: We assume that the insurer wants to create rather stable book value returns (and hence surplus distributions) in order to signal stability to the market. We, therefore, assume that a ratio $d_{\text{pos}}$ of the UGL of stocks is realized annually if unrealized gains exist and a ratio $d_{\text{neg}}$ of the UGL is realized annually if unrealized losses exist. In particular, $d_{\text{neg}} = 100\%$ has to be chosen in a legal framework where unrealized losses on stocks are not possible.
Based on this model, the total asset return on a book value basis can be calculated in each simulation path each year as the sum of coupon payments from bonds, interest payments on the bank account, and the realization of UGL. The split between policyholders and shareholders is driven by the minimum participation parameter \( p \) explained in Sect. 2. If the cumulative required yield on the account values of all policyholders is larger than this share, there is no surplus for the policyholders, and exactly the respective required yield \( z_t \) is credited to every account. Otherwise, surplus is credited, which amounts to the difference between the policyholders’ share of the asset return and the cumulative required yield. Following the typical practice, e.g., in Germany, we assume that this surplus is distributed among the policyholders such that all policyholders receive the same client’s yield (defined by the required yield plus surplus rate), if possible. To achieve that, we apply an algorithm that sorts the accounts by required yield, i.e., \( (z_t^{(1)}, \ldots, z_t^{(k)}) \), \( k \in \mathbb{N} \) in ascending order.

First, all contracts receive their respective required yield. Then, the available surplus is distributed: Starting with the contract(s) with the lowest required yield \( z_t^{(1)} \), the algorithm distributes the available surplus to all these contracts until the gap to the next required yield \( z_t^{(2)} \) is filled. Then, all the contracts with a required yield lower or equal to \( z_t^{(2)} \) receive an equal amount of (relative) surplus until the gap to \( z_t^{(3)} \) is filled, etc. This is continued until the entire surplus is distributed. The result is that all contracts receive the same client’s yield if this unique client’s yield exceeds the required yield of all contracts. Otherwise, there exists a threshold \( z^* \) such that all contracts with a required yield above \( z^* \) receive exactly their required yield (and no surplus) and all contracts with a required yield below \( z^* \) receive \( z^* \) (i.e., they receive some surplus).

From this, the insurer’s profit \( X_t \) results as the difference between the total asset return and the amount credited to all policyholder accounts. If the profit is negative, a shortfall has occurred, which we assume to be compensated by a corresponding capital inflow (e.g., from the insurer’s shareholders) at the beginning of the next year.\(^{10}\) Balance sheet and cash flows are projected over \( \tau \) years until all policies that are in force at time zero have matured.

### 3.3 Key Drivers for Capital Efficiency

The term Capital Efficiency is frequently used in an intuitive sense, in particular among practitioners, to describe the feasibility, profitability, capital requirement, and riskiness of products under risk-based solvency frameworks. However, to the best of our knowledge, no formal definition of this term exists. Nevertheless, it seems obvious that capital requirement alone is not a suitable figure for managing a

\(^{10}\) We do not consider the shareholders’ default put option resulting from their limited liability, which is in line with both, Solvency II valuation standards and the Market Consistent Embedded Value framework (MCEV), cf. e.g., [5] or [10], Sect. 5.3.4.
product portfolio from an insurer’s perspective. Rather, capital requirement and the resulting cost of capital should be considered in relation to profitability.

Therefore, a suitable measure of Capital Efficiency could be some ratio of profitability and capital requirement, e.g., based on the distribution of the random variable

\[
\frac{\sum_{t=1}^{\tau} X_t B_t}{\sum_{t=1}^{\tau} RC_{t-1} \cdot CoC_t B_t}
\]  

(4)

The numerator represents the present value of the insurer’s future profits, whereas the denominator is equal to the present value of future cost of capital: \( RC_t \) denotes the required capital at time \( t \) under some risk-based solvency framework, i.e., the amount of shareholders’ equity needed to support the business in force. The cost of capital is derived by applying the cost of capital rate \( CoC_t \) for year \( t \) on the required capital at the beginning of this year.\(^{11}\) In practical applications, however, the distribution of this ratio might not be easy to calculate. Therefore, moments of this distribution, a separate analysis of (moments of) the numerator and the denominator or even just an analysis of key drivers for that ratio could create some insight.

In this spirit, we will use a Monte Carlo framework to calculate the following key figures using the model described above:

A typical market consistent measure for the insurer’s profitability is the expected present value of future profits (PVFP),\(^{12}\) which corresponds to the expected value of the numerator in (4). The PVFP is estimated based on Monte Carlo simulations:

\[
PVFP = \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{\tau} \frac{X_t^{(n)}}{B_t^{(n)}} = \frac{1}{N} \sum_{n=1}^{N} PVFP^{(n)},
\]

where \( N \) is the number of scenarios, \( X_t^{(n)} \) denotes the insurer’s profit/loss in year \( t \) in scenario \( n \), \( B_t^{(n)} \) is the value of the bank account after \( t \) years in scenario \( n \), and hence \( PVFP^{(n)} \) is the present value of future profits in scenario \( n \).

In addition, the degree of asymmetry of the shareholder’s cash flows can be characterized by the distribution of \( PVFP^{(n)} \) over all scenarios\(^{13}\) and by the time value of options and guarantees (TVOG). Under the MCEV framework,\(^{14}\) the latter is defined by

\[
TVOG = PVFP_{CE} - PVFP
\]

\(^{11}\) This approach is similar to the calculation of the cost of residual nonhedgeable risk as introduced in the MCEV Principles in [9], although \( RC_t \) reflects the total capital requirement including hedgeable risks.

\(^{12}\) The concept of PVFP is introduced as part of the MCEV Principles in [9].

\(^{13}\) Note that this is a distribution under the risk-neutral measure and has to be interpreted carefully. However, it can be useful for explaining differences between products regarding PVFP and TVOG.

\(^{14}\) Cf. [9].
Table 2 Product parameters I

<table>
<thead>
<tr>
<th></th>
<th>Traditional Product (%)</th>
<th>Alternative 1 (%)</th>
<th>Alternative 2 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_p, i_r$</td>
<td>1.75</td>
<td>1.75</td>
<td>1.75</td>
</tr>
<tr>
<td>$i_g$</td>
<td>1.75</td>
<td>0</td>
<td>-100</td>
</tr>
</tbody>
</table>

where $PVFP_{CE} = \sum_{t=1}^{\tau} \frac{X_t^{(CE)}}{B_t^{(CE)}}$ is the present value of future profits in the so-called “certainty equivalent” (CE) scenario. This deterministic scenario reflects the expected development of the capital market under the risk-neutral measure. It can be derived from the initial yield curve $r_0(s)$ based on the assumption that all assets earn the forward rate implied by the initial yield curve.\(^{15}\) The TVOG is also used as an indicator for capital requirement under risk-based solvency frameworks.

Comparing the PVFP for two different interest rate levels—one that we call basic level and a significantly lower one that we call stress level—provides another important key figure for interest rate risk and capital requirements. In the standard formula\(^{16}\) of the Solvency II framework

$$\Delta PVFP = PVFP(\text{basic}) - PVFP(\text{stress})$$

determines the contribution of the respective product to the solvency capital requirement for interest rate risk (SCR\(_{int}\)). Therefore, we also focus on this figure which primarily drives the denominator in (4).

4 Results

4.1 Assumptions

The stochastic valuation model described in the previous section is applied to a portfolio of participating contracts. For simplicity, we assume that all policyholders are 40 years old at inception of the contract and mortality is based on the German standard mortality table (DAV 2008 T). We do not consider surrender. Furthermore, we assume annual charges $c_t$ that are typical in the German market consisting of annual administration charges $\beta \cdot P$ throughout the contract’s lifetime, and acquisition charges $\alpha \cdot T \cdot P$, which are equally distributed over the first 5 years of the contract. Hence, $c_t = \beta \cdot P + \alpha \frac{T \cdot P}{5} \cdot 1_{t \in [0,\ldots,4]}$. Furthermore, we assume that expenses coincide with the charges. Product parameters are given in Tables 2 and 3.

Stochastic projections are performed for a portfolio that was built up in the past 20 years (i.e., before $t = 0$) based on 1,000 new policies per year. Hence, we have a

\(^{15}\) Cf. Oechslin et al. [24].

\(^{16}\) A description of the current version of the standard formula can be found in [12].
Table 3  Product parameters II

<table>
<thead>
<tr>
<th>$G(\text{€})$</th>
<th>$T$ (years)</th>
<th>$P(\text{€})$</th>
<th>$\beta$ (%)</th>
<th>$\alpha$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20,000</td>
<td>20</td>
<td>896.89</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

portfolio at the beginning of the projections with remaining time to maturity between 1 year and 19 years (i.e., $\tau = 19$ years).\(^{17}\) For each contract, the account value at $t = 0$ is derived from a projection in a deterministic scenario. In this deterministic scenario, we use a flat yield curve of 3.0\% (consistent with the mean reversion parameter $\theta$ of the stochastic model after $t = 0$), and parameters for management rules described below. In line with the valuation approach under Solvency II and MCEV, we do not consider new business.

The book value of the asset portfolio at $t = 0$ coincides with the book value of liabilities. We assume a stock ratio of $q = 5$\% with unrealized gains on stocks at $t = 0$ equal to 10\% of the book value of stocks. The coupon bond portfolio consists of bonds with a uniform coupon of 3.0\% where the time to maturity is equally split between 1 year and $M = 10$ years.

Capital market parameters for the basic and stress projections are shown in Table 4. The parameters $\kappa$, $\sigma_r$, $\sigma_S$ and $\rho$ are directly adopted from Graf et al. [16]. The parameters $\theta$ and $r_0$ are chosen such that they are more in line with the current low interest rate level. The capital market stress corresponds to an immediate drop of interest rates by 100 basis points.

The parameters for the management rules are given in Table 5 and are consistent with current regulation and practice in the German insurance market.

For all projections, the number of scenarios is $N = 5,000$. Further analyses showed that this allows for a sufficiently precise estimation of the relevant figures.\(^{18}\)

Table 4  Capital market parameters

<table>
<thead>
<tr>
<th></th>
<th>$r_0$ (%)</th>
<th>$\theta$ (%)</th>
<th>$\kappa$ (%)</th>
<th>$\sigma_r$ (%)</th>
<th>$\sigma_S$ (%)</th>
<th>$\rho$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic</td>
<td>2.5</td>
<td>3.0</td>
<td>30.0</td>
<td>2.0</td>
<td>20.0</td>
<td>15.0</td>
</tr>
<tr>
<td>Stress</td>
<td>1.5</td>
<td>2.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^{17}\) Note that due to mortality before $t = 0$, the number of contracts for the different remaining times to maturity is not the same.

\(^{18}\) In order to reduce variance in the sample an antithetic path selection of the random numbers is applied, cf. e.g., Glasserman [15].
### Table 5  Parameters for management rules

<table>
<thead>
<tr>
<th>q (%)</th>
<th>M (years)</th>
<th>$d_{pos}$ (%)</th>
<th>$d_{neg}$ (%)</th>
<th>p (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>10</td>
<td>20</td>
<td>100</td>
<td>90</td>
</tr>
</tbody>
</table>

### 4.2 Comparison of Product Designs

In Table 6, the PVFP and the TVOG for the base case are compared for the three products. All results are displayed as a percentage of the present value of future premium income from the portfolio. For alternative 1, the PVFP increases from 3.63 to 4.24 %, i.e., by 0.61 percentage points (pp), compared to the traditional contract design (which corresponds to a 17 % increase of profitability). This means that this product with a “maturity only” guarantee and an additional guarantee that the account value will not decrease is, as expected, more profitable than the product with a traditional year-to-year (cliquet-style) guarantee. This difference is mainly caused by the different degree of asymmetry of the shareholders’ cash flows which is characterized by the TVOG. Since PVFP$_{CE}$ amounts to 4.26 % for all products in the base case, the difference of TVOG between the traditional product and alternative 1 is also 0.61 pp. This corresponds to a TVOG reduction of more than 90 % for alternative 1, which shows that the risk resulting from the interest rate guarantee is much lower for the modified product.

Compared to this, the differences between alternative 1 and alternative 2 are almost negligible. The additional increase of the PVFP is only 0.01 pp, which is due to a slightly lower TVOG compared to alternative 1. This shows that the fact that the account value may decrease in some years in alternative 2 does not provide a material additional risk reduction.

Additional insights can be obtained by analyzing the distribution of PVFP$_{(n)}$ (see Fig. 4)$^{19}$: For the traditional contract design, the distribution is highly asymmetric with a strong left tail and a significant risk of negative shareholder cash flows (on a present value basis). In contrast, both alternative contract designs exhibit an almost symmetric distribution of shareholder cash flows which explains the low TVOG. Hence, the new products result in a significantly more stable profit perspective for the shareholders, while for the traditional product the shareholder is exposed to significantly higher shortfall risk.

Ultimately, the results described above can be traced back to differences in the required yield. While for the traditional product, by definition, the required yield always amounts to 1.75 %, it is equal to 0 % in most scenarios for the alternative 1 product. Only in the most adverse scenarios, the required yield rises toward 1.75 %.$^{20}$ For the alternative 2 product, it is even frequently negative.

---

$^{19}$ Cf. Footnote 13.

$^{20}$ Note that here, the required yield in the first projection year reflects the financial buffer available for the considered portfolio of existing contracts at $t = 0$. This is different from the illustrations in Sect. 2, which consider individual contracts from inception to maturity.
Apart from the higher profitability, the alternative contract designs also result in a lower capital requirement for interest rate risk. This is illustrated in Table 7, which displays the PVFP under the interest rate stress and the difference to the basic level. Compared to the basic level, the PVFP for the traditional product decreases by 75%, which corresponds to an SCR\(_{int}\) of 2.73% of the present value of future premium income. In contrast, the PVFP decreases by only around 40% for the alternative contract designs and thus the capital requirement is only 1.66 and 1.65%, respectively.

We have seen that a change in the type of guarantee results in a significant increase of the PVFP. Further analyses show that a traditional product with guaranteed interest rate \(i = 0.9\%\) instead of 1.75% would have the same PVFP (i.e., 4.25%) as the alternative contract designs with \(i_p = 1.75\%\). Hence, although changing only the type of guarantee and leaving the level of guarantee intact might be perceived as a rather small product modification by the policyholder, it has the same effect on the insurer’s profitability as reducing the level of guarantee by a significant amount.

Furthermore, our results indicate that even in an adverse capital market situation the alternative product designs may still provide an acceptable level of profitability: The profitability of the modified products if interest rates were 50 basis points lower roughly coincides with the profitability of the traditional product in the base case.
Table 7  PVFP for stress level and PVFP difference between basic and stress level

<table>
<thead>
<tr>
<th></th>
<th>Traditional product (%)</th>
<th>Alternative 1 (%)</th>
<th>Alternative 2 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PVFP(basic)</td>
<td>3.63</td>
<td>4.24</td>
<td>4.25</td>
</tr>
<tr>
<td>PVFP(stress)</td>
<td>0.90</td>
<td>2.58</td>
<td>2.60</td>
</tr>
<tr>
<td>ΔPVFP</td>
<td>2.73</td>
<td>1.66</td>
<td>1.65</td>
</tr>
</tbody>
</table>

4.3 Sensitivity Analyses

In order to assess the robustness of the results presented in the previous section, we investigate three different sensitivities:

1. **Interest rate sensitivity:** The long-term average $\theta$ and initial rate $r_0$ in Table 4 are replaced by $\theta = 2.0\%$, $r_0 = 1.5\%$ for the basic level, and $\theta = 1.0\%$, $r_0 = 0.5\%$ for the stress level.
2. **Stock ratio sensitivity:** The stock ratio is set to $q = 10\%$ instead of $5\%$.
3. **Initial buffer sensitivity:** The initial bonus reserve $BR_t = AV_t - AR_t$ is doubled for all contracts.

The results are given in Table 8.

**Interest rate sensitivity** If the assumed basic interest rate level is lowered by 100 basis points, the PVFP decreases and the TVOG increases significantly for all products. In particular, the alternative contract designs now also exhibit a significant TVOG. This shows that in an adverse capital market situation, also the guarantees embedded in the alternative contract designs can lead to a significant risk for the shareholder and an asymmetric distribution of profits as illustrated in Fig. 5. Nevertheless, the alternative contract designs are still much more profitable and less volatile than the traditional contract design and the changes in PVFP/TVOG are much less pronounced than for the traditional product: while the TVOG rises from 0.63 to 2.13\%, i.e., by 1.50 pp for the traditional product, it rises by only 0.76 pp (from 0.02 to 0.78\%) for alternative 1.

As expected, an additional interest rate stress now results in a larger SCR$_{int}$. For all product designs, the PVFP after stress is negative and the capital requirement increases significantly. However, as in the base case (cf. Table 7), the SCR$_{int}$ for the traditional product is more than one percentage point larger than for the new products.

**Stock ratio sensitivity** The stock ratio sensitivity also leads to a decrease of PVFP and an increase of TVOG for all products. Again, the effect on the PVFP of the traditional product is much stronger: The profit is about cut in half (from 3.63 to 1.80\%), while for the alternative 1 product the reduction is much smaller (from 4.24 to 3.83\%), and even smaller for alternative 2 (from 4.25 to 3.99\%). It is noteworthy that with a larger stock ratio of $q = 10\%$ the difference between the two alternative

21 The initial book and market values of the assets are increased proportionally to cover this additional reserve.
Table 8 PVFP, TVOG, PVFP under interest rate stress and ΔPVFP for base case and all sensitivities

<table>
<thead>
<tr>
<th>Base case</th>
<th>Traditional product (%)</th>
<th>Alternative 1 (%)</th>
<th>Alternative 2 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PVFP</td>
<td>3.63</td>
<td>4.24</td>
<td>4.25</td>
</tr>
<tr>
<td>TVOG</td>
<td>0.63</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>PVFP(stress)</td>
<td>0.90</td>
<td>2.58</td>
<td>2.60</td>
</tr>
<tr>
<td>ΔPVFP</td>
<td>2.73</td>
<td>1.66</td>
<td>1.65</td>
</tr>
<tr>
<td><strong>Interest rate sensitivity</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PVFP</td>
<td>0.90</td>
<td>2.58</td>
<td>2.60</td>
</tr>
<tr>
<td>TVOG</td>
<td>2.13</td>
<td>0.78</td>
<td>0.76</td>
</tr>
<tr>
<td>PVFP(stress)</td>
<td>−4.66</td>
<td>−1.81</td>
<td>−1.76</td>
</tr>
<tr>
<td>ΔPVFP</td>
<td>5.56</td>
<td>4.39</td>
<td>4.36</td>
</tr>
<tr>
<td><strong>Stock ratio sensitivity</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PVFP</td>
<td>1.80</td>
<td>3.83</td>
<td>3.99</td>
</tr>
<tr>
<td>TVOG</td>
<td>2.45</td>
<td>0.43</td>
<td>0.26</td>
</tr>
<tr>
<td>PVFP(stress)</td>
<td>−1.43</td>
<td>1.65</td>
<td>1.92</td>
</tr>
<tr>
<td>ΔPVFP</td>
<td>3.23</td>
<td>2.18</td>
<td>2.07</td>
</tr>
<tr>
<td><strong>Initial buffer sensitivity</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PVFP</td>
<td>3.74</td>
<td>4.39</td>
<td>4.39</td>
</tr>
<tr>
<td>TVOG</td>
<td>0.64</td>
<td>&lt;0.01</td>
<td>&lt;0.01</td>
</tr>
<tr>
<td>PVFP(stress)</td>
<td>1.02</td>
<td>2.87</td>
<td>2.91</td>
</tr>
<tr>
<td>ΔPVFP</td>
<td>2.72</td>
<td>1.52</td>
<td>1.48</td>
</tr>
</tbody>
</table>

products becomes more pronounced, which is reflected by the differences of the TVOG. Alternative 2 has a lower shortfall risk than alternative 1 since the account value may decrease in some years as long as the account value does not fall below the minimum reserve for the maturity guarantee. Hence, we can conclude that the guarantee that the account value may not decrease becomes more risky if asset returns exhibit a higher volatility.

The results for the stressed PVFPs under the stock ratio sensitivity are in line with these results: First, the traditional product requires even more solvency capital: The SC_{int} is half a percentage point larger than in the base case (3.23 % compared to 2.73 %), and it is also more than one percentage point larger than for the alternative products with 10% stocks (2.18/2.07 %). Second, the interest rate stress shows a more substantial difference between the two different alternative products. While the difference of the SC_{int} between alternative 1 and 2 was 0.01 % in the base case, it is now 0.11 %.

**Initial buffer sensitivity** If the initial buffer is increased, we observe a slight increase of the PVFP for all products. However, there are remarkable differences for the effect on TVOG between the traditional and the alternative products: While for the traditional product the TVOG remains approximately the same, for the alternative products it is essentially reduced to zero. This strongly supports our product
motivation in Sect. 2: For the alternative products, larger surpluses from previous years reduce risk in future years. Furthermore, the stressed PVFPs imply that the decrease of capital requirement is significantly larger for the alternative products: 0.14 % reduction (from 1.66 to 1.52 %) for alternative 1 and 0.17 % reduction (from 1.65 to 1.48 %) for alternative 2, compared to just 0.01 % reduction for the traditional product.

4.4 Reduction in the Level of Guarantee

So far we have only considered contracts with a different type of guarantee. We will now analyze contracts with a lower level of guarantee, i.e., products where \( i_p < i_r \). If we apply a pricing rate of \( i_p = 1.25 \% \) instead of 1.75 \%, the annual premium required to achieve the same guaranteed maturity benefit rises by approx. 5.4 \%, which results in an additional initial buffer for this contract design. For the sake of comparison, we also calculate the results for the traditional product with a lower guaranteed interest rate \( i = 1.25 \% \). The respective portfolios at \( t = 0 \) are derived using the assumptions described in Sect. 4.1.

The results are presented in Table 9. We can see that the PVFP is further increased and the TVOG is very close to 0 for the modified alternative products, which implies an almost symmetric distribution of the PVFP. The TVOG can even become slightly negative due to the additional buffer in all scenarios. Although the risk situation for the traditional product is also improved significantly due to the lower guarantee, the

\[22\] From this, we can conclude that if such alternative products had been sold in the past, the risk situation of the life insurance industry would be significantly better today in spite of the rather high nominal maturity guarantees for products sold in the past.
alternative products can still preserve their advantages. A more remarkable effect can be seen for the \( \text{SCR}_\text{int} \), which amounts to 1.03 and 0.99\% for the alternative products 1 and 2, respectively, compared to 1.69\% for the traditional product. Hence, the buffer leads to a significant additional reduction of solvency capital requirements for the alternative products meaning that these are less affected by interest rate risk.

### 5 Conclusion and Outlook

In this paper, we have analyzed different product designs for traditional participating life insurance contracts with a guaranteed maturity benefit. A particular focus of our analysis was on the impact of product design on capital requirements under risk-based solvency frameworks such as Solvency II and on the insurer’s profitability.

We have performed a market consistent valuation of the different products and have analyzed the key drivers of Capital Efficiency, particularly the value of the embedded options and guarantees and the insurer’s profitability.

As expected, our results confirm that products with a typical year-to-year guarantee are rather risky for the insurer, and hence result in a rather high capital requirement. Our proposed product modifications significantly enhance Capital Efficiency, reduce the insurer’s risk, and increase profitability. Although the design of the modified products makes sure that the policyholder receives less than with the traditional product only in extreme scenarios, these products still provide a massive relief for the insurer since extreme scenarios drive the capital requirements under Solvency II and SST.

It is particularly noteworthy that starting from a standard product where the guaranteed maturity benefit is based on an interest rate of 1.75\%, changing the type of the guarantee to our modified products (but leaving the level of guarantee intact) has the same impact on profitability as reducing the level of guarantee to an interest rate of 0.9\% and not modifying the type of guarantee. Furthermore, it is remarkable that the reduction of \( \text{SCR}_\text{int} \) from the traditional to the alternative contract design is very robust throughout our base case as well as all sensitivities and always amounts to slightly above one percentage point.
We would like to stress that the product design approach presented in this paper is not model arbitrage (hiding risks in “places the model cannot see”), but a real reduction of economic risks. In our opinion, such concepts can be highly relevant in practice if modified products keep the product features that are perceived and desired by the policyholder, preserve the benefits of intertemporal risk sharing, and do away with those options and guarantees of which policyholders often do not even know they exist. Similar modifications are also possible for many other old age provision products like dynamic hybrid products\textsuperscript{23} or annuity payout products. Therefore, we expect that the importance of “risk management by product design” will increase. This is particularly the case since—whenever the same pool of assets is used to back new and old products—new capital efficient products might even help reduce the risk resulting from an “old” book of business by reducing the required yield of the pool of assets.

We, therefore, feel that there is room for additional research: It would be interesting to analyze similar product modifications for the annuity payout phase. Also—since many insurers have sold the traditional product in the past—an analysis of a change in new business strategy might be worthwhile: How would an insurer’s risk and profitability change and how would the modified products interact with the existing business if the insurer has an existing (traditional) book of business in place and starts selling modified products today?

Another interesting question is how the insurer’s optimal strategic asset allocation changes if modified products are being sold: If typical criteria for determining an optimal asset allocation are given (e.g., maximizing profitability under the restriction that some shortfall probability or expected shortfall is not exceeded), then the c.p. lower risk of the modified products might allow for a more risky asset allocation, and hence also higher expected profitability for the insurer and higher expected surplus for the policyholder. So, if this dimension is also considered, the policyholder would be compensated for the fact that he receives a weaker type of guarantee.

Finally, our analysis so far has disregarded the demand side. If some insurers keep selling the traditional product type, there should be little demand for the alternative product designs with reduced guarantees unless they provide some additional benefits. Therefore, the insurer might share the reduced cost of capital with the policyholder, also resulting in higher expected benefits in the alternative product designs.

Since traditional participating life insurance products play a major role in old-age provision in many countries and since these products have come under strong pressure in the current interest environment and under risk-based solvency frameworks, the concept of Capital Efficiency and the analysis of different product designs should be of high significance for insurers, researchers, and regulators to identify sustainable life insurance products. In particular, we would hope that legislators and regulators would embrace sustainable product designs where the insurer’s risk is significantly reduced, but key product features as perceived and requested by policyholders are still present.

\textsuperscript{23} Cf. Kochanski and Karmarski [22].
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References

2.2 Appendix with further analysis to the first paper

In this part, we additionally analyze the percentile plots for the required yields in the base case and in the different sensitivities from sections 4.2 and 4.3 of the first paper “Participating Life Insurance Contracts under Risk Based Solvency Frameworks: How to Increase Capital Efficiency by Product Design”.

The results for alternative 1 are shown in Figures 1 to 4, and for alternative 2 in Figures 5 to 8. In the base case and for all sensitivities, the required yield for alternative 1 product starts at a level of 0% due to surplus distribution before $t = 0$. In the base case (cf. Figure 1), the required yield increases only for the most adverse scenarios such that only for a small share of scenarios the required yield rises to above 1.0% at any time. In the interest rate sensitivity (cf. Figure 2), this ratio is a lot higher, and also the average required yield significantly exceeds 0%. Similarly, for a larger stock ratio (cf. Figure 3), the probability for a higher required yield increases, however to a lesser extent than in the interest rate sensitivity, and it is slightly reduced towards the end of the projection horizon. If the initial buffer is doubled (cf. Figure 4), the increase of the required yield in adverse scenarios is essentially delayed by few years (until buffers are used up).

Figure 1: Percentile plots of the required yield in the base case for alternative 1 product.
Figure 2: Percentile plots of the required yield in the interest rate sensitivity for alternative 1 product.

Figure 3: Percentile plots of the required yield in the stock ratio sensitivity for alternative 1 product.

Figure 4: Percentile plots of the required yield in the initial buffer sensitivity for alternative 1 product.
For the alternative 2 the required yield starts below zero at about \(-4.0\%\) in the base case (cf. Figure 5) which means that the insurer could – in case of a negative asset return – avoid a shortfall by crediting a negative surplus for the next year. Over the analyzed time horizon, the range of possible values of the required yield becomes gradually larger: the 99th percentile increases to 1.75\% (the maximum possible due to contract design) and the 1st percentile decreases to below \(-20\%\). The median and the mean stay close together indicating a rather symmetric distribution of the required yield and decrease gradually to about \(-7.5\%\). For the interest rate sensitivity (cf. Figure 6), the required yield starts at the same level, but then strongly increases in most scenarios. In the last years of the projection the required yield is at 1.75\% for more than 25\% of the scenarios. Hence, the interest rate stress is particularly dangerous in the long run after financial buffers have been absorbed by systematically low returns. For the higher stock ratio (cf. Figure 7), the 1 to 99\% percentile range is larger than in the base case: between \(-25\%\) and 1.75\% in the last year, i.e. the higher volatility of stocks is reflected in a wider distribution of the required yield. With the initial bonus reserve doubled (cf. Figure 8), the required yield at the beginning is as low as \(-9.3\%\). Then it increases slightly as the additional buffer is consumed gradually in cases of an adverse capital market. At the end of the time horizon, the percentiles are almost on the same level as in the base case. This results from the very low bonus reserve at \(t = 0\) of those policies which have been in the portfolio for only a few years when the projection starts. Of course, these are the only policies remaining in the last years of the projection. For those policies doubling the initial bonus reserve has no material effect, and when adverse scenarios materialize close to the end of the time horizon, the policies with the larger additional reserves have already left.
Figure 5: Percentile plots of the required yield in the base case for alternative 2 product.

Figure 6: Percentile plots of the required yield in the interest rate sensitivity for alternative 2 product.

Figure 7: Percentile plots of the required yield in the stock ratio sensitivity for alternative 2 product.
Figure 8: Percentile plots of the required yield in the initial buffer sensitivity for alternative 2 product.
2.3 Participating Life Insurance Products with Alternative Guarantees: Reconciling Policyholders’ and Insurers’ Interests

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Participating Life Insurance Products with Alternative Guarantees: Reconciling Policyholders’ and Insurers’ Interests

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Abstract: Traditional participating life insurance contracts with year-to-year (cliquet-style) guarantees have come under pressure in the current situation of low interest rates and volatile capital markets, in particular when priced in a market-consistent valuation framework. In addition, such guarantees lead to rather high capital requirements under risk-based solvency frameworks such as Solvency II or the Swiss Solvency Test (SST). Therefore, insurers in several countries have developed new forms of participating products with alternative (typically weaker and/or lower) guarantees that are less risky for the insurer. In a previous paper, it has been shown that such alternative product designs can lead to higher capital efficiency, i.e., higher and more stable profits and reduced capital requirements. As a result, the financial risk for the insurer is significantly reduced while the main guarantee features perceived and requested by the policyholder are preserved. Based on these findings, this paper now combines the insurer’s and the policyholder’s perspective by analyzing product versions that compensate policyholders for the less valuable guarantees. We particularly identify combinations of asset allocation and profit participation rate for the different product designs that lead to an identical expected profit for the insurer (and identical risk-neutral value for the policyholder), but differ with respect to the insurer’s risk and solvency capital requirements as well as with respect to the real-world return distribution for the policyholder. We show that alternative products can be designed in a way that the insurer’s expected profitability remains unchanged, the insurer’s risk and hence capital requirement is substantially reduced and the policyholder’s expected return is increased. This illustrates that such products might be able to reconcile insurers’ and policyholders’ interests and serve as an alternative to the rather risky cliquet-style products.

Keywords: participating life insurance; interest rate guarantees; capital efficiency; asset allocation; profit participation rate; policyholder’s expected return; solvency capital requirements; Solvency II; SST; market-consistent valuation

1. Introduction

Traditional participating (i.e., non-linked) life insurance products have come under significant pressure in the current environment with low interest rates and capital requirements based on risk-based solvency frameworks such as Solvency II or the Swiss Solvency Test (SST). This is due to the fact that these products usually come with very long-term and year-by-year (cliquet-style) guarantees which make them rather risky (and hence capital intensive) from an insurer’s perspective. For this reason, participating products that come with alternative forms of guarantees have been developed in several countries and are currently discussed intensively.

Different aspects like the financial risk and the fair valuation of interest rate guarantees in participating life insurance products have been analyzed, e.g., in Briys and de Varenne [1], Grosen...
Some authors analyze participating life insurance contracts also from a policyholder’s perspective. Bohnert and Gatzert [14] examine the impact of three typical surplus distribution schemes on the insurer’s shortfall risk and the policyholder’s net present value. They conclude that, even though the amount of surplus is always calculated the same way, the surplus distribution scheme has a substantial impact. Gatzert et al. [15] compare the different perspectives of policyholders and insurers concerning the value of a contract. They identify contracts that maximize customer value under certain risk preferences while keeping the contract value for the insurer fixed.

Finally, Reuß et al. [13] introduce participating products with alternative forms of guarantees. They analyze the impact of alternative guarantees on the capital requirement under risk-based solvency frameworks and introduce the concept of Capital Efficiency which relates profits to capital requirements.

Introducing such alternative guarantees primarily attempts to reduce the insurer’s risk. Typically, this would ceteris paribus make such contracts less attractive from a policyholder’s perspective. Since in the current market environment some insurers find it rather difficult to continue offering contracts with guarantees at all (and some have already stopped new business in participating contracts or switched to products with a lower protection level),1 products with somewhat weaker forms of guarantees might be a way to at least continue offering some products that are attractive to risk-averse policyholders seeking guarantees. In this paper, we therefore analyze how participating products can be modified in terms of surplus participation and asset allocation with the objective of balancing the interests of policyholders and the insurer. We particularly take into account that policyholders may demand some kind of “compensation” for the modified guarantees since these may lead to lower benefits than the traditional product in certain adverse scenarios.

The remainder of this paper provides a possible approach for this objective. In Section 2, we present the three considered contract designs from Reuß et al. [13] that all come with the same level of guaranteed maturity benefit but with different types of guarantee. As a reference, we consider a traditional contract with a cliquet-style guarantee based on a guaranteed interest rate >0%. The first alternative product has the same guaranteed maturity benefit which is, however, valid only at maturity; additionally, there is a 0% year-to-year guarantee on the account value, meaning that the account value cannot decrease from one year to the next. The second alternative product finally only has the (same) guaranteed maturity benefit. But in this product there is no year-to-year guarantee on the account value at all, meaning that the account value may decrease in some years. On top of the different types of guarantees, all three products include a surplus participation depending on the insurer’s realized investment return.

In Section 3, we introduce our stochastic model for the stock market return and the short-rate process. We then describe how the evolution of the insurance portfolio and the insurer’s balance sheet are projected in our asset-liability model. The considered asset allocation consists of bonds with different maturities and an equity investment. The model also incorporates management rules as well as typical intertemporal risk-sharing mechanisms (e.g., building and dissolving unrealized gains and losses), which are an integral part of participating contracts in many countries and should therefore not be neglected.

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1 For example, Zurich Deutscher Herold Lebensversicherung stopped new business in participating life insurance in 2013, and now offers so-called “select products” (cf. Alexandrova et al. [16]). Generali Deutschland announced in a press release in May 2015 their target to discontinue participating life insurance and focus on offering unit-linked insurance. The Talanx group announced in July 2015 to sell new business from the end of 2016 on with only a return-of-premium guarantee (instead of a guaranteed interest rate).
In Section 4, we present the results of our analyses. First, for all considered product types, we determine combinations of asset allocation and surplus participation rate that all come with the same expected profit from the insurer’s perspective (“iso-profit” products). Hence, from a policyholder’s view, the risk-neutral values of all these contracts are also identical, i.e., all these products would be considered equally fair in a fair value framework. Then, we have a closer look at those iso-profit product designs with alternative guarantees that come with the same asset allocation as the traditional product. We find that for the alternative products, the insurer’s risk measured by the Solvency II capital requirement is significantly reduced. Unfortunately, they might appear less attractive from the policyholder’s perspective. Therefore, we also consider iso-profit products with alternative guarantees that come with a higher equity ratio. For this set of product designs, the insurer’s risk can lie anywhere between the reduced risk and the risk of the traditional product. We then analyze the real-world risk-return distribution of products from the policyholder’s perspective and find that products can be designed which only slightly increase the policyholder’s risk (although they significantly reduce the insurer’s capital requirement) and significantly increase the policyholder’s real-world expected return (although the insurer’s risk-neutral expected profit remains unchanged). We therefore conclude that carefully designed participating products with modified guarantees might be suitable for reconciling the insurer’s and policyholders’ interests.

Section 5 concludes and provides an outlook on further research.

2. Considered Products

The three product designs that will be analyzed are the same as in Reuß et al. [13]. We therefore only briefly describe the most important product features and refer to that paper for more details.

All three considered products provide a guaranteed benefit $G$ at maturity $T$ based on regular (annual) premium payments $P$. The prospective actuarial reserves for the guaranteed benefit that the insurer has to set up at time $t$ is given by $\text{AR}_t$. Furthermore, $\text{AV}_t$ denotes the client’s account value at time $t$ consisting of the sum of the actuarial reserve $\text{AR}_t$ and any surplus (explained below) that has already been credited to the policyholder. At maturity, $\text{AV}_T$ is paid out as maturity benefit.

We do not assume one single “technical interest rate”, but rather define three different interest rates: a pricing interest rate $i_p$ that determines the ratio between the annual premium $P$ and the guaranteed maturity benefit $G$, a reserving interest rate $i_r$, that is used for the calculation of the actuarial reserve $\text{AR}_t$ and a year-to-year guaranteed interest rate $i_g$, which corresponds to the minimum return that the client has to receive each year on the account value $\text{AV}_t$.

With annual charges $c_t$, the actuarial principle of equivalence\textsuperscript{5} yields
\[
\sum_{t=0}^{T-1} (P - c_t) \cdot (1 + i_p)^{T-t} = G
\] (1)

Based on the reserving rate $i_r$, the actuarial reserve $\text{AR}_t$ at time $t$ is given by
\[
\text{AR}_t = G \cdot \left( \frac{1}{1 + i_r} \right)^{T-t} - \sum_{k=1}^{T-1} (P - c_k) \cdot \left( \frac{1}{1 + i_r} \right)^{k-t}
\] (2)

\textsuperscript{2} Note that these products have different surplus participation rates due to the aforementioned construction of the iso-profit products.

\textsuperscript{3} For these types of products—different than, for example, for US-style variable annuities—typically no hedging strategies for the guarantees are in place. Under current regulation, all policyholders (who have contracts with different levels of guarantee that started at different points in time and will mature at different points in time) participate in the return of the same pool of assets. Hence, hedge assets could not be attributed to certain guarantees that mature at a certain time, which limits the potential for micro-hedging. Therefore, we do not consider any hedging strategies in our paper.

\textsuperscript{4} Reuß et al. [13] pointed out some restrictions on the choice of the three interest rates: “only combinations fulfilling $i_g \leq i_p \leq i_r$ result in suitable products: If the first inequality is violated, then the year-to-year minimum guaranteed interest rate results in a higher (implicitly) guaranteed maturity benefit than the (explicit) guarantee resulting from the pricing rate. If the second inequality is violated then at $t = 0$, additional reserves (exceeding the first premium) are required.”

\textsuperscript{5} The equivalence principle is explained, for example, in Saxer [17] and Wolthuis [18].
As in Reuß et al. [13], we assume that in case of death or surrender in year \( t \), the current account value \( AV_t \) is paid at the end of year \( t \). Therefore, mortality rates are not relevant in the formulae above.

Annual surplus is typically credited to such policies according to country-specific regulation. In Germany, at least \( p = 90\% \) of the (local GAAP book value) investment income on the insurer’s assets (but not less than \( z_t \) defined below) has to be credited to the policyholders’ accounts.

In previous years, in many countries so-called cliquet-style guarantees prevailed, where all three interest rates introduced above coincide and this single rate is referred to as guaranteed rate or technical rate. In such products, typically, any surplus credited to the contract leads to an increase of the guaranteed maturity benefit and this increase is also calculated based on the same technical rate. Our more general setting includes this product as the special case where \( i_p = i_t = i_g \).

Note that the regulatory requirements regarding reserving and minimum surplus participation limit the potential for diversification over time.\(^7\)

The “required yield” on the account value in year \( t \) is given by

\[
z_t = \max \left\{ \frac{\max \{ AR_t, 0 \}}{(AV_{t-1} + P - c_{t-1})} - 1, i_g \right\}
\]

This definition makes sure that the account value remains non-negative, never falls below the actuarial reserve, and earns at least the year-by-year guaranteed interest rate. With \( s_t \) denoting the annual surplus, the account value evolves according to

\[
AV_t = (AV_{t-1} + P - c_{t-1}) \cdot (1 + z_t) + s_t.
\]

If the pricing rate exceeds the year-by-year guaranteed interest rate, the required yield decreases if surplus (which is included in \( AV_{t-1} \)) has been credited in previous years. Hence, for such products (contrary to the traditional product), distributing surplus to the client decreases the insurer’s risk in future years.

We consider three concrete product designs that all come with the same level of the maturity guarantee (based on a pricing rate of 1.75%) but with a different type of guarantee.\(^8\)

- Traditional, cliquet-style product: \( i_g = i_p = i_t = 1.75\% \)
- Alternative 1 product with a 0% year-by-year guarantee: \( i_p = i_t = 1.75\%, i_g = 0\% \)
- Alternative 2 product without any year-by-year guarantee: \( i_p = i_t = 1.75\%, i_g = -100\% \)

Although all three products come with the same guaranteed maturity benefit, they come with a different risk for the policyholder in the sense that for the alternative products it is more likely that the actual maturity benefit will be at or close to the guaranteed value. To illustrate this, consider the simplified example of a contract with two years term to maturity and a maturity guarantee based on an interest rate of 1.75%. Now, assume that the asset return to be credited to the policyholder’s account (\( p = 90\% \) of the book value return as described above) would be 6% in year one and 0% in year two. In the traditional design, the policyholder would receive 6% in year one and still the year-by-year guaranteed rate of 1.75% in year two. In the alternative designs, the policyholder would receive 6%.

---

\(^6\) Since for all product designs the account value (and hence the surrender value) never falls below the prospective reserve for the guaranteed maturity benefit, this is, in our opinion, consistent with guaranteed minimum surrender benefits specified by German insurance contract law (§169 “Versicherungsvertragsgesetz” (VVG)).

\(^7\) Without these regulatory requirements, the insurer might make even better use of time diversification of asset returns (see, for example, the results on optimal pension insurance and time diversification in the life cycle model in Aase [19]). However, the alternative products proposed in this paper are designed to allow a higher degree of time diversification than the traditional product. The Alternative 2 product even comes with the maximum degree of time diversification possible under existing regulation for participating contracts.

\(^8\) Note that 1.75% is the maximum reserving rate allowed in Germany until 31 December 2014. On 1 January 2015, it has been lowered to 1.25%. In order to make our results comparable to the results in Reuß et al. [13], we still use a reserving rate of 1.75%.
in year one. The required yield would then drop to zero and the policyholder would receive 0% in year two.

In our numerical analyses, we assume that all policyholders are 40 years old at inception of the respective contract, that the considered pool of policies decreases due to mortality, which is based on the German standard mortality table (DAV 2008 T), and that no surrender occurs. Furthermore, we assume annual administration charges \( \beta \cdot P \) throughout the contract’s lifetime, and acquisition charges \( \alpha \cdot T \cdot P \) which are equally distributed over the first five years of the contract. Hence, \( c_t = \beta \cdot P + \alpha \cdot \frac{T \cdot P}{5} \cdot 1_{t \in [0, \ldots, 4]} \). Furthermore, we assume that expenses coincide with the charges. The product parameters are given in Table 1.

### Table 1. Product parameters.

<table>
<thead>
<tr>
<th>G</th>
<th>T</th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( P )</th>
</tr>
</thead>
<tbody>
<tr>
<td>20,000 €</td>
<td>20 years</td>
<td>4%</td>
<td>3%</td>
<td>896.89 €</td>
</tr>
</tbody>
</table>

### 3. Stochastic Modeling and Assumptions

The framework for the financial market model and for the projection of the insurer’s balance sheet and cash flows, including management rules and surplus distribution (which is based on local GAAP book values), is taken from Reuß et al. [13]. We therefore keep the following subsections brief and refer to that paper for more details.

#### 3.1. The Financial Market Model

We assume that assets are invested in coupon bonds and equity. Cash flows arising between annual re-allocation dates are invested in a riskless bank account. Since we will perform analyses in both a risk-neutral and a real-world framework, we specify dynamics under both measures. We let the short rate process \( r_t \) follow a Vasicek\(^9\) model, and the equity index \( S_t \) follow a geometric Brownian motion and get the following risk-neutral dynamics:

\[
\begin{align*}
    dr_t &= \kappa (\theta - r_t) \, dt + \sigma_r \, dW_t^{(1)} \\
    \frac{dS_t}{S_t} &= r_t \, dt + \rho \sigma_S \, dW_t^{(1)} + \sqrt{1 - \rho^2} \sigma_S \, dW_t^{(2)}
\end{align*}
\]

where \( W_t^{(1)} \) and \( W_t^{(2)} \) are independent Wiener processes on some probability space \((\Omega, \mathcal{F}, \mathbb{P}, \mathbb{Q})\) with a risk-neutral measure \( \mathbb{Q} \) and the natural filtration \( \mathcal{F} = \mathcal{F}_t = \sigma \left( \left( W_s^{(1)}, W_s^{(2)} \right), s < t \right) \). Assuming a constant market price of interest rate risk \( \lambda \), the corresponding real-world dynamics are given by:

\[
\begin{align*}
    dr_t &= \kappa (\theta^* - r_t) \, dt + \sigma_r \, dW_t^{*(1)} \\
    \frac{dS_t}{S_t} &= \mu dt + \rho \sigma_S \, dW_t^{*(1)} + \sqrt{1 - \rho^2} \sigma_S \, dW_t^{*(2)}
\end{align*}
\]

where \( \theta^* = \theta + \lambda \sigma_r^2 \), \( \mu \) includes an equity risk premium and \( W_t^{*(1)}, W_t^{*(2)} \) are independent Wiener processes under a real-world measure \( \mathbb{P} \).

\(^9\) cf. Vasicek [20].
The parameters $\theta$, $\kappa$, $\sigma_r$, $\lambda$, $\mu$, $\sigma_S$, and $\rho$ are deterministic and constant. For the purpose of performing Monte Carlo simulations, the above equations can be solved to

$$S_t = S_{t-1} \exp \left( \int_{t-1}^t r_u du - \frac{\sigma_S^2}{2} + \int_{t-1}^t \rho \sigma_S dW_u^{(1)} + \int_{t-1}^t \sqrt{1 - \rho^2} \sigma_S dW_u^{(2)} \right)$$

and

$$r_t = e^{-\kappa} r_{t-1} + \theta (1 - e^{-\kappa}) + \int_{t-1}^t \sigma_r e^{-\kappa(t-u)} dW_u^{(1)}$$

in the risk-neutral case. It can be shown that the four integrals in the formulae above follow a joint normal distribution. Monte Carlo paths are calculated using random realizations of this multidimensional distribution. Similarly, we obtain

$$S_t = S_{t-1} \exp \left( \mu - \frac{\sigma_S^2}{2} + \int_{t-1}^t \rho \sigma_S dW_u^{(1)} + \int_{t-1}^t \sqrt{1 - \rho^2} \sigma_S dW_u^{(2)} \right)$$

and

$$r_t = e^{-\kappa} r_{t-1} + \theta^* \left(1 - e^{-\kappa}\right) + \int_{t-1}^t \sigma_r e^{-\kappa(t-u)} dW_u^{(1)}$$

for the real-world approach. In both settings, the initial value of the equity index $S_0 = 1$ and the initial short rate $r_0$ are deterministic parameters.

The bank account is given by $B_t = \exp \left( \int_{t_0}^t r_u du \right)$ and the discretely compounded yield curve at time $t$ by

$$r_1(s) = \exp \left[ \frac{1}{s} \left( \frac{1 - e^{-\kappa s}}{\kappa} r_t + \left( s - \frac{1 - e^{-\kappa s}}{\kappa} \right) \left( \theta - \frac{\sigma^2_r}{2\kappa^2} \right) + \left( \frac{1 - e^{-\kappa s}}{\kappa} \right) ^2 \left( \frac{\sigma^2_S}{4\kappa} \right) \right) \right] - 1$$

for any time $t$ and term $s > 0$. Based on the yield curve, we can calculate the par-yield that determines the coupon rate of the considered coupon bond.

In our numerical analyses, we use market parameters shown in Table 2. The parameters $\kappa$, $\sigma_r$, $\lambda$, $\sigma_S$, and $\rho$ are directly adopted from Graf et al. [12]. The choice of the parameters $r_0$, $\theta$, and $\mu$ reflects lower interest rate and equity risk premium levels.

<table>
<thead>
<tr>
<th>$r_0$</th>
<th>$\theta$</th>
<th>$\kappa$</th>
<th>$\sigma_r$</th>
<th>$\lambda$</th>
<th>$\mu$</th>
<th>$\sigma_S$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5%</td>
<td>3.0%</td>
<td>30.0%</td>
<td>2.0%</td>
<td>-23.0%</td>
<td>6.0%</td>
<td>20.0%</td>
<td>15.0%</td>
</tr>
</tbody>
</table>

### 3.2. The Asset-Liability Model

The insurer’s simplified balance sheet at time $t$ is given by Table 3 (the rather simple structure is justified in Reuß et al. [13]).

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$BV_t^\gamma$</td>
<td>$X_t$</td>
</tr>
<tr>
<td>$BV_t^\delta$</td>
<td>$AV_t$</td>
</tr>
</tbody>
</table>

---

10 cf. Zaglauer and Bauer [21]. A comprehensive explanation of this property is included in Bergmann [22].
11 See Seyboth [23] as well as Branger and Schlag [24].
The liability side of the balance sheet consists of the account value $AV_t$ (defined in Section 2) and the shareholders’ profit or loss $X_t$ in year $t$.\(^\text{12}\)

On the asset side, we have the book value of bonds $BV_t^B$, which coincides with the nominal amount under German GAAP since we assume that bonds are considered as held to maturity. For the book value of the equity investment $BV_t^S$, the insurer has more discretion. We assume that the insurer wants to create rather stable book value returns (and hence surplus distributions) in order to signal stability to the market. Therefore, a ratio $d_{pos}$ of the unrealized gains or losses (UGL) of equity is realized annually if $UGL > 0$ (i.e., in case of unrealized gains) and a ratio $d_{neg}$ of the UGL is realized annually if $UGL < 0$. In particular, $d_{neg} = 100\%$ has to be chosen in a legal framework, where unrealized losses on equity investments are not possible.

At the end of the year, the following rebalancing is implemented: market values of all assets (including the bank account) are derived and a constant ratio $q$ is invested in equity. The remainder is invested in bonds.

For new bond investments, coupon bonds yielding at par with a given term $M$ are used until all insurance contracts’ remaining terms are less than $M$ years. Then, we invest in bonds with a term that coincides with the longest remaining insurance contracts. If bonds need to be sold, they are sold proportionally to the market values of the different bonds in the existing portfolio.

Finally, the book value return on assets is calculated for each year in each simulation path as the sum of coupon payments from bonds, interest payments on the bank account, and the realization of UGL. The split between policyholders and shareholders is driven by the participation rate $p$, introduced in Section 2. If the policyholders’ share is not sufficient to pay the required yields to all policyholders, there is then no surplus for the policyholders, and all policies receive exactly the respective required yield $z_j$. Otherwise, surplus is credited which amounts to the difference between the policyholders’ share of the asset return and the cumulative required yield. Following the typical practice, as, e.g., in Germany, we assume that this surplus is distributed among the policyholders such that all policyholders receive the same client’s yield (defined by the required yield plus surplus rate), if possible.\(^\text{13}\)

The insurer’s profit/loss $X_t$ results as the difference between the total investment income and the amount credited to all policyholder accounts. We assume that $X_t$ leads to a corresponding cash flow to or from the shareholders at the beginning of the next year; that means, particularly, that a loss is compensated by the insurer’s shareholders.\(^\text{14}\)

In our numerical analyses we let $M = 10$ years, $d_{pos} = 20\%$, and $d_{neg} = 100\%$.

3.3. The Projection Setup

We use a deterministic projection for the past (i.e., until $t = 0$) to build up a portfolio of policies for the analysis. This portfolio consists of 1000 policies that had been sold each year in the past 20 years. Hence, at $t = 0$, we have a portfolio with remaining times to maturity between one year and 19 years.\(^\text{15}\)

Therefore the time horizon for the stochastic projection starting at $t = 0$ amounts to $\tau = 19$ years.

\(^{12}\) As in Reuß et al. [13] we perform our analyses for the insurance portfolio on a stand-alone basis, and therefore do not explicitly consider the shareholders’ equity or other reserves on the liability side. This is due to the fact that the valuation of liabilities from insurance contracts is typically independent of the amount of shareholders’ equity held by the insurance company. Consequently, our framework measures the contribution of a specific portfolio of insurance contracts to the own funds of the insurance company in a risk-based solvency framework. Of course, shareholder’s equity is another important component of the own funds (but does not depend on the product design).

\(^{13}\) The distribution algorithm is explained in more detail in Reuß et al. [13].

\(^{14}\) As stated in Reuß et al. ([13], p. 196): “We do not consider the shareholders’ default put option resulting from their limited liability, which is in line with both, Solvency II valuation standards and the Market Consistent Embedded Value framework (MCEV), cf. e.g., Bauer et al. [25] or DAV [26], Section 5.3.4 (p. 30ff).”

\(^{15}\) cf. Reuß et al. ([13], p. 199): “Note that due to mortality before $t = 0$, the number of contracts for the different remaining times to maturity is not the same.”
In the deterministic projection before $t = 0$, we use a flat yield curve of 3.0% (consistent with the mean reversion parameter $\theta$ of the stochastic model after $t = 0$), and management rules described above.

Then, starting at $t = 0$, stochastic projections are performed for this portfolio. In line with the valuation approach under Solvency II and MCEV, we do not consider new business after $t = 0$ for the calculation of the insurer’s profitability and risk. We do, however, consider new business when calculating the risk-return characteristics from the policyholder’s perspective.

We assume that the book value of the asset portfolio at $t = 0$ coincides with the book value of liabilities. The initial amount of UGL is derived from a base case projection of the traditional product with an equity ratio of $q = 5\%$ and a participation rate of $p = 90\%$. This value is used as the initial UGL (before solvency stresses or sensitivities) for the projections of all products. The coupon bond portfolio at $t = 0$ consists of bonds with a uniform coupon of 3.0%, where the time to maturity is equally split between one year and $M = 10$ years.

For all projections, the number of scenarios is $N = 5,000$. Further analyses showed that this allows for stable results.

4. Results

In Reuß et al. [13], it was shown that a modification of the products reduces the insurer’s risk and increases the insurer’s profitability. Obviously, such products are less attractive for the policyholder since they pay lower benefits at least in some possible scenarios. Therefore, it is currently intensively discussed among practitioners how policyholders can be compensated for this fact and whether the resulting products are then still attractive for the insurer. In this section, we show how products can be designed that “give back” some or all of the increased profitability and the reduced risk to the policyholder.

4.1. Analysis of the Insurer’s Profit

In a first step, we consider alternative products that achieve the same profitability (“iso-profit”) as the traditional product by using suitable combinations of the equity ratio $q$ and the profit participation rate $p$.

To measure the insurer’s profitability in a market-consistent framework we use the expected present value of future profits (PVFP) under the risk-neutral measure $Q$.

The Monte Carlo estimate for the PVFP is calculated by

$$ PVFP = \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{\tau} \frac{X_t^{(n)}}{B_t^{(n)}} = \frac{1}{N} \sum_{n=1}^{N} PVFP^{(n)} $$

where $N$ is the number of scenarios, $X_t^{(n)}$ denotes the insurer’s profit/loss in year $t$ in scenario $n$, $B_t^{(n)}$ is the value of the bank account after $t$ years in scenario $n$, and hence $PVFP^{(n)}$ is the present value of future profits in scenario $n$.

The PVFP for the traditional product in our base case scenario is given by 3.62% (as a percentage of the present value of future premium income) using $p = 90\%$ (which is the minimum profit participation

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16 As in Reuß et al. [13], we apply an antithetic path selection of the random numbers in order to reduce variance in the sample, cf. e.g., Glasserman [27].

17 For instance, the German life insurer Allianz has introduced a product with alternative guarantees in the German market that compensates the policyholder for lower and weaker guarantees by an increase in surplus distribution. Also, several insurers have introduced products that are similar to our Alternative 1 product with $i_e = 0\%$, $i_p = x\%$, and $i_r = 1.75\%$, where $x$ is chosen such that the guaranteed benefit coincides with the sum of all premiums paid. In these products, as a compensation for the lower and weaker guarantee, the policyholders may choose annually to invest their surplus distribution in some equity option generating an annual return on the policy that depends on some equity index, cf. Alexandrova et al. [16].

18 The concept of PVFP is introduced as part of the MCEV Principles in the CFO-Forum [28].
rate required under German regulation) and an equity ratio of $q = 5\%$. Figure 1 shows combinations of $p$ and $q$ that lead to the same PVFP for all three considered products.

![Iso-profit curves under risk-neutral measure Q [PVFP=3.62%]](image)

**Figure 1.** Iso-profit curves with $PVFP = 3.62\%$ (based on the traditional product with $p = 90\%$ and $q = 5\%$).

Obviously, for all products, the insurer’s risk resulting from an increased equity ratio has to be compensated by a reduced participation rate in order to keep the PVFP unchanged. Only for very low equity ratios (below $0.5\%$), we observe the opposite effect. This is caused by the missing diversification effects between bonds and stocks which leads to an increase in risk if the equity ratio is further reduced.

We find that for a given participation rate, the alternative products allow for a significantly higher equity ratio if the insurer wants to keep the PVFP unchanged. As expected, the effect is stronger for the Alternative 2 product. Note that the difference between Alternative 1 and 2 is negligible for equity ratios below $7\%$ since for low equity ratios the probability for a year with negative client’s yield in the Alternative 2 product (which is the only situation where the two alternative products differ) is very low.

If the insurer intends to keep the participation rate at the legally required minimum of $90\%$, the equity ratio could be increased from $5\%$ to roughly $10.75\%$ for the Alternative 1 product and to $12.75\%$ for the Alternative 2 product. This would increase the policyholder’s expected return without affecting the insurer’s expected profit.

Conversely, if the equity ratio remains unchanged at $5\%$, the participation rate could be increased to $91.48\%$, with unchanged PVFP.

Note that, in our setting, products with identical PVFP from the insurer’s perspective automatically have the same fair value (in a risk-neutral framework) from the policyholder’s perspective. Therefore, if a fair value approach is taken to analyze products from the policyholder’s perspective, all “iso-profit” products are equally fair. However, following, e.g., Graf et al. [29], we will also analyze the products’ risk-return characteristics in a real-world framework in Section 4.3.

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19 In our numerical analyses, we always vary the equity ratio $q$ in steps of $0.25\%$ and then calculate the PVFP for a given profit participation rate $p$ or the profit participation rate $p$ for a given PVFP. Therefore, in what follows, equity ratios are always given as multiples of $0.25\%$. 

4.2. Analysis of the Insurer’s Risk

Of course, the products from the previous subsection, which all come with the same expected profit from the insurer’s perspective, differ in terms of risk for the insurer. Therefore, we now analyze the insurer’s risk resulting from these “iso-profit” products. We use the insurer’s Solvency Capital Requirement for market risk ($SCR_{mkt}$) as a measure for risk and consider only interest rate and equity risk as part of the market risk. To determine $SCR_{mkt}$, we calculate the PVFP under an interest rate stress of 100 bps ($PVFP_{int}$), i.e., using $r_0 = 1.5\%$ and $\theta = 2.0\%$. Furthermore, we calculate the PVFP under a stress of the initial market value of equities ($PVFP_{eq}$), using a reduction of 39% according to the Solvency II standard formula. Then, the Solvency Capital Requirement for interest rate risk is determined by $SCR_{int} = (PVFP - PVFP_{int})$ and the Solvency Capital Requirement for equity risk is determined by $SCR_{eq} = (PVFP - PVFP_{eq})$. According to the standard formula of the Solvency II framework, the aggregated SCR for market risk is then calculated by

$$SCR_{mkt} = \sqrt{(SCR_{int})^2 + (SCR_{eq})^2 + 2 \cdot \rho_m \cdot SCR_{int} \cdot SCR_{eq}}$$

(16)

with a correlation of $\rho_m = 0.5$ between the interest rate and equity risk.

Figure 2 shows SCR$_{mkt}$ for the iso-profit products from Figure 1. Note that the x-axis here only shows the equity ratio $q$. As explained in the previous subsection, this value $q$ also defines the value of $p$ for the corresponding iso-profit products. Therefore, for a given value of $q$, the corresponding value of $p$ is different for the three products. We can observe that all three products require more solvency capital with an increasing equity ratio.

![SCR(mkt)-curves of iso-profit products (under Q)](image)

**Figure 2.** SCR$_{mkt}$ curves of the iso-profit products.

If we intend to design products with the same profitability and the same equity ratio, the alternative products will significantly reduce the insurer’s risk. For instance, if we keep the expected profit unchanged at 3.62% and we keep the equity ratio unchanged at 5%, the risk (measured by $SCR_{mkt}$) is reduced from 3.41% (for the traditional product) to 1.66% or 1.64% for Alternatives 1 or

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20 Note that other market risk modules, such as property risk and spread risk, are not relevant in our simplified asset-liability model. However, the analysis could be extended using more complex asset models.

21 A description of the version of the standard formula that has been applied during the preparatory phase of Solvency II can be found in EIOPA [30].
2, respectively. Note that in this case, the alternative products come with a higher participation rate (91.48%) than the traditional product (as explained in the previous subsection and Figure 1). We would like to stress again that these alternative products—when compared to the traditional product—come with the same profitability for the insurer and hence have the same fair value (and the same maturity guarantee) from the policyholder’s perspective. Still, they significantly reduce the insurer’s capital requirement. Of course, products without guarantees at all or with a lower guaranteed benefit at maturity could further reduce the insurer’s risk (measured by $\text{SCR}_{\text{mkt}}$). However, such products would not be attractive to the (large) share of risk-averse policyholders that typically seek a certain level of protection. Therefore, we do not consider such products in this paper.

We can also see from Figure 2 that the alternative products allow for a significantly higher equity ratio if we consider products with identical profitability and identical risk.

4.3. Analysis of Policyholder’s Risk-Return Profiles

Now, we compare the different product designs from a policyholder’s perspective using risk-return profiles, cf. Graf et al. [29]. For this, we perform projections under the real-world measure $\mathbb{P}$ (including annual new business of 1000 new policies per year) and analyze the policyholder’s risk and return on the policies taken out in the first year.

Following typical practice in the German market, the policyholders’ return is measured by the internal rate of return (IRR) and the policyholders’ risk is measured by the conditional tail expectation of the return on the lowest 20% of scenarios ($\text{CTE}_{20}$).

As a reference point, we again use the traditional product with $q = 5\%$ and $p = 90\%$. From an insurer’s perspective, this product has an expected profit of 3.62% and a Solvency Capital Requirement for market risk of about 3.4%.

As in the previous subsection, we first consider alternative products with the same insurer’s profitability and the same asset allocation (which significantly reduce the insurer’s risk, as stated before). The risk-return characteristics from the policyholder’s perspective are shown in Table 4 (in row “Same asset allocation”). While the expected return barely changes (2.49% for the traditional and 2.47/2.48% for the two alternative products), the $\text{CTE}_{20}$ is reduced from 1.96% to 1.83%. So, from the policyholder’s perspective, such alternative products appear less attractive than the traditional product: they come with a slightly lower return and a moderately increased risk.

Table 4. IRR and CTE20 for products with the same asset allocation, with the same $\text{SCR}_{\text{mkt}}$, and with reduced $\text{SCR}_{\text{mkt}}$ for alternative products, based on the traditional product with $q = 5\%$ ($\text{PVFP} = 3.62\%$).

<table>
<thead>
<tr>
<th>IRR/CTE20</th>
<th>Traditional Product</th>
<th>Alternative 1</th>
<th>Alternative 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Same asset allocation</td>
<td>2.49%/1.96%</td>
<td>2.47%/1.83%</td>
<td>2.48%/1.83%</td>
</tr>
<tr>
<td>Same risk level ($\text{SCR}_{\text{mkt}} = 3.4%$)</td>
<td>2.65%/1.87%</td>
<td>2.75%/1.85%</td>
<td></td>
</tr>
<tr>
<td>Reduced risk for alternatives 1/2 ($\text{SCR}_{\text{mkt}} = 2.5%$)</td>
<td>2.59%/1.86%</td>
<td>2.66%/1.85%</td>
<td></td>
</tr>
</tbody>
</table>

22 It is common practice in some insurance markets to use risk-return profiles in order to present the characteristics of insurance products to policyholders. E.g., German regulation requires a risk-return classification of government subsidized old-age provision products based on risk-return profiles derived from a “simulation model” (cf. “Altersvorsorgevertrag-Zertifizierungsgesetz” (AltZertG), §7). Therefore, we will focus on risk-return profiles. Of course, it would also be interesting to perform utility optimizations or to analyze for which types of clients (characterized by their utility functions and parameters) certain product designs are particularly appealing.

23 The following measures for risk and return are being used in the framework mentioned in Footnote 22.

24 It must be noted that the observed changes and their size generally depend on the choice of the risk measures. The $\text{CTE}_{20}$ was chosen as a measure for policyholders’ risk because it has been used in a framework implemented by product rating firms and will be used in the risk-return classifications required by German regulation. Therefore, the focus of market participants is on this measure and it might coincide with perceived risk. However, it does not reflect all aspects of risk, and by applying other risk measures different size changes might be observed.
Since iso-profit products that strongly reduce the insurer’s risk do not seem very attractive from the policyholder’s perspective, we now consider iso-profit products that have the same risk from the insurer’s perspective as the traditional product. Such products are identified by the dashed horizontal line in Figure 2. With Alternative 1 and 2 products, the equity ratio \( q \) can be increased to 10.0% and 13.0%, respectively. The corresponding values for \( p \) are 90.45% and 89.98%, respectively.

Figure 3 shows the probability distribution of the policyholder’s terminal benefit of these products. We can see that, roughly up to the 25th percentile, the traditional product yields higher benefits than the alternative products. This results from the fact that in adverse years, the traditional product provides higher returns due to the year-by-year cliquet-style guarantees. Moreover, there are some scenarios where the alternative products only pay the guaranteed benefit, which is essentially impossible for the traditional product. For instance, in the fifth percentile, the alternative products pay only the guaranteed benefit of 20,000 €, whereas the traditional product pays 20,314 €. On the other hand, the alternative products provide a higher return in most scenarios. For instance, in the 50th percentile, the payoff of the traditional product is 21,490 € whereas it amounts to 21,848 € or 22,060 € for the alternative products, respectively. In the 95th percentile, the alternative products even pay 1,120 € or 1,817 € more than the traditional product. Also, the mean payoff is higher for the alternative products.

![Benefit distribution (G = 20,000 €)](image)

**Figure 3.** Benefit distribution of products with the same PVFP and the same SCR\(_{\text{mkt}}\), based on the traditional product with \( q = 5\% \).

The risk-return characteristics of these products are summarized in Table 4 (in row “Same risk level”). It demonstrates that the traditional product comes with a lower risk for the policyholder (CTE20 is larger), but the alternative products provide significantly higher expected returns. The increase in risk appears rather small when compared to the increase in expected return by 16 and 26 bps, respectively. This could be due to the fact that a policyholder of an alternative product only receives less than a policyholder of the traditional product if one or several "bad" years occur. On the other hand, such bad years are the key driver for the insurer’s solvency capital requirement. The pure possibility to give less to the policyholder in such bad years reduces the capital requirement, even if no such years occur.

We have now analyzed alternative products that appear very attractive for the insurer but might not appeal to policyholders, and also alternative products that come with interesting risk-return characteristics from the policyholder’s perspective but do not reduce the insurer’s risk. Since the insurer’s primary incentive to develop alternative guarantees is de-risking, the latter would not be appealing to the insurer. We therefore consider alternative product designs that lie between these extremes. We still assume that the insurer designs the products such that the expected profitability (and hence the fair value from the policyholder’s perspective) remains unchanged. However, only 50%
of the reduction in risk that would result from a pure modification of guarantees (with unchanged asset allocation) is "given back" to the client in the form of a higher equity ratio and hence more upside potential. This is illustrated on Figure 4. The alternative products with unchanged profitability and unchanged equity ratio would reduce the SCR\textsubscript{mkt} from 3.4\% to about 1.65\%. Now, we increase the equity ratio such that the resulting products come with 50\% of this reduction, i.e., an SCR\textsubscript{mkt} of about 2.5\%. The corresponding equity ratios are 8.25\% or 10.0\%, for Alternative 1 and 2 products, respectively.

![SCR(mkt)-curves of iso-profit products (under Q)](image)

**Figure 4.** Comparison of products with the same PVFP and reduced SCR\textsubscript{mkt}, based on the traditional product with \( q = 5\% \).

Figure 5 shows the benefit distribution of the resulting products from the policyholder’s perspective. We can see that these products, which leave the insurer’s profitability unchanged and provide a significant reduction of the insurer’s risk, provide higher benefits for the policyholder in most scenarios. Again, the alternative products’ benefits are below the traditional product’s benefit only up to the 25th to 28th percentile. Due to the guaranteed maturity benefit that is the same for all three products, however, the difference is limited.

Compared to the previous case where the insurer’s risk was the same for all products, naturally, the expected returns of the alternative products are smaller here (see last row of Table 4). Compared to the traditional product however, they are still remarkably larger by 10 and 17 bps, respectively. The CTE20 of the alternative products does not change significantly between the two latter risk levels.\(^{25}\) Therefore, these products might be attractive to both the insurer and the policyholder.

Of course, we cannot conclude that the alternative products would be more beneficial for both the policyholder and the insurer under every measure. Our results are possible since insurers and policyholders focus on different risk measures. Solvency regulation makes insurers focus on market consistent valuation and corresponding risk-based capital requirements whilst regulation with respect to product information disclosure for the policyholder highlights a CTE-measure of the real-world benefit distribution.

\(^{25}\) We particularly observe that the CTE20 for the Alternative 2 product shows very little variation (also in the values shown in Tables 5 and 6). This is the result of two opposing effects that occur if the risk level (and hence also the equity ratio) is increased: on the one hand, this increases the returns also for some scenarios in the lower tail (causing the higher CTE20 for Alternative 1 in the two lower rows of Table 4 since for the Alternative 1 product, this effect dominates). On the other hand, it causes a higher volatility in the asset portfolio which increases the number of years where a negative return is credited to the account for the Alternative 2 contract. As a result, in a larger portion of scenarios, only the guaranteed benefit is paid, which reduces the CTE20. For Alternative 2, these two effects almost exactly cancel each other out.
4.4. Sensitivity Analyses

In the remainder of this section, we will explain the results of several sensitivity analyses.

4.4.1. Asset Allocation

We first perform our analyses for product designs with different asset allocations. We still keep the profitability fixed at a PVFP of 3.62%. For this, we compare traditional products with \( q = 2.5\% \) (resulting in a \( \text{SCR}_{\text{mkt}} \) of 2.3\%) and \( q = 7.5\% \) (which implies a \( \text{SCR}_{\text{mkt}} \) of 4.9\%) with alternative products with the same profit and risk level. Figure 6 shows that in the case of \( \text{SCR}_{\text{mkt}} = 2.3\% \), the equity ratios can be increased from 2.5\% to 7.5\% or 9.5\% for Alternative 1 and 2, respectively; in the case of \( \text{SCR}_{\text{mkt}} = 4.9\% \), they increase from 7.5\% to 12.5\% or 16.5\%, respectively.

![Figure 5. Benefit distribution of products with the same PVFP and reduced SCR\(_{\text{mkt}}\), based on the traditional product with \( q = 5\% \).](image)

![Figure 6. Comparison of products with the same PVFP and the same SCR\(_{\text{mkt}}\), based on traditional products with \( q = 2.5\% \) (SCR\(_{\text{mkt}} = 2.3\% \)) and \( q = 7.5\% \) (SCR\(_{\text{mkt}} = 4.9\% \)).](image)
The values marked “Same risk level” in Table 5 show the risk-return characteristics of these products from the policyholder’s perspective. We can observe that a higher equity ratio (and thus higher risk for the insurer) leads to a higher expected return from a policyholder perspective and vice versa. However, the alternative products always provide a significant additional expected return. For all three considered asset allocation levels (base case and sensitivities), the additional expected return is approximately 15 bps for Alternative 1, and between 21 and 30 bps for Alternative 2. Hence, as one would expect, Alternative 2 is more sensitive with respect to the equity ratio level.

Table 5. IRR and CTE20 for products with the same SCR\textsubscript{mkt}, and with reduced SCR\textsubscript{mkt} for alternative products, for base case and sensitivities of asset allocation.

<table>
<thead>
<tr>
<th>IRR/CTE20</th>
<th>Risk Level</th>
<th>Traditional Product</th>
<th>Alternative 1</th>
<th>Alternative 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>“Less equity” (q = 2.5%)</td>
<td>Same risk level (SCR\textsubscript{mkt} = 2.3%)</td>
<td>2.41%/1.91%</td>
<td>2.56%/1.85%</td>
<td>2.62%/1.85%</td>
</tr>
<tr>
<td></td>
<td>Reduced risk for Alternatives 1/2 (SCR\textsubscript{mkt} = 1.8%)</td>
<td>2.51%/1.84%</td>
<td>2.52%/1.84%</td>
<td></td>
</tr>
<tr>
<td>Base case (q = 5%)</td>
<td>Same risk level (SCR\textsubscript{mkt} = 3.4%)</td>
<td>2.49%/1.96%</td>
<td>2.65%/1.87%</td>
<td>2.75%/1.85%</td>
</tr>
<tr>
<td></td>
<td>Reduced risk for Alternatives 1/2 (SCR\textsubscript{mkt} = 2.5%)</td>
<td>2.59%/1.86%</td>
<td>2.66%/1.85%</td>
<td></td>
</tr>
<tr>
<td>“More equity” (q = 7.5%)</td>
<td>Same risk level (SCR\textsubscript{mkt} = 4.9%)</td>
<td>2.56%/2.00%</td>
<td>2.72%/1.88%</td>
<td>2.86%/1.85%</td>
</tr>
<tr>
<td></td>
<td>Reduced risk for Alternatives 1/2 (SCR\textsubscript{mkt} = 3.5%)</td>
<td>2.65%/1.87%</td>
<td>2.76%/1.85%</td>
<td></td>
</tr>
</tbody>
</table>

It is worth noting that the CTE20 of the traditional product increases significantly with a larger equity ratio, while there is no difference for Alternative 2 and only a slight difference for Alternative 1.

We also performed the analyses for the case that only 50% of the risk reduction is “given back” to the policyholder which in the “less equity” sensitivity leads to an SCR\textsubscript{mkt} of 1.8% for the alternative products. They come with equity ratios of 6.0% and 6.25%, respectively. In the “more equity” case, SCR\textsubscript{mkt} amounts to 3.5%, and equity ratios increase to 10.25% or 13.25%, respectively. The resulting risk-return profiles (cf. rows marked “Reduced risk for Alternatives 1/2” in Table 5) are consistent with the previous observations: As expected, the expected return is lower than in the “Same risk level” case, but still significantly higher than for the traditional product (by approximately 10 bps for Alternative 1, and between 11 and 20 bps for Alternative 2).

4.4.2. Capital Market Assumptions

We now perform analyses for different interest rate levels and expected returns on equity investment: the parameters $r_0$, $\theta$ and $\mu$ (as defined in Section 3) are simultaneously reduced or increased by 50 bps. As a reference point, we again use the traditional product with $q = 5\%$ and $p = 90\%$.

In the case of lower capital market returns, the PVFP amounts to 2.48% and comes with an SCR\textsubscript{mkt} of 4.71%. If the insurer intends to keep the same profit and the same risk for Alternatives 1 and 2, the equity ratio can be increased to 10.75% or 13.5%, respectively. If the insurer intends to keep the same profit, but to reduce the risk to 3.6% (again by “giving back” 50% of the risk reduction), the equity ratio can be increased to 8.5% or 10.25%, respectively. The resulting risk-return profiles for the policyholder are summarized in Table 6.
Table 6. IRR and CTE20 for products with the same SCR\textsubscript{mkt}, and with reduced SCR\textsubscript{mkt} for alternative products, for base case and capital market sensitivities (± 50 bps).

<table>
<thead>
<tr>
<th>IRR/CTE20</th>
<th>Risk Level</th>
<th>Traditional Product</th>
<th>Alternative 1</th>
<th>Alternative 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>“Cap.Mkt. ~50 bps”</td>
<td>Same risk level (SCR\textsubscript{mkt} = 4.7%)</td>
<td>2.28%/1.93%</td>
<td>2.42%/1.84%</td>
<td>2.51%/1.84%</td>
</tr>
<tr>
<td></td>
<td>Reduced risk for Alternatives 1/2 (SCR\textsubscript{mkt} = 3.6%)</td>
<td>2.35%/1.84%</td>
<td>2.41%/1.84%</td>
<td></td>
</tr>
<tr>
<td>Base case</td>
<td>Same risk level (SCR\textsubscript{mkt} = 3.4%)</td>
<td>2.49%/1.96%</td>
<td>2.65%/1.87%</td>
<td>2.75%/1.85%</td>
</tr>
<tr>
<td></td>
<td>Reduced risk for Alternatives 1/2 (SCR\textsubscript{mkt} = 2.5%)</td>
<td>2.59%/1.86%</td>
<td>2.66%/1.85%</td>
<td></td>
</tr>
<tr>
<td>“Cap.Mkt. +50 bps”</td>
<td>Same risk level (SCR\textsubscript{mkt} = 2.7%)</td>
<td>2.86%/2.21%</td>
<td>3.02%/2.15%</td>
<td>3.13%/2.13%</td>
</tr>
<tr>
<td></td>
<td>Reduced risk for Alternatives 1/2 (SCR\textsubscript{mkt} = 2.0%)</td>
<td>2.97%/2.15%</td>
<td>3.04%/2.14%</td>
<td></td>
</tr>
</tbody>
</table>

Given the same risk for the insurer, the expected returns of the Alternative 1 and 2 products are 14 or 23 bps higher than for the traditional product. In case of a reduced SCR\textsubscript{mkt}, the increase is still 7 or 13 bps. The CTE20 is always 9 bps lower.

In the case of higher capital market returns, the starting point corresponds to a PVFP of 4.43% and an SCR\textsubscript{mkt} of 2.65%. For unchanged risk level, the resulting equity ratios for Alternatives 1 and 2 are 9.5% and 12.5%, respectively. Assuming a reduced SCR\textsubscript{mkt} of 2.0% (again according to the method outlined above), the equity ratios are 8.0% or 9.75%. The risk-return profiles show additional expected returns for the alternative products of between 11 and 27 bps. Overall, the results for different capital market assumptions prove to be consistent with the base case.

5. Conclusions and Outlook

In this paper, we have discussed profitability and risk of participating products with alternative guarantees, both from the policyholder’s and the insurer’s perspectives. We have considered three different product designs: a traditional product with year-to-year cliquet-style guarantees which is common in Continental Europe, and two products with alternative guarantees. In Reuß et al. [13], it has already been shown that such modified guarantees significantly improve the insurer’s profitability while reducing solvency capital requirements. However, in order to keep the alternative products attractive in a market where traditional products continue to be offered, it is expected that the policyholder would demand an additional benefit as a compensation for the somewhat weaker alternative guarantees.

In our analyses we have shown that surplus participation rate and asset allocation of the different products can be adjusted such that all products result in the same profitability from the insurer’s perspective (and hence in the same fair value from the policyholder’s perspective). Even on the same profitability level, the alternative products are significantly less risky from the insurer’s perspective (if the insurer’s asset allocation is not changed) and hence require less solvency capital. If we allow for a change of the asset allocation, products can be designed where the insurer’s risk lies anywhere between this reduced risk and the risk of the traditional product—still without changing the insurer’s profitability or the fair value from the policyholder’s perspective. Nevertheless, the products differ significantly with respect to the risk-return profiles from the policyholder’s perspective.

Since the primary incentive for the insurer to develop products with alternative guarantees is de-risking, products with the same profitability and the same risk are probably not desired. On the other hand, products with alternative guarantees that come with the same profitability and the same asset allocation are significantly less risky from the insurer’s perspective, but appear to be
less attractive from the policyholder’s perspective. Therefore, we have focused on product designs that lie between these two cases. If, e.g., 50% of the reduction in risk that would result from a pure modification of guarantees is “given back” to the policyholder by increasing the equity ratio, the policyholder has a significantly higher expected return than with the traditional product design, whereas the policyholder’s risk is only moderately increased. Sensitivity analyses show that similar effects can be achieved also in different capital market environments.

So far, we have separately analyzed portfolios of the traditional or the alternative products. For further research, it would be particularly interesting to see how such products interact when they are combined in an insurer’s book of business: e.g., it might be interesting to investigate the effects on the profitability and risk of an insurer that has sold the traditional product in the past and starts selling alternative products now. Furthermore, we have focused on risk-return profiles to assess the policyholders’ view in this paper. An extension to utility-based analyses also seems worthwhile. Here, at least two questions seem interesting: Which product design (fulfilling certain restrictions defined by the insurer) maximizes utility for a certain type of policyholder? To which type of policyholder (characterized by a utility function and parameters) would a certain product be most appealing? Finally, it would be interesting to analyze how alternative guarantees can be integrated in the annuity payout phase.

In conclusion, products with alternative guarantees allow for a large variety of product designs that might be suitable for reconciling policyholder’s and insurer’s interests, in particular, in a market environment with low interest rates. Hence, when designed properly, participating products with modified guarantees could be of interest to all market participants.

Author Contributions: The authors Andreas Reuß, Jochen Ruß, and Jochen Wieland developed the ideas to this research, analyzed the results and wrote the paper in close collaboration. Jochen Wieland engineered the projection model and performed the calculations.

Conflicts of Interest: The authors declare no conflict of interest.

References

26 Such cross-generational effects are similar to those analyzed by Hieber et al. [31]. They analyze the attractiveness of traditional policies for different generations of policyholders in a case where all assets are pooled, and in a case where assets covering the different generations are segregated.

27 cf. e.g., the results in Aase [19], where a life cycle model of a consumer with recursive utility is used, and conclusions on optimal pensions and life insurance contracts are drawn that lead to smoother consumption paths.
2.4 Runoff or Redesign? Alternative Guarantees and New Business Strategies for Participating Life Insurance

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Runoff or Redesign? Alternative Guarantees and New Business Strategies for Participating Life Insurance

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Abstract

Portfolios of traditional participating life insurance contracts with year-to-year (cliquet-style) guarantees are under pressure in the current situation of persistently low interest rates when valued in a market consistent valuation framework.

For a portfolio with a fixed technical interest rate it has been shown in Reuß et al. [2015] that product designs with alternative guarantees are able to reduce the insurer's risk and increase capital efficiency.

The objective of this paper is to analyze interactions between new contracts and an existing book of insurance contracts. We consider an insurer that has built up a portfolio in the past under changing guaranteed interest rates and market conditions. Then, we analyze different new business strategies for this insurer and the resulting risk exposure and capital requirement. We show that – if all contracts are covered by the same pool of assets – switching to carefully designed participating contracts with alternative guarantees is typically preferable to a runoff scenario and can substantially reduce financial risk in future years.

Keywords: Participating Life Insurance, Interest Rate Guarantees, Capital Efficiency, Portfolio Mix, Solvency Capital Requirements, Solvency II, SST, Market Consistent Valuation, New Business Strategy, ORSA
1 Introduction

Participating life insurance contracts have been and still are a main pillar of old-age provision and the life insurance business in Germany and many other countries. Since they traditionally come with a guaranteed minimum interest rate that is fixed for each contract until maturity, and this rate has been rather high in the last decades, most insurance companies have long-term interest rate guarantees in their books that exceed today’s interest rate levels. This has developed to a substantial financial burden. E.g. in Germany, insurers are required to set up additional reserves under local GAAP for contracts with higher guarantee levels,¹ and the capital requirements under market-consistent solvency frameworks (like Solvency II) increase strongly for portfolios of participating contracts. The requirements become particularly high in case of contracts with long maturities. Furthermore, participating products might be perceived unattractive by the client when priced under market consistent valuation. Therefore, many insurers question whether it is reasonable to continue new business in participating life insurance. Some have already started to search for modified participating products or to focus on different product types like unit-linked insurance.² The uncertainty on the profit and risk development of their portfolios in the future is an important issue for the insurance companies in this context.

¹ Since 2011 German life insurers have to set up an additional reserve (so-called “Zinszusatzreserve”) for contracts with a technical interest rate above a certain reference rate. This reference rate is updated every year by a prescribed mechanism based on Euro swap rates of the current and previous years (cf. “Deckungsrückstellungsverordnung” (DeckRV), §5), and amounts to 2.88% for year-end 2015.

² For example, Zurich Deutscher Herold Lebensversicherung stopped new business in participating life insurance in 2013, and now offers so-called “select products”. Generali Deutschland announced in a press release in May 2015 their target to discontinue participating life insurance and focus on offering unit-linked insurance. The Talanx group announced in July 2015 to sell new business from the end of 2016 on only with a return-of-premium guarantee (instead of a guaranteed interest rate). Even though such announcements are perceived in public often as a full withdrawal from traditional participating contracts, the new product strategies are mostly redesigned traditional contracts that are similar to the concepts discussed in Reuß et al. [2015] and Alexandrova et al. [2015].
There are several papers that have analyzed questions like the valuation and financial risk of interest rate guarantees in participating life insurance products e.g. Briys and de Varenne [1997], Grosen and Jorgensen [2000], Grosen et al. [2001], Grosen and Jorgensen [2002], Miltersen and Persson [2003], Bauer et al. [2006], Kling et al. [2007a], Kling et al. [2007b], Barbarin and Devolder [2005], Gatzert and Kling [2007], Gatzert [2008], and Graf et al. [2011]. For further details on this literature, see the literature overview in Reuß et al. [2015].

In the context of the low interest rate environment, Berdin and Gründl [2015] investigate the effects of the current low interest rate phase on the balance sheet of a representative German life insurer. Hieber et al. [2015] analyze the effects of prevailing low interest rates on investment decisions and surplus participation of an insurance portfolio with different annual interest rate guarantees, in particular focusing on cross-subsidizing effects between different generations of contracts.

Reuß et al. [2015] analyze interest rate guarantees from a viewpoint of product design and solvency regulation. It is shown how modified participating products can reduce the insurer’s risk, particularly in terms of capital requirement under solvency frameworks. Moreover, the concept of Capital Efficiency is introduced which relates profit to capital requirement. Based on this, Reuß et al. [2016] combine the insurer’s and the policyholder’s perspective by identifying and analyzing product designs that apply certain asset allocations and surplus participation rules such that policyholders are compensated for alternative, less valuable guarantees, but the risk-reducing character for the insurer remains in place.

So far, however, no analysis exists on the interaction of new business contracts that apply alternative guarantees with an existing book of business in a situation where all contracts are covered by a single pool of assets. Therefore, this paper first analyzes
a representative portfolio of existing traditional participating contracts that has been built up in the past decades, and compares it with corresponding portfolios consisting of contracts with alternative guarantees. Stochastic risk-neutral valuation techniques are used to measure the profitability and risk of the portfolios from today's perspective. Then, calculations are performed under the assumption that the insurer changed the product strategy at some point in time. This allows an analysis of different new business strategies considering interactions with old books of business. In particular, we investigate the future development of profitability and risk exposure under different assumed business strategies which is also a main requirement for insurance companies in the Solvency II framework (forward-looking perspective in the Own Risk and Solvency Assessment (ORSA)\(^3\)).

The remainder of this paper is structured as follows: In section 2, we explain the considered products and the modeling of the analyzed insurance portfolios. We also present the assumptions on the stochastic projections, particularly the financial dynamics. In section 3, we first present the key measures applied for valuation. Then, the different analyses are specified in detail and the results are interpreted. Section 4 summarizes and concludes.

2 The products and the modeling of the insurance portfolio

2.1 The considered products

The three product designs that will be analyzed are the same as in Reuß et al. [2015]. Therefore, we only briefly describe the most important product features and refer to that paper for more details.

\(^3\) Cf. EIOPA [2009], Article 45.
The common characteristic of all three products is that they provide a guaranteed benefit $G$ at maturity $T$, and an additional benefit from a bonus reserve (if surpluses from asset returns have occurred during the contract’s lifetime). Based on annual premium payments $P$ prospective actuarial reserves $AR_t$ ($t = 0, ..., T$) for the guaranteed benefit $G$ are built up. In case of sufficient asset returns in year $t$, annual surplus $s_t$ is credited to the bonus reserve. Actuarial reserve and bonus reserve sum up to the client’s account value $AV_t$.

At maturity, $AV_T$ is paid out as maturity benefit. In case of death or surrender in year $t$, the current account value $AV_t$ is paid at the end of year $t$. Therefore, the formulae for the calculation of premiums and actuarial reserves (given below) do not need to include biometric components.\(^4\)

With annual charges $c_t$, pricing interest rate $i_p$ and reserving interest rate $i_r$, the following equations determine premiums and reserves:

\[
\sum_{t=0}^{T-1}(P - c_t) \cdot (1 + i_p)^{T-t} = G
\]

\[
AR_t = G \cdot \left(\frac{1}{1+i_r}\right)^{T-t} - \sum_{k=t}^{T-1}(P - c_k) \cdot \left(\frac{1}{1+i_r}\right)^{k-t}
\]

In this paper we assume that the pricing and reserving rate always coincide for a certain contract, i.e. $i_p = i_r$. The regulation on determining the annual surplus is the same for all three products and described in section 2.2.\(^5\)

The distinguishing characteristic of traditional participating contracts (as they are common in Continental Europe) is that the pricing rate $i_p$ is also applied as a year-to-

\(^4\) However, mortality influences the development of the modeled insurance portfolio (explained in the following sections), and is therefore taken into account for the projections.

\(^5\) Annual surplus is typically credited to such policies according to country specific regulation. In Germany, at least 90% of the (local GAAP book value) investment income on the insurer’s assets has to be credited to the policyholders’ accounts.
year guaranteed interest rate $i_g$ on the account value $AV_t$, i.e. it acts as a minimum return that the client has to receive each year on both actuarial reserve and bonus reserve.

The idea of the analyzed alternative products is to detach the year-to-year guarantee from the pricing and reserving rate. As in Reuß et al. [2015] we consider an Alternative 1 product with a 0% year-to-year guarantee, i.e. $i_g = 0\%$, and an Alternative 2 product without any year-to-year guarantee, i.e. $i_g = -100\%$.

It follows that for all product types the required yield $z_t$ on $AV_t$ is given by\(^6\)

$$z_t = \max \left\{ \frac{\max (AR_t, 0)}{(AV_t - P - c_{t-1})} - 1, i_g \right\},$$  

(1)

and the account value evolves according to

$$AV_t = (AV_{t-1} + P - c_{t-1}) \cdot (1 + z_t) + s_t.$$

Since with the alternative product designs the year-to-year guaranteed interest rate is lower than the pricing rate, the required yield $z_t$ and hence the insurer’s financial risk decreases if surplus (which is included in $AV_{t-1}$) has been credited in previous years.

For our analyses, we assume that all products come with the same maturity guarantee $G$, annual administration charges $\beta \cdot P$ throughout the contract’s lifetime, and acquisition charges $\alpha \cdot T \cdot P$ which are equally distributed over the first 5 years of the contract. Hence, $c_t = \beta \cdot P + \alpha \cdot \frac{T \cdot P}{5} \cdot 1_{t\in\{0,...,4\}}$. Furthermore, we assume that expenses coincide with the charges. The product parameters are given in Table 2-1.

<table>
<thead>
<tr>
<th>$G$</th>
<th>$T$</th>
<th>$\beta$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20,000 €</td>
<td>20 years</td>
<td>3%</td>
<td>4%</td>
</tr>
</tbody>
</table>

Table 2-1: Product parameters

---

\(^6\) This definition makes sure that the account value remains non-negative, never falls below the actuarial reserve and earns at least the year-to-year guaranteed interest rate.
2.2 The asset-liability model

The insurer’s simplified balance sheet at time $t$ is given by Table 2-2.\textsuperscript{7}

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$BV_t^S$</td>
<td>$X_t$</td>
</tr>
<tr>
<td>$BV_t^B$</td>
<td>$AV_t$</td>
</tr>
</tbody>
</table>

Table 2-2: Balance sheet at time $t$.

The liability side of the balance sheet consists of the account values $AV_t$ (defined in section 2.1) of all contracts, and $X_t$, which denotes the shareholders’ profit or loss in year $t$, with corresponding cash flow at the beginning of the next year.\textsuperscript{8}

We assume that assets are invested in coupon bonds and equity. Only cash flows arising between the beginning of the year and the re-allocation of assets at the end of each year are invested in a riskless bank account. Hence, in the balance sheet we have the book value of bonds $BV_t^B$, and the book value of equity investments $BV_t^S$.

Under German GAAP $BV_t^B$ coincides with the nominal amount since we assume that they are considered as held to maturity. For $BV_t^S$, the insurer has more discretion. Since we assume that the insurer wants to create rather stable book value returns (and hence surplus distributions), the following management rules apply: a ratio $d_{pos}$ of the unrealized gains or losses (UGL) of the equity investment is realized annually if $UGL > 0$ (i.e. in case of unrealized gains) and a ratio $d_{neg}$ of the UGL is realized annually if $UGL < 0$.

The financial dynamics of the assets are described in the following sections. For the rebalancing of assets at the end of the year, market values of all assets are derived and a constant ratio $q$ is invested in equity. The remainder is invested in bonds. If

\textsuperscript{7} Further explanations on the applied asset-liability model are given in Reuß et al. [2015] and Reuß et al. [2016].

\textsuperscript{8} As in Reuß et al. [2015] we do not explicitly consider the shareholders’ equity or other reserves on the liability side since our analyses for the insurance portfolio are performed on a stand-alone basis.
bonds need to be bought, coupon bonds yielding at par with a given term $M$ are used until all insurance contracts' remaining term is less than $M$ years. After that, we invest in bonds with a term that coincides with the longest remaining insurance contracts. If bonds need to be sold, they are sold proportionally to the market values of the different bonds in the existing portfolio.

For each year, the book value return on assets is calculated as the sum of coupon payments from bonds, interest payments on the bank account and the realization of UGL. The policyholders' share of profit is determined by the participation rate $p$. If the share is not sufficient to credit the required yields to all policyholder accounts, there is no surplus for the policyholders, and all policies receive exactly the respective required yield $z_t$. Otherwise, surplus is credited which amounts to the difference between the policyholders' share of profit and the cumulative required yield.

Following the currently typical practice in Germany, we assume that surplus is distributed such that all contracts receive the same client's yield (defined by the required yield plus surplus rate), if possible. To achieve that, we apply the algorithm explained in Reuß et al. [2015]: 'The accounts are sorted by required yield, i.e. $(z_t^{(1)}, \ldots, z_t^{(k)})$, $k \in \mathbb{N}$ in ascending order. First, all contracts receive their respective required yield. Then the available surplus is distributed: Starting with the contract(s) with the lowest required yield $z_t^{(1)}$, the algorithm distributes the available surplus to all these contracts until the gap to the next required yield $z_t^{(2)}$ is filled. Then all the contracts with a required yield lower or equal to $z_t^{(2)}$ receive an equal amount of (relative) surplus until the gap to $z_t^{(3)}$ is filled, etc. This is continued until the entire surplus is distributed.'
As a result, the insurer's profit \( X_t \) is given by the difference between the total asset return and the amount credited to all policyholder accounts. We assume that a negative profit is compensated by the insurer's shareholders at the beginning of the next year.\(^9\)

In our numerical analyses we let \( M = 15 \) years, \( q = 5\% \), \( p = 90\% \), \( d_{pos} = 20\% \) and \( d_{neg} = 100\% \).

### 2.3 The history of the representative insurance portfolio

The main objective of this paper is to perform analyses from today's perspective based on an insurance portfolio that has been built up over the past decades. For that purpose, we consider a representative insurance portfolio consisting of traditional cliquet-style contracts, assuming the insurer started business at the beginning of 1988 (\( =: \tau_0 \)) and the portfolio is valued at the beginning of 2014 (\( =: \tau_1 \))\(^{10}\). The parameters given in the previous sections are applied. We assume a constant new business volume of 1,000 contracts every year from 1988 until 2013. We assume that all policyholders are 40 years old at inception of the respective contract, that no surrender occurs, and that mortality in the portfolio incurred according to the German standard mortality table (DAV 2008T). For the premiums and reserves of the contracts that are sold in a certain year, the maximum technical

---

\(^9\) As stated in Reuß et al. [2015], p. 196: ‘We do not consider the shareholders’ default put option resulting from their limited liability, which is in line with both, Solvency II valuation standards and the Market Consistent Embedded Value framework (MCEV), cf. e.g. Bauer et al. [2012] or DAV [2011], section 5.3.4.’

\(^{10}\) The valuation dates are generally set at the beginning of the year (and not at the end) to simplify the presentation of the results only. The valuation does not include new business of that year unless stated otherwise.
rate allowed by German regulation at the beginning of the respective year is applied.\textsuperscript{11} These rates are shown in Table 2-3.\textsuperscript{12}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
\hline
$i_p = i_r$ & 3.50\% & 4.00\% & 3.25\% & 2.75\% & 2.25\% & 1.75\% & 1.25\% \\
\hline
\end{tabular}
\caption{Technical rates}
\end{table}

In the modeled historical scenario, balance sheets and profit and loss statements (P\&L) of this portfolio are projected from 1987 to 2013 using book and market values. We refer to the following data for deriving the coupon rates and the returns on equity investment: For modeling the yield curves, we apply the zero-coupon Euro swap curves from 2002 to 2013 published by the German Federal Bank.\textsuperscript{13} Since those are not available for the years before 2002, we use the yield curves of the German government bonds for the years 1987 to 2001. These are calculated and published by the German Federal Bank as well.\textsuperscript{14} The returns of the equity investment correspond with the annual returns of the DAX performance index of every year between 1987 and 2013.

\textsuperscript{11} Note that since 2011 an additional reserve, so-called “Zinszusatzreserve” (as outlined in Footnote 1), exists under German regulation for contracts with a high technical rate that is calculated by replacing the original technical rate by a certain (lower) reference rate for a certain period of time. However, this rule is not considered in the model of this paper. We believe that in order to provide a more universal analysis (applicable also for different settings) and pure comparison of the different portfolios and product types, the effects of this country specific regulation should be left out of consideration. However, we want to point out that taking into account this additional reserve (which has been introduced to handle the current low interest rate environment) could change the surplus amount and therefore reduce risks in traditional contracts in certain scenarios, and lead to somewhat different results.

\textsuperscript{12} In reality, in 1994 and 2000, the maximal technical rate was changed by July 1\textsuperscript{st}. Since we assume that all new contracts are sold at the beginning of the year, the rate valid until June 30\textsuperscript{th} is applied for the respective years. From 2016 on, there might not be any maximal technical rate regulated by law, but we will assume for our new business analyses that the insurer keeps this level for future contracts.

\textsuperscript{13} Cf. Bundesbank [2015].

\textsuperscript{14} We note that swap rates can deviate from the rates of government bonds, and are thus not fully comparable. However, for our purpose of modeling a representative insurance portfolio the difference is negligible.
The one-year and 15-year spot rates of the yield curve as well as the resulting coupons of 15-year-bonds are shown in Figure 2-1. The yield curves and coupons show a strong decrease from about 8% in 1990 to about 2% in 2013 which leads to today’s difficult low interest rate environment. The returns on equity investment shown in Figure 2-2 exhibit a strong volatility, particularly between the end of the 1990s and 2010.

![Yield curves and coupons](image1)

**Figure 2-1:** Yield curves and coupons used for the modeled historical scenario.

![Annual return of DAX performance index](image2)

**Figure 2-2:** Annual return of the DAX performance index used for the modeled historical scenario.

### 2.4 The financial dynamics for the projections

For the stochastic valuation of the portfolio, we specify dynamics for the short rate and the equity index in a risk-neutral framework. We let the short rate process $r_t$
follow a Vasicek\textsuperscript{15} model, and the equity index $S_t$ follow a geometric Brownian motion, and get the following dynamics:

\[ dr_t = \kappa(\theta - r_t)dt + \sigma_r dW_t^{(1)} \]

\[ \frac{dS_t}{S_t} = r_t dt + \rho \sigma_S dW_t^{(1)} + \sqrt{1 - \rho^2} \sigma_S dW_t^{(2)}, \]

where $W_t^{(1)}$ and $W_t^{(2)}$ are independent Wiener processes on some probability space $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{Q})$ with a risk-neutral measure $\mathbb{Q}$ and the natural filtration $\mathbb{F} = \mathcal{F}_t = \sigma\left(\left(W_s^{(1)}, W_s^{(2)}\right), s < t\right)$.

The parameters $\kappa$, $\theta$, $\sigma_r$, $\sigma_S$ and $\rho$ are deterministic and constant. The above equations can be solved to

\[ S_t = S_{t-1} \cdot \exp\left(\int_{t-1}^t r_u du - \frac{\sigma_S^2}{2} + \int_{t-1}^t \rho \sigma_S dW_u^{(1)} + \int_{t-1}^t \sqrt{1 - \rho^2} \sigma_S dW_u^{(2)}\right) \]

\[ r_t = e^{-\kappa} \cdot r_{t-1} + \theta (1 - e^{-\kappa}) + \int_{t-1}^t \sigma_r \cdot e^{-\kappa(t-u)} dW_u^{(1)}. \]

The initial value of the equity index $S_0 = 1$ and the initial short rate $r_0$ are deterministic as well. It can be shown that the four integrals in the formulae above follow a joint normal distribution\textsuperscript{16}. By using random realizations of this multidimensional distribution, Monte Carlo paths are calculated.

The bank account is given by $B_t = \exp\left(\int_0^t r_u du\right)$, and the discretely compounded yield curve at time $t$ by\textsuperscript{17}

\[ r_t(s) = \exp\left[\frac{1}{s} \left(\frac{1-e^{-\kappa s}}{\kappa} r_t + \left(s - \frac{1-e^{-\kappa s}}{\kappa}\right) \cdot \left(\theta - \frac{\sigma_r^2}{2\kappa^2} + \left(\frac{1-e^{-\kappa s}}{\kappa}\right)^2 \frac{\sigma_S^2}{4\kappa} \right)\right) - 1\right].\]

\textsuperscript{15} Cf. Vasicek [1977].

\textsuperscript{16} Cf. Zaglauer and Bauer [2008].

\textsuperscript{17} See Seyboth [2011] as well as Branger and Schlag [2004].
for any time $t$ and term $s > 0$. Based on the yield curve, we can calculate the par yield that determines the coupon rates of the considered coupon bond.

For our calculations, we use market parameters shown in Table 2-4. The parameters $\kappa$, $\sigma_r$, $\sigma_S$ and $\rho$ are directly adopted from Graf et al. [2011]. For the initial short rate $r_0$ and the long term mean of short rates $\theta$, we consider two different sets of parameters: one for a basic level which is specified such that it reflects the interest rate level applicable for Solvency II calculations at year-end 2013; and one for a stress level where $r_0$ and $\theta$ are reduced by 100 bps compared to the basic level.

<table>
<thead>
<tr>
<th></th>
<th>$r_0$</th>
<th>$\theta$</th>
<th>$\kappa$</th>
<th>$\sigma_r$</th>
<th>$\sigma_S$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>basic</td>
<td>1.5%</td>
<td>3.0%</td>
<td>30.0%</td>
<td>3.0%</td>
<td>20.0%</td>
<td>15.0%</td>
</tr>
<tr>
<td>stress</td>
<td>0.5%</td>
<td>2.0%</td>
<td></td>
<td>2.0%</td>
<td>20.0%</td>
<td>15.0%</td>
</tr>
</tbody>
</table>

Table 2-4: Capital market parameters

Besides the stochastic scenarios, a so-called “certainty equivalent (CE)” scenario will be used for certain risk measures (cf. section 3.1). This deterministic scenario reflects the expected development of the capital market under the risk-neutral measure. It can be derived from the initial yield curve $r_0(s)$\(^1\) based on the assumption that all assets earn the forward rate implied by the initial yield curve.

3 Analyses and results

In this section, we present the valuation measures, the analyses and the results of the considered insurance portfolios.

For all projections, the number of scenarios is $N = 5,000$. Our analyses showed that this creates stable results.\(^1\) For the initial balance sheet (used as a starting point for the projections), the book values of the asset and liability portfolio as well as the

\(^1\) Cf. Oechslin et al. [2007].

\(^1\) As in Reuß et al. [2015], we apply an antithetic path selection of the random numbers in order to reduce variance in the sample, cf. e.g. Glasserman [2003].
market value of the equity investment are taken from the final (end of 2013) balance sheet of the historical scenario.

### 3.1 Valuation measures

For the market consistent valuation of the profitability of an insurance portfolio, we use the expected present value of future profits ($PVFP$)\(^{20}\). Based on Monte Carlo simulations, the $PVFP$ is estimated by

$$PVFP = \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{\tau} \frac{X_t^{(n)}}{B_t^{(n)}} = \frac{1}{N} \sum_{n=1}^{N} PVFP^{(n)},$$

where $N$ is the number of scenarios, $\tau$ is the time horizon of the projection, $X_t^{(n)}$ denotes the insurer's profit/loss in year $t$ in scenario $n$, and $B_t^{(n)}$ is the value of the bank account in year $t$ in scenario $n$. We calculate the $PVFP$ under both, the basic and the stress parameter level (as presented in Table 2-4); the latter will be denoted by $PVFP_{stress}$. While in the MCEV framework the $PVFP$ is usually considered without new business, we will for some analyses also consider the $PVFP$ including new business (denoted by $PVFP^{(NB)}$).

In addition, the (weighted) average of the required yield of all contracts determines the minimum yield to be earned on the asset portfolio in order to avoid a loss for the insurer. For year $t$ the average required yield is defined by

$$ARY_t = \frac{\sum_{k=0}^{T-1} AV^{(k)}_{(t-1)+} \cdot z_t^{(k)}}{\sum_{k=0}^{T-1} AV^{(k)}_{(t-1)+}},$$

where $z_t^{(k)}$ denotes the required yield in year $t$ of contracts with a remaining time to maturity of $k$ years, and $AV^{(k)}_{(t-1)+}$ is the account value at the beginning of year $t$ (after

---

\(^{20}\) The concept of PVFP is introduced as part of the MCEV Principles in CFO-Forum [2009].
premium payment) of all contracts with a remaining time to maturity of $k$ years. If the resulting $ARY_t$ is lower than the average technical interest rate in the insurance portfolio, a financial buffer has been built up by surplus distribution to contracts with alternative guarantees in the past.

Furthermore, the degree of asymmetry of the shareholder’s cash flows is characterized by the time value of options and guarantees (TVOG). Under the MCEV framework\textsuperscript{21}, it is defined by

$$TVOG = PVFP_{CE} - PVFP,$$

where $PVFP_{CE} = \sum_{t=1}^{T} \frac{X_t^{(CE)}}{R_t^{(CE)}}$ is the present value of future profits in the CE scenario explained in section 2.4. The TVOG is a very useful measure for the insurer’s financial risk resulting from the portfolio.

We also analyze the solvency capital requirement which is the key risk measure under risk based solvency regulation like Solvency II. In the standard formula\textsuperscript{22} of the Solvency II framework the difference of $PVFP$ between the basic and the stress level

$$\Delta PVFP = PVFP - PVFP_{stress}$$

determines the solvency capital requirement for interest rate risk ($SCR_{int}$). In the Solvency II framework the capital requirement is generally defined based on a runoff portfolio (i.e. without new business). In our analyses, we will additionally consider $\Delta PVFP^{(NB)}$ as a measure for capital requirement in setups that include new business.

The term “capital efficiency”, which is also discussed in Reuß et al. [2015], is frequently used by practitioners intuitively meaning the ratio of profitability and capital requirement under risk based solvency frameworks. However, to the best of our

\textsuperscript{21} Cf. CFO-Forum [2009].

\textsuperscript{22} A description of the standard formula can be found in the Delegated Regulation (EU) 2015/35 (cf. EIOPA [2015]).
knowledge, no formal definition of this term exists. Also, since both, profit and capital requirement over time are stochastic, it is not clear, whether a formal definition should only consider expected profitability and expected capital requirement or the expected ratio or also other characteristics of the distributions. However, we think it is reasonable to define a simple and easy to calculate indicator to assess and compare capital efficient behavior of different participating products. Therefore, we will use the following measure that reflects the basic intuition of capital efficiency:

\[
iCE := \frac{PVFP}{PVFP - \Delta PVFP} \approx \frac{PVFP}{SCR_{int}}
\]

Note that this measure is only reasonable as long as \(PVFP\) and \(\Delta PVFP\) are both positive; therefore, we will only consider it in this case. Moreover, this measure only considers interest rate risk; however, it can easily be extended to other sources of risk. We refrain from doing so since the focus of our analyses is on interest rate risk.

In the settings where we consider new business, the new business margin

\[
NBM = \frac{\sum_{n=1}^{N} (PVFP^{(n)}(NB) - PVFP^{(n)})}{\sum_{n=1}^{N} PVFPremNB^{(n)}}
\]

is a suitable measure for the profitability of new business. Here, \(PVFP^{(n)}(NB)\) denotes the present value of future profits including new business in scenario \(n\), and \(PVFPremNB^{(n)}\) is the present value of future premiums from new business in scenario \(n\). This measure will generally include profit and premiums from all new business cohorts that are considered in the respective setting. The new business margin calculated under the stress parameter level (as presented in Table 2-4) is denoted by \(NBM_{stress}\) (analogously to \(PVFP_{stress}\)).
3.2 Analysis of portfolios with traditional and alternative products in the past

In section 2.3 we have described how the insurance portfolio with contracts of maturity $T = 20$ years is built up in the historical scenario. Hence, when stochastic projections start at the beginning of 2014 ($= \tau_1$), we have a portfolio with remaining time to maturity between 1 year and 19 years$^{23}$. For the valuation of the existing portfolio (in line with Solvency II and MCEV), we do not consider new business after $\tau_1$. Hence, the time horizon for the stochastic projection is $\tau = 19$ years.

First, we consider an insurance portfolio consisting only of the traditional product. We compare this with the portfolios that consist only of products with alternative guarantees assuming that such products had already existed in the past. The results (given as percentages of the present value of future premium income) are presented in Table 3-1. We observe that if the insurer had sold contracts with alternative guarantees instead of cliquet-style contracts, the present portfolios would be significantly more profitable: The $PVFP$ would be about 80% higher. Furthermore, the average required yield of the portfolio at $\tau_1$ (i.e. when the stochastic projection starts) would be close to zero with Alternative 1 or even be remarkably below zero with Alternative 2, whereas it amounts to 3.46% with the traditional guarantees. Even considering rather high coupon payments from old bonds, a required yield on this level is difficult to earn under the current market conditions. As a consequence, the $TVOG$, which indicates the asymmetry of profits in the projected scenarios, is substantial with the traditional product, but close to zero with the alternative guarantees. Further insights can be gained by analyzing the $PVFP_{stress}$ assuming a more adverse capital market (using the stress parameters in Table 2-4). The

$^{23}$ Note that due to mortality before 2014, the number of contracts for the different remaining times to maturity is not the same.
for the traditional product falls significantly below zero, while with alternative guarantees it is still positive and higher than the $PVFP$ with traditional guarantees before stress. In consequence, today’s solvency capital requirement for interest rate risk (as a percentage of the present value of future premiums before stress) would be about 60% lower if alternative guarantees had been sold in the past. Finally, the $iCE$ values are multiple times larger with alternative products; this indicates that the capital efficiency of the portfolio – relating profit to capital requirement – would be much higher with alternative guarantees.

<table>
<thead>
<tr>
<th></th>
<th>Traditional</th>
<th>Alternative 1</th>
<th>Alternative 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$PVFP$</td>
<td>2.87%</td>
<td>5.09%</td>
<td>5.17%</td>
</tr>
<tr>
<td>$ARY_{2013}$</td>
<td>3.46%</td>
<td>0.02%</td>
<td>-3.39%</td>
</tr>
<tr>
<td>$TVOG$</td>
<td>2.31%</td>
<td>0.13%</td>
<td>0.12%</td>
</tr>
<tr>
<td>$PVFP_{stress}$</td>
<td>-1.59%</td>
<td>3.05%</td>
<td>3.24%</td>
</tr>
<tr>
<td>$SCR_{int}$</td>
<td>4.56%</td>
<td>1.87%</td>
<td>1.74%</td>
</tr>
<tr>
<td>$iCE$</td>
<td>0.63</td>
<td>2.73</td>
<td>2.97</td>
</tr>
</tbody>
</table>

Table 3-1: Valuation measures at $\tau_1 = 2014$ (values given as percentages of the present value of future premium income).

In Reuß et al. [2015] alternative guarantees have been shown to increase capital efficiency in a situation where a portfolio of contracts with a technical rate of 1.75% was built up under recent market conditions. In extension to that, the results of this section show that the alternative products are even more capital efficient for an insurance portfolio built up over decades with changing technical (and hence guaranteed) interest rates and changing market conditions.

In the next step, we analyze scenarios where the insurer started with selling traditional contracts in $\tau_0 = 1988$, but then switched to one of the alternative products
at some point in time. The traditional and the alternative contracts are covered by the same pool of assets. Table 3-2 shows the results for switches in 2000, 2004, 2008 and 2012. We can see that the insurer’s profit and risk situation improves substantially even if the alternative contracts have been sold for only a few years. The alternative guarantees relieve the risk resulting from interest rate guarantees since the average required yield declines. Not surprisingly, the earlier the insurer has changed the product strategy to alternatives, the stronger are the effects. For example, compared to a TVOG of 2.31% if the insurer has continued to sell the traditional product, the TVOG is reduced to 1.96% if Alternative 1 has been sold for only two years (from 2012 on), but to 0.56% if the switch happened eight years earlier (in 2004). Furthermore, we observe that the different measures take different amounts of time to react to the change in product strategy: While there is a significant increase in PVFP and a corresponding decrease in TVOG already within few years, $SCR_{int}$ and the average required yield need more time to adjust. For example, PVFP increases already from 2.87% to 3.28% if the insurer has sold the Alternative 2 product for only two years (from 2012 on); on the other hand, $ARY_{2013}$ decreases by only 0.01 percentage points (from 3.46% to 3.45%) and $SCR_{int}$ decreases by only 0.04 percentage points (from 4.56% to 4.52%). After a longer period with alternative guarantees in the portfolio, the effects in the average required yield and in the $SCR_{int}$ become stronger as well. These delays in the effects of the two measures are linked to each other: As long as the required yield is high when compared to the asset return level in the stressed scenario, $PVFP_{stress}$ will be very low keeping the solvency requirement for interest risk on a high level. Altogether, the $iCE$ values increase disproportionately compared to the time since the change in product strategy; this
allows the conclusion that capital efficiency improves even more with a longer history of selling alternative contracts.

<table>
<thead>
<tr>
<th>switch to…</th>
<th>2012</th>
<th>2008</th>
<th>2004</th>
<th>2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alt. 1</td>
<td>Alt. 2</td>
<td>Alt. 1</td>
<td>Alt. 2</td>
<td>Alt. 1</td>
</tr>
<tr>
<td>$PVFP$</td>
<td>3.22%</td>
<td>3.28%</td>
<td>3.91%</td>
<td>3.96%</td>
</tr>
<tr>
<td>$ARY_{2013}$</td>
<td>3.45%</td>
<td>3.45%</td>
<td>3.28%</td>
<td>3.18%</td>
</tr>
<tr>
<td>$TVOG$</td>
<td>1.96%</td>
<td>1.91%</td>
<td>1.27%</td>
<td>1.23%</td>
</tr>
<tr>
<td>$SCR_{int}$</td>
<td>4.48%</td>
<td>4.52%</td>
<td>4.04%</td>
<td>4.07%</td>
</tr>
<tr>
<td>$iCE$</td>
<td>0.72</td>
<td>0.73</td>
<td>0.97</td>
<td>0.97</td>
</tr>
</tbody>
</table>

Table 3-2: Valuation measures at $\tau_1 = 2014$ (values given as percentages of the present value of future premium income). Results for scenarios where the insurer starts business with the traditional product, and switches to an alternative product in the respective year.

Figure 3-1 illustrates how the unfavorable combination of low future profits and high capital requirements observed for the traditional product can be reversed by an early change of the product strategy. $PVFP$ and $SCR_{int}$ are compared in absolute numbers for our portfolio in the case where the insurer has continued to sell the traditional product until 2013, and the cases of a switch to Alternative 1 in 2004, 2008 or 2012, respectively. Note that selling alternative products already from 2004 on results in a significant change in the relation of future profits and capital requirement: the level of $PVFP$ is higher, the level of $SCR_{int}$ is lower then. But also a later switch (in 2008 or 2012) shows strong improvements in the balance of profitability and capital requirement.
3.3 Analysis of new business strategies

In this section, we investigate the effects on profit and risk if the insurer changes the product strategy for new business, i.e. in $\tau_1 = 2014$ when stochastic projections start. Different new business strategies are compared by a present-value analysis. We assume that the insurer has sold traditional contracts until 2013, and then has to decide between the following options: Runoff (i.e. no new business), continue selling traditional contracts, or start selling new contracts with alternative guarantees. Except for the runoff strategy, we consider new business for 5 years (from 2014 to 2018) of 1,000 contracts per year, and conduct projections running for $\tau = 24$ years until the maturity of the last contracts. Remember that for contracts sold from 2015 on, the pricing and reserving rate is $i_p = i_r = 1.25\%$ (and for the traditional contracts this is also the year-to-year guaranteed rate).

Figure 3-2 shows that with selling new business the $PVFP^{(NB)}$ as well as the capital requirement (measured by $\Delta PVFP^{(NB)}$) grow. However, the relation of profitability and capital requirement is better than in the runoff scenario. This shows that stopping...
new business in the current situation of difficult capital markets might not be beneficial if one considers profit and financial risk resulting from interest rate guarantees. The difference of $\Delta PVFP^{(NB)}$ and $PVFP^{(NB)}$ also gives an indication for the required shareholders’ equity: this value is significantly smaller with alternative contracts. However, we have to stress that the model does not consider certain factors that can affect the profitability of runoff portfolios and new business in reality, like fixed costs, changing asset allocation or advance financing of acquisition expenses.

In Table 3-3 we see that under the basic parameter set the new business margins with alternative products (3.59% and 3.70% for Alternative 1 / 2) are significantly higher than with the traditional product (2.93%) which in turn is only slightly higher than the profitability of the traditional in-force business (2.87%). In the valuation under stress parameters, the new business margins for all product strategies are positive in contrast to the negative profitability of the in-force business; however, with alternative guarantees the new business margin is more than tripled. Also the $iCE$ values indicate that capital efficiency increases most with a new business strategy.
based on alternative contracts. If we compare Alternative 1 and 2 products, there are only marginal differences. This is consistent with results in Reuß et al. [2015].

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<td>$NBM$</td>
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<td>$NBM_{stress}$</td>
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<td>0.36%</td>
<td>1.20%</td>
<td>1.25%</td>
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Table 3-3: Valuation measures at $\tau_1$ for scenarios where the insurer starts business with the traditional product, and with either runoff from 2014 on, or new business with the traditional / Alternative 1 / Alternative 2 product (values given as percentages of the present value of future premium income).

### 3.4 Development of profitability and risk exposure in the future (ORSA perspective)

In the previous section, we compared different new business strategies by a present-value analysis (in $\tau_1 = 2014$) of stochastic scenarios including new business. In this section, we complete the new business analysis by considering explicit projections of the in-force and new business portfolio under so-called planning scenarios and analyze the resulting profit and risk situation. This is similar to the so-called Own risk and solvency assessment (ORSA) that is required under the Solvency II framework. For the planning scenario, we will assume that in the next 5 years (from 2014 on) coupon bonds with maturity $M = 15$ years and a coupon rate of 2.6% will be

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24 Cf. EIOPA [2009], Article 45.

25 This value is derived from the par yield under risk-neutral returns in the CE scenario of the stochastic projections with basic parameters described in section 2.4.
available in the market, and that equity investments will yield returns of 5.6% (i.e. the risk premium is 3 percentage points). Since the analysis of the previous section indicated that the differences in the results between new business with Alternative 1 and 2 are rather small, we will restrict focus on Alternative 1 here, and analyze the following settings:

1. Runoff, i.e. the insurer has sold traditional contracts in the past, and stops new business from 2014 on.
2. The insurer has sold and continues selling traditional contracts.
3. The insurer has sold traditional contracts in the past, but sells the Alternative 1 product from 2014 on.
4. The insurer has already switched from selling traditional to Alternative 1 contracts in 2008, and continues selling Alternative 1.

Except for the runoff scenario, we assume a new business volume of 1,000 contracts per year during the planning scenario (starting in $\tau_1 = 2014$). We calculate the valuation measures for the years 2015, 2016 and 2019, i.e. after a 1-year, 2-year and 5-year run of the deterministic planning scenario. The valuation is done by stochastic scenarios with basic parameters and assuming no more new business after the valuation date (i.e. 2015, 2016, or 2019, respectively).

Figure 3-3 shows the development of the valuation measures. The $PVFP$ decreases permanently in the runoff scenario. It stays at about the same level if the insurer continues selling the traditional product, and it increases by approx. 14% over 5 years if the insurer changes the product strategy in 2014, and starts selling Alternative 1 contracts. If the change in product strategy happened in 2008, it would already be at a significantly larger level and would stay roughly at that level. As with the analyses in section 3.2, we observe that the average required yield needs more
time to adjust, i.e. it decreases stronger with a longer history of alternative contracts in the portfolio. Looking at the risk measures, we see that the $TVOG$ decreases in all settings with new business, but stronger with alternative guarantees in new contracts, whereas it increases if the insurer stops new business in 2014. The solvency requirement shows an almost parallel decrease for all settings. This effect can be explained by different causes: in the runoff setting by a generally decreasing interest rate sensitivity in a portfolio that runs off, but in the other settings due to the decreasing average guarantee level in the portfolio caused by the lower guarantees in new business contracts. Note, that in all settings more and more contracts with high guaranteed interest rates of 3.5% or 4.0%, that have been sold until the year 2000, will mature during the planning scenario which reduces the $SCR_{int}$. However, we observe a lower risk level if alternative contracts have already been sold since 2008, because the required yield in the portfolio has been reduced since then.

In summary, the $iCE$ values increase during the planning scenario for all four settings (due to the $SCR_{int}$ reduction), but they develop particularly strong after a few years when there are alternative contracts in the portfolio. Hence, it can be concluded that switching to alternative guarantees now will have a sustainable impact on capital efficiency in the typical planning horizon. It should be noted that the results will depend on an insurer’s specific situation, i.e. they may be different for an insurer with an existing book of business or asset portfolio that significantly deviates from our assumptions.
Figure 3-3: Development of valuation measures for the four settings in the planning scenario (values given as percentages of the present value of future premium income).
Finally, we also consider a planning scenario under stress, where we assume 15-year coupon bonds with a coupon rate of 1.6%\textsuperscript{26} and equity investment returns of 4.6%. The four business strategy settings are as before and the valuation is done with stochastic scenarios under the stress parameters given in Table 2-4. In the results shown in Figure 3-4 we observe that the profitability is negative for all strategies under the stress assumption, but the projected loss develops worst in the runoff scenario and least severe if the insurer switches to alternative guarantees. The SCR\textsubscript{int} now shows a stronger decrease for the runoff portfolio in the first setting; however, considering this together with the strongly increasing loss in this setting, stopping new business still does not appear beneficial. Since the PVFP is always negative here, calculating iCE values is not reasonable.

Overall, we see that contracts with alternative guarantees can significantly help an insurer with a traditional participating portfolio to improve future profitability and reduce risk. However, under persistently low interest rates the help of capital efficient products in new business for the portfolio is limited, and the problems resulting from existing business cannot be fully solved, but only somewhat reduced.

\textsuperscript{26} This value is derived from the par yield under risk-neutral returns in the CE scenario of the stochastic projections with stress parameters described in section 2.4.
4 Conclusion and Outlook

In this paper we have studied the profitability and risk situation from today’s perspective for an insurer with a representative portfolio of participating contracts that has been built up in the past. We have first considered a portfolio with a traditional product (which is common in Continental Europe) with year-to-year cliquet-style guarantees, and have compared it to corresponding portfolios with contracts that apply alternative types of guarantees (but with the same level of guaranteed benefit). Extending the findings of Reuß et al. [2015], we have shown that the alternative products are more capital efficient, i.e. they increase profitability and reduce capital requirement, also in a framework that reflects the market environment of the past decades. In a next step, we have analyzed the impacts of contracts with alternative guarantees that are added to an existing portfolio of traditional contracts from a
certain point in time on. We have seen a significant effect for all profit and risk measures, albeit with a different speed and magnitude. Particularly for reducing solvency capital requirement, effects will only be observable after alternative products have been sold for some years.

We have also analyzed possible new business strategies for an insurer with an existing book of traditional business. We found that new business – even with the traditional product – can beat a runoff scenario since average guaranteed rates will be reduced by adding new contracts with today’s guaranteed rates. However, we remind that the model does not reflect all factors that can be relevant in reality, such as expenses caused by new business. Moreover, we observed that new business margins of alternative contracts are significantly larger than the profitability of the traditional business, in particular in a more adverse scenario.

Similar to the ORSA requirements, we have finally studied the development of the insurer’s profit and risk in a so-called planning scenario for the future applying different new business strategies. In line with the previous results, we found that alternative guarantees can cause a substantial increase of capital efficiency; however, the risk reducing effect on capital requirement needs some time to emerge. In a more adverse scenario where all considered product strategies are unprofitable, alternative guarantees are at least able to reduce the insurer’s risks.

The improved capital efficiency induced by alternative guarantees can give the insurer also some room to compensate policyholders for contracts with alternative (and somewhat weaker) guarantees. Therefore, it would be interesting for further research to analyze possible management decisions like a different asset allocation or a larger surplus participation that can lead to a more attractive maturity benefit for
the alternative new business contracts.\textsuperscript{27} Due to the weaker guarantees alternative contracts will typically pay less in the most adverse scenarios compared to traditional contracts.\textsuperscript{28} In a mixed portfolio (consisting of traditional and alternative contracts), the insurer might therefore consider a so-called surplus spread giving a larger share of surplus to alternative contracts (in case of sufficient asset returns) and a lower share to traditional contracts. The effects of such a mechanism would be particularly interesting to analyze.

Furthermore, integrating alternative guarantees in the annuity payout phase would be an exciting field of research. This is particularly interesting in insurance markets where the guaranteed interest rate given at the beginning of the accumulation phase also holds for the annuity payout phase.

Altogether, we conclude that in the recent and current market environment changing the product strategy to alternative contracts can significantly help a traditional participating portfolio to attain a sustainable perspective in terms of profits and capital requirements – the earlier, the better.

5 References


\textsuperscript{27} Compare for example the approach in Reuß et al. [2016].

\textsuperscript{28} See Reuß et al. [2016].


3 Zusammenfassung


Klassische überschussbeteiligte Lebensversicherungsprodukte haben sich während der vergangenen Jahrzehnte als eine bedeutende Säule für eine stabile und nachhaltige Altersvorsorge in vielen europäischen Ländern, und im Besonderen in Deutschland, erwiesen. Bei diesen Produkten erhält der Versicherungsnehmer eine garantierte Leistung auf Basis einer vorsichtigen aktuariellen Berechnung und zusätzlich eine Beteiligung an Kapitalanlage- und anderen Überschüssen (wobei Mindestgrenzen meist gesetzlich festgelegt sind). Die gezahlten Prämien werden während der Vertragslaufzeit
Zusammenfassung

in einem kollektiven Bestand angesammelt, der damit ein wirksames Instrument zum Risikoausgleich im Versicherungskollektiv und in der Zeit darstellt.

Zunehmend sieht sich die klassische Lebensversicherung jedoch ernsthaften Schwierigkeiten ausgesetzt: Anhaltend niedrige Kapitalmarktzinsen erschweren es, eine Rendite zu erzielen, die für Garantieversprechen älterer Verträge ausreicht, was zu hohen Kapitalanforderungen unter Solvency II führt. Starre Zinsgarantien verteuern die Produkte unter marktkonsistenter Bewertung, während durch das niedrige Zinsniveau Neugeschäftsverträge für die Kunden an Attraktivität verlieren. Dadurch hinterfragen immer mehr Versicherer, ob es sinnvoll ist weiterhin klassische Verträge anzubieten. Zwar wird nach wie vor diskutiert, wie die marktkonsistente Bewertung und Solvabilitätsvorschriften angepasst werden können, sodass sie den speziellen, langfristigen Charakter der klassischen Verträge berücksichtigen. Weitaus wichtiger erscheint jedoch über neue und modifizierte Produktvarianten nachzudenken, die das finanzielle Risiko des Versicherers wirklich reduzieren können und zu einer besseren Profitabilitäts- und Risikoperspektive führen.

Daher zielt diese Arbeit auf eine tiefgreifende Analyse der Zinsgarantien in klassischen Verträgen und des darauf basierenden finanziellen Risikos ab. Auf Grundlage dieser Analyse sollen alternative Produktdesigns mit modifizierten Garantien abgeleitet werden, die für den Versicherer Nutzen in Form von Rentabilität, reduziertem Risiko und finanzieller Entlastung schaffen, aber auch zu Vorteilen für den Versicherungsnehmer im Sinne von attraktiven Auszahlungsprofilen führen.

Um die Wirkungsweisen der unterschiedlichen Produkte in einem Versicherungsbestand vergleichen zu können, sind umfassende Analysen von repräsentativen Beständen mit Hilfe von Profitabilitäts- und Risikomaßen, insbesondere in Bezug auf die neuen Solvabilitätsrahmenwerke, notwendig. Um mögliche Vorteile für die Versicherungsnehmer zu untersuchen, ist es sinnvoll, Steuerungsmöglichkeiten zur Überschussbeteiligung und Kapitalanlageallokation zu analysieren. Um schließlich Schlussfolgerungen für zu-
kunftsfähige Strategien zur Bestandsentwicklung ziehen zu können, sollten nicht nur Versicherungsportfolios mit bestehendem Geschäft betrachtet werden, sondern auch repräsentative Bestände, die Neugeschäft mit abbilden. Insgesamt werden daher folgende neue Fragestellungen in den drei Forschungsarbeiten thematisiert:

- Welche Funktionen haben Zinssätze und Zinsgarantien in traditionellen, überschussbeteiligten Verträgen und wie beeinflussen sie die Verzinsungsanforderung, die jährlich auf die Kapitalanlagen verdient werden muss? Wie können Produkte mit modifizierten Garantien gestaltet werden, sodass sie das Risiko für den Versicherer senken?

- Wie können Versicherungsverträge unter Berücksichtigung von Profitabilität und finanziellem Risiko für den Versicherer bewertet werden? Was ist eine sinnvolle Kennzahl für das Verhältnis der beiden Größen?

- Wie können auch Versicherungsnehmer von alternativen Produktgestaltungen profitieren? Wie können alternative Garantien mit ihrem risikoreduzierenden Potential dazu beitragen, attraktive Renditechancen für die Versicherungsnehmer zu erzeugen?

- Welche Aus- und Wechselwirkungen zeigen Produkte mit alternativen Garantien, die in ein bestehendes Portfolio mit traditionellen Verträgen eingebracht werden? Welche Schlussfolgerungen für Neugeschäftsstrategien können gezogen werden, und wie entwickelt sich die Risikosituation zukünftig in Abhängigkeit der Strategie?


samtleistung als die traditionellen Verträge; dieses Risiko ist aber aufgrund der anfangs garantierten Ablaufleistung für den Kunden begrenzt. Die alternativen Verträge können damit insgesamt eine Reduktion der Solvenzkapitalanforderung und interessante Chance-Risiko-Profile bieten. Die Forschungsarbeit umfasst auch Sensitivitätsanalysen mit unterschiedlichen Annahmen in der Kapitalanlageallokation bzw. den Kapitalmarktrenditen, welche mit den bisherigen Resultaten in Einklang stehen.


Die Ergebnisse stützen und erweitern die Erkenntnisse aus dem ersten Artikel, und zeigen, dass Produkte mit alternativen Garantien auch in einem Modell, welches das Marktumfeld der vergangenen Jahre wiederspiegelt, kapitaleffizienter sind, d.h. Profitabilität erhöhen und Kapitalanforderungen senken. Es zeigt sich aber, dass die Effekte der alternativen Verträge, die in den traditionellen Bestand eingebracht werden, auf die verschiedenen Profitabilitäts- und Risikokennzahlen mit unterschiedlicher Stärke und Geschwindigkeit wirken: So müssen alternative Garantien einige Jahre im Bestand sein, bis sich eine signifikante Reduktion der Solvenzkapitalanforderung ergibt.
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JOCHEN ALOIS GEORG WIELAND

DATE AND PLACE OF BIRTH
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EDUCATION

SINCE 03/2016  DEUTSCHE AKTUARVEREINIGUNG E.V. (DAV)
Membership of the German Actuarial Society „Aktuar (DAV)“

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Dissertation for a doctoral degree in economic sciences (Dr. rer. pol.)
- Subject: Product Design and Capital Efficiency in participating life insurance under risk
  based Solvency frameworks

08/2009 – 07/2010  SAN DIEGO STATE UNIVERSITY, USA
Studies in Applied Mathematics (M.Sc.)
- Final grade: GPA 4.0 (Scale: 4...0)
- Major fields of study: Financial mathematics and Statistics
- Scientific assistant and Master thesis in the valuation of weather derivatives applying
  stochastic processes and time series (Jump Diffusion, ARIMA, et al.)

Studies in Mathematics and Management (Dipl. math. oec.)
- Final grade: 1.0 (Scale: 1...5)
- Major fields of study: Actuarial and Financial mathematics
- Final thesis „Curve Fitting – Efficient methods for calculating Solvency Capital“ (Solvency
  II), awarded with the GAUSS-Nachwuchspreis 2012 of the Deutsche Gesellschaft für
  Versicherungs- und Finanzmathematik (DGVFM)
- Award for outstanding merits toward the Faculty of Mathematics and Economics, endowed
  by PricewaterhouseCoopers

High School (Abitur)
- Final grade: 1.2 (Scale: 1...6)
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  and for extraordinary social commitment
- First prize at the „Bundeswettbewerb Mathematik“
PROFESSIONAL EXPERIENCE

SINCE 12/2011  INSTITUT FÜR FINANZ- UND AKTUARWISSENSCHAFTEN (IFA) GMBH, ULM
Actuarial Consultant
- Actuarial consulting in life insurance for a large multinational insurance group
- Requirement analyses, actuarial and technical concepts for the product development
- Enhancing actuarial processes and technical routines in cooperation with IT, accounting and operating departments
- Analyses of the insurance portfolio (using SAS, SQL and VBA), and supporting system and data migrations
- Managing sub-projects, e.g. development and implementation of a machinable annual adjustment process of pension scheme contracts and of automated processes for validity checks

Intern and student trainee for actuarial consulting in risk management
- Projects on risk management requirements and preparation for Solvency II
- Methods for assessing risk, and implementation of a limit system (MaRisk)
- Valuations and developing tools for market risks, underwriting risks and reserves of life and non-life insurers along Solvency II specifications (QIS 5)

Visiting Associate in management consulting

08/2009 – 07/2010  SAN DIEGO STATE UNIVERSITY, USA
Teaching Associate for statistics classes

08/2008 – 09/2008  MERCER DEUTSCHLAND GMBH, DÜSSELDORF
Intern for actuarial consulting in pension management

02/2008 – 05/2008  AXA SERVICE AG, KÖLN
Intern for the actuarial controlling of life insurance products

Lecturing tutor for statistics classes

04/2005 – 08/2005  DAIMLER CHRYSLER AG, STUTTGART
Summer job in the development department of truck engines

07/2004 – 03/2005  MALTESER HILFDIENST GMBH, SCHWÄBISCH GMÜND
Civilian service for disabled people

09/2001 – 03/2005  FREELANCE
Private tutor in high school
ACADEMIC WORK

SCIENTIFIC PAPERS


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Ulm, June 2016

Jochen Alois Georg Wieland