High-Resolution Ultrasonic Sensing for Autonomous Mobile Systems

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Dirk Bank
aus Birkenfeld / Nahe

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Amtierender Dekan: Prof. Dr. H. Partsch
1. Gutachter: Prof. Dr. G. Palm
2. Gutachter: Prof. Dr. Dr. F. J. Radermacher
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New approaches in the research fields of ultrasonic sensing, environment mapping and self-localization, as well as fault detection, diagnosis, and recovery for autonomous mobile systems are presented.

A concept of high-resolution ultrasonic sensing by a multi-aural sensor configuration is proposed, which incorporates cross echoes between neighbor sensors as well as multiple echoes per sensor. For implementation, a novel ultrasonic sensing system has been developed. As a result, by a given number of sensors a significantly higher number of echoes can be utilized in comparison with conventional ultrasonic sensing systems for mobile robots.

In order to benefit from the increased sensor information, algorithms for adequate sensor data processing and sensor data fusion have been developed. In this context, it is described how local environment models can be created at different robot locations by extracting geometric primitives from laser range finder data and from ultrasonic sensor data. Two new algorithms, called parameter space clustering and tangential regression, have been developed for deriving local environment models from ultrasonic sensor data. Additionally, the application of an extended Kalman filter to mobile robot self-localization based on previously modeled and newly extracted geometric primitives is explained. Furthermore, it is demonstrated how local environment models can be merged for building a global environment map. Fusion methods have been developed for spatially integrating geometric primitives extracted at different robot locations in order to create detailed maps of complex environments.

As a supplement for monitoring the state of environmental sensors, a fault detection model has been developed, which consists of sub-models for data from laser range finders and ultrasonic sensors. A two-step fault detection method compares in the first step real with simulated distance readings, and in the second step real distance readings among one another. Based on these two steps, reliable overall sensor assessment can be performed.
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1 Introduction

This monograph presents an approach to safe navigation of autonomous mobile systems within partially known or unknown dynamic environments. Considering mobile robot navigation, the complete contact-less sensory coverage of the workspace represents a fundamental difficulty, i.e. many objects in real environments like homes, industrial plants, or concourses cannot be reliably detected by the commonly employed distance sensing systems (e.g. mirrors, panes, shiny metal surfaces, table edges, fences, clotheslines, stair-steps, show-cases, shelves, or racks). In such cases, tactile sensors have to serve for safety deactivation, whereas they often also cannot cover the whole potential collision range.

The limited perceptive faculty of mobile robots may be regarded as a major reason why service robots have only partially entered private and public surroundings so far. Objects that cannot be reliably detected reduce the performance of service robots and increase the potential risk involved with their operation. Consequently, the necessity results to achieve robust distance sensing according to the intended operation environments.

To overcome the perception problem, several sensor systems relying on physically different measurement principles and having various properties can be used in combination. For instance, ultrasonic sensors cannot receive echoes from smooth surfaces at unfavorable detection angles and from edges of very small radii. Optical sensors (e.g. laser range finders, infrared sensors, or video systems) are unable to properly detect glass or mirrors. Moreover, laser range finders cannot observe objects below and above the scanning level, infrared sensors provide only punctual distance information, and video systems require suitable light conditions. Additionally, optical sensors have disadvantages compared with radar sensors in adverse weather and environmental conditions (e.g. rain or dust). Nevertheless, radar sensors show deficiencies compared with ultrasonic sensors in detecting plastic surfaces, since plastic provides low reflectivity for radio frequency energy and high reflectivity for acoustical energy, while metal surfaces reflect both radio frequency and acoustical energy well. Finally, different kinds of sensors have different accuracies, detection angles, and measurement ranges.
Sensor redundancy as well as spatial redundancy can be exploited for object perception by performing appropriate sensor data processing and sensor data fusion. Although some materials and surfaces are difficult to detect with certain sensors and from certain perspectives, various classes of objects (e.g. smooth, rough, reflecting, absorbing, transparent, or opaque objects) can be detected with diverse sensors and from diverse perspectives.

One objective of this dissertation is to improve the perception of objects with sonar sensors in environments consisting of various materials and surfaces that are partly difficult to detect. The capability of detecting glass and mirrors is an essential advantage of ultrasonic sensors compared with optical sensors. Additionally, the broad beam width of ultrasonic sensors in comparison with the narrow beam width of laser range finders or infrared sensors represents an important spatial characteristic concerning three-dimensional environment coverage. Thus, although laser range finders have become the favorite sensors in robotics, ultrasonic sensors are still indispensable as a complement for safety reasons.

Further subjects of the dissertation are environment mapping and self-localization, which are fundamental requirements for mobile robots with regard to navigation purposes such as path planning and trajectory execution. On the one hand, the location of the robot must be accurately known to update the environment model exactly, and on the other hand, the environment must be exactly modeled to estimate the robot location accurately. For this reason, the tasks of environment mapping and self-localization must be solved concurrently.

Another concern of the dissertation are failures or impairments of sensor systems. For instance, it frequently occurs that individual sensors are covered or otherwise influenced (e.g. by interferences from other sensing systems or disturbing ultrasonic noise). Since collision-free navigation of autonomous mobile systems within the above mentioned environments is required, faulty, maladjusted, covered, and otherwise impaired sensors should be recognized, and adequate measures for failure correction should be applied.

The research fields of the dissertation, which comprise ultrasonic sensing, environment mapping and self-localization, as well as fault detection, diagnosis, and recovery, are introduced in Section 1.1, Section 1.2, as well as Section 1.3, respectively. The robot platforms on which the developed algorithms have been implemented are explained in Section 1.4, and the evaluation environment in which the novel approaches have been verified is described in Section 1.5. An outline of the dissertation is given in Section 1.6.
1.1 Ultrasonic Sensing

In Chapter 3, a newly developed ultrasonic sensing system for autonomous mobile systems will be presented. It will also be described how wide-angled ultrasonic transducers can be used to obtain substantial information of the environment. This can be achieved by exploiting the overlapping of detection cones from neighbor sensors and by receiving cross echoes between them. The novel ultrasonic sensing system allows the detection of multiple echoes from different echo paths for each sensor. In this way, a significantly higher number of echoes can be obtained in comparison with conventional ultrasonic sensing systems for mobile robots.

1.2 Environment Mapping and Self-Localization

In Chapter 4, it will be described how local environment models can be created at different robot locations by extracting straight line segments from laser range finder data and from ultrasonic sensor data. In Chapter 5, mobile robot self-localization based on previously modeled and newly extracted straight line segments will be explained in order to accurately project newly extracted straight line segments into the world coordinate system. In Chapter 6, it will be demonstrated how local environment models created at different robot locations can be merged for building a global environment map. Figure 1.1 illustrates the interaction of the developed environment mapping and self-localization methods.
Creating local environment models at different robot locations by extracting straight line segments from laser range finder data (Section 4.2)

Creating local environment models at different robot locations by extracting straight line segments from ultrasonic sensor data (Section 4.3)

Parameter Space Clustering (Subsection 4.3.1)

Tangential Regression (Subsection 4.3.2)

Mobile robot self-localization by multi-target tracking based on previously modeled and newly extracted straight line segments (Section 5.3)

Building a global environment map from laser range finder data and from ultrasonic sensor data by fusing straight line segments extracted at different robot locations (Section 6.4)

Clustering Approach (Subsection 6.4.1)

Regression Approach (Subsection 6.4.2)

Figure 1.1: Interaction of the developed environment mapping and self-localization methods
1.3 Fault Detection, Diagnosis, and Recovery

In Chapter 7, fault diagnosis for the perceptual system of mobile robots will be covered. As general basis for monitoring the state of environmental sensors, a fault detection model has been developed, which consists of sub-models for data from laser range finders and ultrasonic sensors. A two-step fault detection method compares in the first step real with simulated distance readings, and in the second step real distance readings among one another. Figure 1.2 demonstrates the operation of the developed two-step fault detection method.

![Operation of the developed two-step fault detection method](image)

**Figure 1.2:** Operation of the developed two-step fault detection method

1.4 Robot Platforms

The developed algorithms have been implemented on an intelligent wheelchair (MAid) developed at FAW as well as on an experimental robot Nomad XR4000 from Nomadic Technologies. MAid (mobility aid for elderly and disabled people) is designed to autonomously or semi-autonomously transport people through private and public surroundings [137, 138, 139]. The experimental robot Nomad XR4000 at FAW serves as a platform to simulate an automatically guided hospital bed (AutoBed) [126, 127]. For applications with a person on board of an autonomous mobile system, safe navigation is particularly important. Within this monograph, the developed algorithms, which improve navigation safety, will be described for the circular sensor arrangement on the experimental robot Nomad XR4000.
The intelligent wheelchair (Figure 1.3, Figure 1.4) represents a transportation platform for elderly and disabled people with severely impaired motion skills and insufficient fine motor manipulations. The robotic vehicle is based on a commercially available wheelchair, which has been equipped with an intelligent control and navigation system. MAid has three modes of operation. In the first mode, it can autonomously navigate within narrow environments such as private homes. The second mode allows deliberative autonomous navigation in rapidly changing public surroundings, such as pedestrian areas, shopping malls, and railway stations. Additionally, the robotic vehicle can be used as a passenger transportation system for exhibitions, airports, and parks. In the third mode, the robotic wheelchair can accompany a person through crowded concourses. MAid’s hardware design and control architecture as well as the navigation in dynamic environments are explained in [137]. The motion coordination between a human and a mobile robot is described in [138, 139].

Figure 1.3: Intelligent wheelchair (MAid)
Figure 1.4: MAid navigating through crowded public surroundings
The study of an automatically guided hospital bed (Figure 1.5, Figure 1.6) demonstrates another application of service robots in health care. AutoBed’s hardware consists of a commercial robot, equipped with two laser range finders and a bed frame mounted on top of the robot. It is shown in [126, 127] how AutoBed can find its way in partially known environments by exactly determining its position and orientation, planning trajectories from an arbitrary start location to any desired goal location in these environments, executing planned paths, and avoiding obstacles possibly blocking the way. The holonomic drive system of the used platform enhances the accomplishments to navigate precisely with a bulky bed frame in very narrow environments such as small elevators and rooms. AutoBed is an initial step towards the development of a mobile platform for transporting existent sick beds in hospitals.

Figure 1.5: Simulation of an automatically guided hospital bed (AutoBed)
For evaluation purposes, both platforms have been equipped with a conventional ultrasonic sensing system utilizing sensors from Polaroid Corporation as well as with the newly developed ultrasonic sensing system using wide-angled sensors from Robert Bosch GmbH.

Figure 1.6: AutoBed navigating into an elevator
1.5 Evaluation Environment

The novel approaches have been verified in a hallway and in a room of a typical office environment. The experimental results obtained in the office room will be demonstrated in detail within the dissertation. Figure 1.7 and Figure 1.8 display photos of the office room serving as evaluation environment. The photos show the same scene from opposite points of view. The office room contains various kinds of objects that are partly difficult to detect either by laser range finders or by ultrasonic sensors (e.g. desks, chairs, racks, window-glass, mirror, movable wall with ingrain wallpaper, and movable wall of styrofoam).

![Figure 1.7: Evaluation environment (view in positive y-direction)](image)
Figure 1.8: Evaluation environment (view in negative y-direction)

Figure 1.9 presents a detailed reference map of the evaluation environment. The numbered crosses in the reference map mark a path of 17 robot locations with distances of 0.25 m, 0.30 m, or 0.50 m to each other. The robot has been controlled to these locations in order to model the environment with the aid of laser range finders and ultrasonic sensors.
Figure 1.9: Reference map of the evaluation environment
1.6 Outline of the Dissertation

The state of the art in the research fields of the dissertation is explicated in Chapter 2. The novel ultrasonic sensing system for autonomous mobile systems is presented in Chapter 3. Local environment modeling by extracting straight line segments is described in Chapter 4. Mobile robot self-localization based on straight line segments is explained in Chapter 5. Global environment mapping by merging local environment models is demonstrated in Chapter 6. Fault diagnosis for the perceptual system of mobile robots is covered in Chapter 7. The conclusion of this research work is provided in Chapter 8.
2 State of the Art

Within this chapter, the state of the art in the research fields of the dissertation is explicated. The review of related work is divided into three sections, namely ultrasonic sensing in Section 2.1, environment mapping and self-localization in Section 2.2, as well as fault detection, diagnosis, and recovery in Section 2.3.

2.1 Ultrasonic Sensing

Sensors for mobile robots can be categorized into “internal sensors” (e.g. wheel encoders, gyroscope) for measuring the internal parameters of the robot, and “external sensors” (e.g. ultrasonic sensors, laser range finders, infrared sensors, vision) for perceiving the external environment of the robot. Internal sensors serve to compute the robot’s motion by dead-reckoning, and external sensors measure the positions of objects with respect to the robot. Information from both categories of sensors can be combined to estimate the location of the robot within the environment and to create a map of the environment.

Range sensors are based on two physical principles: time of flight and triangulation. Time of flight sensors emit a pulse (sound wave or electromagnetic wave) and measure the time between the transmission of the pulse and the reception of a possible echo from an object. Triangulation sensors measure range by geometric means, which requires the detection of an object from two different viewpoints with known distance from each other.

2.1.1 Reflection Properties of Sound and Light

Sound waves differ from electromagnetic waves in three important physical characteristics, namely the medium, the velocity, and the wavelength [114]. Sound waves in contrast to electromagnetic waves require a medium, such as air or water, for transmission. The velocity of sound is much lower than the velocity of light, which allows on the one hand a simple
measurement process with sound waves and on the other hand a short measurement period with electromagnetic waves. Most planar objects in man-made environments reflect sound specularly and light diffusely due to differences in wavelength.

Acoustical and optical waves striking an object will be subject to reflection, absorption, or transmission. When an acoustical wave is incident on the boundary between two media, a portion of the wave undergoes reflection and a portion of the wave crosses the boundary. The degree of reflection depends on the difference in acoustic impedance (impedance mismatch) between the two media. Hard surfaces predominantly appear as acoustic reflectors, since the acoustic impedance of air is quite low in comparison with typical acoustic impedances of solid objects. Attenuation (i.e., absorption or transmission loss) occurs when sound energy is converted into vibrational energy within an object (e.g., in a soft material). Acoustical energy which is not reflected or absorbed, is transmitted through the object. When a light wave of a given frequency hits a material with electrons having the same vibrational frequencies, then those electrons absorb the energy of the light wave by converting it into thermal energy. Reflection or transmission of light waves occurs when the frequencies of the light waves do not match the vibrational frequencies of the material. In this case, if the material is opaque, the energy is re-emitted on the same side of the object as a reflected light wave, or if the material is transparent, the energy is emitted on the opposite side of the object as a transmitted light wave. For the explained reasons, acoustical sensors show deficiencies in detecting sound absorbing or transmitting objects (e.g., soft materials, clothes, curtains), while optical sensors have disadvantages in detecting light absorbing or transmitting objects (e.g., dark materials, glass, perspex).

Whether reflections occur specularly or diffusely depends on the extent of wave interference due to phase differences caused by irregularities of the reflecting surface. The phase differences can vary between $\Delta\phi = 0$ for smooth surfaces and $\Delta\phi = \pi$ for rough surfaces. The same surface may appear smooth for some wavelengths and rough for others, or for constant wavelength it may appear either smooth or rough for different angles of incidence [104].

The wavelength of sound waves is much larger than the roughness of most indoor surfaces (e.g., $\lambda = 6.872$ mm at $f = 50$ kHz) [114]. As a result, an ultrasonic beam is reflected on a smooth surface with an angle of reflection equal to the angle of incidence. This unidirectional reflection is called specular reflection. In contrast, when the beam falls on a rough surface the energy is scattered in various directions. This multidirectional reflection is called diffuse reflection.
The described effect becomes obvious at optical wavelengths when the beam of a flashlight is pointed towards a mirror at a non-perpendicular angle. There will be no illumination visible on the mirror surface itself, because the light energy will be specularly reflected away. If the flashlight is deflected by the mirror towards a nearby wall, the flashlight spot will be visible on the wall surface. Thus, for optical energy the mirror is a specular reflector and the wall is a diffuse reflector [110].

Specular reflection will only occur if the average depth of the surface irregularities is substantially less than the wavelength of the incident beam. Also, the transverse dimensions of the reflecting surface must be substantially larger than the wavelength of the incident beam. For this reason, a smooth round pole will produce diffuse reflection, particularly when the diameter of the pole is less than the beam width [114].

Generally, acoustical and optical waves specularly reflected from planar surfaces at non-perpendicular angles propagate away from the emitting sensor. Such energy may only return to the point of emission via a path of multiple reflections, i.e. an acoustical or optical sensor would observe a mirrored image. Diffuse reflections of both kinds of waves can be detected by the emitting sensor if the returned energy is sufficiently high. Considering the majority of planar objects in man-made environments, which reflect sound specularly and light diffusely, ultrasonic sensors receive the specular reflections of sound only from a perpendicular perspective, while laser sensors receive the diffuse reflections of light from various perspectives.

### 2.1.2 Echolocation

“Sound navigation and ranging” (sonar) utilizes the principle of echolocation. The distance between a sonar sensor and a reflecting object can be calculated by multiplying the speed of sound in air by the measured time of flight (TOF) of a short ultrasonic pulse traveling to the object and its echo traveling back to the sensor:

\[ d = \frac{1}{2} c_{\text{sound}} \Delta t \]  

\( c_{\text{sound}} \) = speed of sound in air  
\( \Delta t \) = time of flight (TOF)  
\( d \) = object distance
2.1 Ultrasonic Sensing

The speed of sound in air is temperature dependent and can be calculated according to the following formula [111]:

\[
c_{\text{sound}} = \sqrt{\kappa RT}
\]  

(2.2)

\(\kappa\) = adiabatic exponent \((\kappa_{\text{air}} = 1.402)\)

\(R\) = gas constant \((R_{\text{air}} = 287 \text{ J/KgK} = 287 \text{ m}^2/\text{s}^2\text{K})\)

\(T\) = absolute temperature of the gas

\((T_{\text{air}} = T_0 + t_{\text{air}} = 273.15 \text{ K} + 20^\circ \text{C} = 293.15 \text{ K})\)

2.1.3 Ultrasonic Sensor Model

While there are significant differences between sound waves and electromagnetic waves, the modeling of them is similar [114]. Ultrasonic transducers can be modeled as a plane circular piston set in an infinite baffle, yielding the following radiation characteristic function [112, 115]:

\[
P(\theta) = \frac{2J_1(kr \sin(\theta))}{kr \sin(\theta)}
\]  

(2.3)

\(J_1\) = Bessel function of the first kind and first order

\(k\) = wave number \((k = 2\pi f/c_{\text{sound}} = 2\pi/\lambda)\)

\(r\) = radius of the ultrasonic transducer

\(\theta\) = azimuth angle with respect to the transducer axis

\(P(\theta)\) = radiation characteristic function of the ultrasonic transducer

By increasing the ratio of the transducer diameter to the ultrasonic wavelength, the radiation of the aperture can be changed from a spherical intensity distribution to a directed conical beam with side lobes, similar to a radio frequency beam. Thus, the width of the beam is determined by the transducer diameter and the operating frequency.

2.1.4 Beam Formation

Ultrasonic transducers are designed to concentrate most of the energy in a main lobe in order to increase the range and lateral resolution of the echo-location system. While the transducer still emits a spherical wave front, the
intensity varies around the wave front as a function of the polar angle (with respect to the beam axis). This intensity variation can be modeled with a Bessel function of the first kind and first order (cp. Eq. 2.3). The variation in intensity is caused by interference between the waves generated by different parts of the transducer surface. Intensity maxima occur in directions where the interference is constructive, and minima occur at angles relative to the acoustic axis for which waves from opposite points of the transducer surface are 180° out of phase and thus destructively interfere. Between the minima the interference varies resulting in a beam shape [19].

The angles at which the minima in the radiation characteristic occur are defined by the zeroes of the Bessel function:

\[ J_1(kr \sin(\theta)) = 0 \quad (2.4) \]

Solving Eq. 2.4 for the first zero of the Bessel function yields the angle of the first intensity minimum in the radiation characteristic:

\[ J_1(3.832) = 0 \quad (2.5) \]

\[ kr \sin(\theta_1) = \frac{2\pi}{\lambda} r \sin(\theta_1) = 3.832 \quad (2.6) \]

\[ \theta_1 = \sin^{-1}\left(\frac{0.61k}{r}\right) \quad (2.7) \]

\( \lambda \) = wavelength of the sound wave
\( \theta_1 \) = angle of the first intensity minimum

Due to similar interference effects, the receive sensitivity of the transducer is almost identical to the transmit characteristic. Consequently, a high energy signal coming from the direction of an intensity minimum may produce very little response in the transducer and may not be detected [19].

The beam width of a transducer can be defined as the angle between the two points at which the sound power has been reduced to half of the peak value (-3 dB). However, what is generally of more concern are the beam constraints within which objects can be reliably detected. This effective beam width depends on the characteristics of the object (size, shape, orien-
Ultrasonic Sensing

The main lobe can be regarded as a beam of waves radiating out from the transducer. In the near field, the main intensity fluctuates rapidly, and peaks at a distance $l$ from the transducer. Within this region, the beam is cylindrical. Within a transition region between $l$ and $2l$ the beam changes from cylindrical to conical. For distances greater $2l$, the beam is conical and the intensity decreases inversely with the square of distance. The point of transition $l$ from the near field (Fresnel region) to the far field (Fraunhofer region) can be calculated [114]:

$$l = \frac{r^2}{\lambda} - \frac{\lambda}{4}$$

$I = \text{length of the near field}$

2.1.5 Sound Propagation

The intensity of a propagating sound pulse decreases due to atmospheric attenuation, spherical divergence, and surface reflection.

Atmospheric Attenuation

As an acoustical wave travels away from its source, there is an exponential loss associated with molecular absorption of sound energy by the medium [110]:

$$I = I_0 e^{-2\alpha R}$$

$I_0 = \text{maximum (initial) intensity}$
$\alpha = \text{attenuation coefficient for the medium}$
$R = \text{traveled distance}$
$I = \text{intensity (power per unit area) at distance R}$

The value of $\alpha$ varies slightly with the humidity and dust content of the air and is also a function of the operating frequency (transmissions of higher frequencies attenuate faster). The maximum detection range for an ultrasonic sensor is thus dependent on both the emitted power and the frequency of operation.
**Spherical Divergence**

As the waves are spherical, the surface area of the wave front is proportional to the square of the distance from the point source. Consequently, the signal power decreases according to the inverse square of distance [19, 110]:

\[
I = \frac{I_0}{4\pi R^2} \quad (2.10)
\]

Combining the attenuation through molecular absorption and spherical divergence results in the following equation for intensity as a function of distance \( R \) from the source [110]:

\[
I = \frac{I_0 e^{-2\alpha R}}{4\pi R^2} \quad (2.11)
\]

**Surface Reflection**

If the wave is totally reflected by a purely specular surface, the beam spread and thus the equation for intensity remain the same. In contrast, if an object scatters the signal randomly, the emanating echo forms a new spherical wave that again dissipates in accordance with the inverse square of distance, and the equation for intensity becomes [19, 110]:

\[
I = \frac{I_0 e^{-2\alpha R}}{16\pi^2 R^4} \quad (2.12)
\]

Considering that all energy incident upon a target object is either reflected, absorbed, or transmitted, be it acoustical, optical, or radio frequency in nature, a reflection coefficient can be introduced to account for the reflectivity of the target. However, the amount of energy returning to a transducer also depends on the directivity of the target surface.

**2.1.6 Ultrasonic Sensor Technologies**

Two different types of ultrasonic transducers are commonly used in mobile robotics, electrostatic and piezoceramic (also known as piezoelectric).
Pulses of ultrasonic energy are generated by vibration of a membrane or a piezoelectric crystal. Electrostatic transducers transmit an outgoing signal and act as an electrostatic microphone in order to receive the reflected waveform. Piezoceramic transducers are electrically similar to quartz crystals and resonant at only two frequencies: the resonant and antiresonant frequencies. Transmission is most effective at the resonant frequency while optimum receiving sensitivity occurs at the antiresonant frequency. Electrostatic transducers generate small forces but have a fairly large displacement amplitude, and therefore couple more efficiently to a compressible medium such as air. Piezoelectric crystals change dimension under the influence of an external electrical potential and begin to vibrate if the applied potential is made to oscillate at the crystal’s resonant frequency. While the force generated can be significant, the displacement of the oscillations is typically very small, and so piezoelectric transducers tend to couple well to solids and liquids but rather poorly to low-density compressible media such as air [110].

2.1.7 Review of Related Work

Kuc from Yale University and his associates have a long research record in ultrasonic object localization and shape recognition. In a well-known paper from 1987, Kuc and Siegel [16] described a physically based simulation model that combines concepts from the fields of acoustic, linear system theory, and digital signal processing. By virtually separating the transducer for analytical purposes into transmitter and receiver apertures, and by assuming mirror-like reflectors, closed-form solutions for reflections from planes, corners, and edges are determined as a function of transducer size, position, and orientation. Kuc and Siegel conclude that from a single reflected signal it is impossible to differentiate planes, corners, and edges, since the forms of their impulse responses are identical.

Barshan and Kuc [3] described in 1990 a two-transducer system that differentiates reflections from planes and right-angled corners by exploiting the physical properties of sound propagation, and by processing the amplitudes and ranges of reflected signals for the different transmitter and receiver pairs. Reflections from edges are not considered in this paper.

Edges are only detectable at close range, since reflection amplitudes from edges are due to diffraction much smaller than those from planes or corners. For this reason, Kuc [17] described in 1990 a spatial sampling criterion to determine the sonar scanning density required to detect weak echoes from edge-like reflectors, which therefore assures that all possible obstacles can be detected.
Bozma and Kuc [6] described in 1991 a differentiation procedure for planes, corners, and edges using the physical properties of reflection and diffraction. Edges can be differentiated from planes and corners by the angular extent of reflector visibility when scanning from a single position, whereas planes and corners can be differentiated by scanning from two separate positions.

Barshan and Kuc [4] described in 1992 a “bat-like” three-transducer array (one transmitter and two receivers) for estimating range and azimuth of an obstacle by calculating the intersection of two ellipses.

Bozma and Kuc [7] presented in 1994 a physical model-based procedure of processing echo energy, echo duration, and range measurements by template matching for interpreting sonar data and identifying the roughness, position, and orientation of a reflecting surface.

Kleeman and Kuc [13] presented in 1995 a processing approach for echo shapes obtained with a sonar array consisting of two transmitters and two receivers. Without sensor movement, planes, corners, and edges are statistically classified by maximum likelihood estimators.

McKerrow and Hallam [19] from the University of Wollongong and the University of Edinburgh presented in 1990 an introduction to the physics of echolocation. The paper describes sound propagation, beam forming, echo formation, visibility criteria, feature extraction, ultrasonic scanning, and sensor rings. The authors conclude that environment mapping with ultrasonic sensors involves robot motion, as motion provides the additional information required to define the shape and extent of surfaces.

Borenstein and Koren [5] from the University of Michigan introduced in 1992 a method for firing multiple ultrasonic sensors at high rates and reducing the number of erroneous readings due to environmental ultrasonic noise, ultrasonic noise from other mobile robots operating in the same environment, and crosstalk between onboard ultrasonic sensors. The described method is based on comparison of consecutive readings as well as employment of alternating delays before firing each sensor. The latter measure artificially creates differences between consecutive crosstalk readings, while leaving direct readings unaltered.

Nagashima and Yuta [20] at the University of Tsukuba proposed in 1992 a linear sensor arrangement of one transmitter between two receivers to estimate the distance and normal direction of walls by measuring the difference in TOF from the transmitter to both receivers. In order to cover a wide range of possible normal directions, wide beam width transducers
are used. By detecting surface elements at many different robot positions a map can be constructed.


Radar techniques (coded waveform, matched filter, peak detector) are used to determine the arrival times of multiple, possibly overlapping echoes by processing the entire received signal. The triaural sensor system consists of one transmitter and three receivers (one transceiver and two additional receivers). By triangulation, the position (distance and bearing) of reflecting objects can be determined. The triangulation approach allows the usage of transducers with a broad beam width to increase the angular field of view and at the same time to achieve high angular accuracy. With the aid of the sensor system, reflectors can also be discriminated into planes, corners, and edges.

Finding a circle tangent to three given measurement circles defines the reflector’s position and its radius. The determined radius is the inverse of the curvature of the reflector. Curvature information can be used to discriminate between different types of reflectors, in particular between planes and edges \( r_c = \infty \) for a plane and \( r_c = 0 \) for an edge. Corners cannot be distinguished from planes basing the decision solely on arrival times. However, by moving the triaural sensor system and by combining measurements from different positions, it becomes possible to discriminate (based on arrival times) between planes and corners. Introducing motion has the additional benefit of making reflector type distinction between planes and edges more robust.

Wilkes et al. [29] from the University of Toronto, McGill University, and York University presented in 1993 an algorithm that uses multiple peaks in the return signals from several transducers with broad beams and overlapping fields of view. Cross echoes from neighbor sensors are not exploited. A grid-based representation and a Bayesian update scheme are used to integrate the multiple return signals from multiple transducers and multiple robot positions.

Sabatini and Di Benedetto [25], and Sabatini [26] at Pisa Scuola Superiore Sant’Anna proposed in 1994 a linear array of three sequentially fired ultrasonic transducers for performing a full scan of the environment. A
geometric relocation approach based on an extended Kalman filter is adopted for estimating the geometrical parameters (distance, direction, and radius) of generalized cylindrical targets (including cylinders with infinite radius and zero radius).

Sabatini and Spinielli [27] described in 1994 a correlation-based pulse-echo ranging system with completely digital signal processing in the receiver. The usage of appropriate sampling techniques and signal processing algorithms allows to benefit from the advantage of correlation-based detection methods in detecting weak echo signals and accurate ranging of multiple objects.

Hanebeck and Schmidt [9] at the Technical University Munich presented in 1994 an actively sensing multi-sonar system composed of 24 transmitter and receiver elements arranged on a horizontal plane around a mobile robot. The sensor elements can be electronically configured to sensor arrays. By appropriately phasing adjacent physical transmitters, a virtual point source in the center of the robot can be formed. Assuming that there is only one single pulse stemming from the origin, it is not necessary to decide which physical transmitter produced the pulse detected by an individual receiver.

Hanebeck [10] proposed in 1998 fast sampling strategies for pulse-echo sonar tracking systems. Using time multiplexing, several measurement streams are interlaced in such a way that additional pulses are already transmitted while other measurement sequences are still in progress. Thus, the proposed scheme of pipelined sampling makes maximum use of the available channel capacity.

Lawitzky et al. [18] from Siemens AG compared in 1995 monaural (one transmitter, one receiver), binaural (one transmitter, two receivers), and triaural (one transmitter, three receivers) sensor system configurations. The authors state that binaural or triaural sensing allows to get more information from a single measurement by improving the angular resolution or distinguishing the object type. For this reason, they conclude that these principles have a large potential for increasing speed and precision of environment mapping for obstacle avoidance and navigation.

Jörg and Berg [11, 12] from the University of Kaiserslautern presented in 1996 and 1998 a method to eliminate misreadings caused through crosstalk or external ultrasound sources by applying pseudo-random sequences and a matched filter technique. The approach allows to fire multiple sonar sensors simultaneously by emitting random noise, i.e. the transmitted burst of each sonar sensor is a pseudo-random sequence. In case the
sonar sensors are fired in parallel, the received signals may be a superposition of multiple echoes. However, if each of the transmitted pseudo-random sequences has a sharp auto-correlation function and if they are not cross-correlated, then the individual echo of each sonar sensor can be identified by applying a matched filter technique. Polaroid series transducers are used and crosstalk can be either eliminated or exploited to perform triangulation.

Politis and Probert [24] from the University of Oxford described in 1998 the usage of a continuous transmission frequency modulated (CTFM) sonar to obtain information about the location and type (planes, corners, edges) of reflectors. CTFM sonars emit energy continuously and sweep over a range of frequencies. The algorithms exploit both range and amplitude information.

Wirnitzer et al. [30] from Mannheim University of Applied Sciences and Robert Bosch GmbH presented in 1998 a method for interference canceling by stochastic coding of the transmitted signals and adaptive filtering of the received signals to avoid disturbances between sensors within the same array and between different ultrasonic sensor systems. The target application is an automotive low-range detection system such as a car parking assistant. Wide-angled sensors are employed and cross echoes are explicitly used. An array of several sensors allows, additional to distance measurements, to localize an obstacle inside the operation range of at least two sensors of the array. Additional shape information can be obtained if the sensors operate in the cross echo mode.

Schmidt et al. [28] from Robert Bosch GmbH described in 1999 based on the above described ultrasonic sensor system an approach to classify objects into planar and cylindrical shapes.

Di Miro et al. [8] from Mannheim University of Applied Sciences and Robert Bosch GmbH presented in 2002 an extension of the pervious work by an object tracking algorithm and by automatic calibration of sensor positions.

Yata et al. [31] from the University of Tsukuba described in 1999 the design of a sonar ring system with wide directivity transducers for simultaneous pulse transmissions in all directions, which corresponds to a transmission from a single point source. Distances and bearing angles of reflecting objects can be calculated by using time of flight differences between neighboring receivers. The authors state that echo overlapping causes hiding of nearby reflecting points, which makes it difficult to apply the system to complicated environments.
Kleeman [14, 15] at Monash University presented in 1999 and 2001 an approach of interference rejection between different sensors based on encoding transmission signals by sending double pulses with time separations unique to particular transmitters. Thus, received echo signals can be identified by delayed pulse subtraction.

Ait Oufroukh et al. [1] from the University of Evry and I3S-CNRS compared in 2002 various classification methods (k-nearest neighbors, linear discriminant analysis, quadratic discriminant analysis, Parzen window, and neural network) for discriminating objects (planes, corners, edges, and cylinders).

2.2 Environment Mapping and Self-Localization

Environment mapping and self-localization are fundamental requirements for mobile robots in order to be able to perform productive assignments. Environment mapping is the process of building a model of the environment that represents objects and free space. Self-localization is the procedure of estimating the location (position and orientation) of the robot within the environment. On the one hand, the location of the robot must be accurately known to update the environment model exactly, and on the other hand, the environment must be exactly modeled to estimate the robot location accurately. For this reason, the tasks of environment mapping and self-localization must be solved concurrently. Concepts of “simultaneous localization and mapping” [57, 67] are widely known as SLAM.

2.2.1 Common Map Representations

The following map representations are commonly used for environment mapping and self-localization with ultrasonic range data:

- Grid-based maps
- Feature-based maps
- Topological maps

Grid-based maps are spatially discretized environment models (occupancy grids, certainty grids) which are constructed by projecting sensor readings into grid cells. The sensor readings are modeled as probability profiles in order to represent the uncertainty in the spatial information about the existence of objects at individual grid points. Feature-based maps
(e.g. line segment maps, polygonal maps) describe the environment by geometric primitives (e.g. line segments). The geometric primitives are extracted from the sensor data and composed to an environment model. Updates of grid-based maps and feature-based maps rely on estimates of the vehicle location in the environment in order to model objects at their absolute positions. Topological maps record the relative geometric relationships between observed features (landmarks) rather than their absolute positions. Consequently, this approach allows integration of large area maps without degradation through accumulated odometric location errors. The resulting representation takes the form of a graph where the nodes represent the observed landmarks and the edges represent the relationships between the landmarks.

### 2.2.2 Map-Based Localization

Local grid maps or local feature maps derived from sensor data can be matched against a global map of the environment in order to compute the position and orientation of the robot. The global map may be a priori provided as a floor plan or can be previously constructed from sensor data. The recognition problem in a topological map can be reformulated as a graph-matching problem where the objective is to find a set of landmarks in the relational map such that the relationships between these landmarks match the relationships between the observed landmarks.

### 2.2.3 Sensor Data Fusion

Since a real environment cannot be adequately perceived with a single sensor modality, information from various kinds of sensors must be combined by multi-sensor fusion. Thus, the basic problem regarding multi-sensor systems is to integrate a sequence of observations from a number of different sensors into a single estimate of the state of the environment. The general estimation problem is well known and many powerful techniques exist for its solution under various conditions [109].

The fusion of data from multiple sensors or a single sensor over time can be performed at the signal, pixel, feature, and symbol level of representation. Most of the sensors typically employed in practice provide data that can be fused at one or more of these levels. The different levels can provide information to a system for a variety of purposes. Signal level fusion can be used in real-time applications and can be considered as an additional step in the overall processing of the signals. Pixel level fusion can improve the performance of many image processing tasks such as segmen-
2 State of the Art

2.2.4 The Kalman Filter

The most widely employed method for sensor fusion and location estimation in mobile robot applications is the Kalman filter. The linear Kalman filter provides a statistically optimal estimate for the fused data, if the system can be described with a linear model and the system error as well as the sensor error can be modeled as white, Gaussian noise.

The Kalman filter is calculated recursively, i.e. in each iteration step only the newest measurement and the latest estimate are used. Thus, it is not necessary to store previous measurements and estimates. Measurements from a number of sensors can be fused by a Kalman filter to provide an estimate of the current state of a system and a prediction of the future state of the system.

The inputs to a Kalman filter are the measurements, the a priori information are the system dynamics and noise properties of system and sensors, and the outputs are the innovation as well as the estimated system state. The innovation (measurement residual) is the difference between a predicted and an observed measurement, by which the performance of the Kalman filter may be quantified.

At each step, the Kalman filter generates a state estimate by computing a weighted average of the predicted state (obtained from the system model) and the innovation. The weight used for computing the weighted average is determined by the covariance matrix, which is a direct indication of the error in state estimation.

The extended Kalman filter is used instead of the conventional (linear) Kalman filter if the system model is potentially numerically instable or if the system model is not approximately linear. The extended Kalman filter is a version of the Kalman filter that can handle nonlinear dynamics and nonlinear measurement equations [107].

The original approach by Kalman to linear filtering and prediction problems is formulated in [120]. A historical survey on least squares estimation theory from the Gaussian concept to the Kalman filter is provided in [123]. A detailed explanation of the Kalman filter can be found in a well-known book by Maybeck about stochastic models, estimation, and control [113]. The Kalman filter description by Bar-Shalom and Fortmann in their book on tracking and data association [102] is often referred to in the context of location estimation for autonomous mobile systems.
2.2 Environment Mapping and Self-Localization

2.2.5 Bayesian Estimation

Bayesian estimation provides a formalism for multi-sensor fusion that allows redundant information from a number of sensors to be combined according to the rules of probability theory. The description of Bayesian estimation presented within this subsection follows the approach of estimating an occupancy grid as presented in [47]. A comprehensive explanation of Bayesian analysis can be found in [106].

**Distance Estimation from Range Data**

A range sensor be characterized by a sensor model defined by a probability density function of the form \( p(r|z) \), relating a range measurement \( r \) to the actual distance \( z \). An optimal estimate \( \hat{z} \) of the distance to the object detected by the range sensor is determined using Bayes’ theorem and the maximum a posteriori decision rule. Bayes’ theorem is applied according to the following abbreviated formula [47]:

\[
p(z|r) = \frac{p(r|z)p(z)}{p(r)}
\]

(2.13)

where \( p(z) \) is the a priori probability distribution of an object being located at distance \( z \), \( p(r|z) \) is the conditional probability distribution of receiving range measurement \( r \) given the actual distance \( z \), \( p(r) \) is a normalizing constant representing the distribution of possible sensor readings \( r \), and \( p(z|r) \) is the a posteriori probability distribution that the object is located at distance \( z \) given the information provided by range measurement \( r \). Using the maximum a posteriori decision rule, the optimal estimate \( \hat{z} \) of the distance to the detected object is obtained as the value that maximizes the probability density function \( p(z|r) \).

**Estimation of an Occupancy Grid**

Estimating an occupancy grid leads to a more complex estimation problem, since this involves recovering a spatial model of the environment, rather than determining the distance to a single detected object. In an occupancy grid, the state variable \( s(C) \) associated with a grid cells \( C \) is defined as a discrete random variable with two states, occupied and empty (denoted occ. and emp.). To avoid a combinatorial explosion in the number of possible world configurations, it is assumed in the estimation of the
occupancy grid that the cell states are independent random variables, so that the state of the grid can be determined by estimating the state of each cell individually. To determine how a sensor reading $r$ is used in estimating the cell states $s(C_i)$ of the occupancy grid, Bayes’ theorem is applied to single cells $C_i$ as follows [47]:

$$P[s(C_i) = \text{occ}., l|r] = \frac{P[r|s(C_i) = \text{occ}.]P[s(C_i) = \text{occ}.]}{\sum_{s(C_i)} P[r|s(C_i)]P[s(C_i)]}$$ \hspace{1cm} (2.14)

The sensor related terms $p[r|s(C_i)]$ in this equation do not correspond directly to the sensor model $p(r|z)$, since the sensor model implicitly relates the range reading to the detection of a single object. The sensor model can be rewritten as [47]:

$$p(r|z) = p[r|s(C_i) = \text{occ}.] \land (s(C_k) = \text{emp}.), k < i]$$ \hspace{1cm} (2.15)

### 2.2.6 Dempster-Shafer Evidential Reasoning

The use of Dempster-Shafer evidential reasoning [117] for multi-sensor fusion allows each sensor to contribute information at its own level of detail. E.g., one sensor may be able to provide information that can be used to distinguish individual objects, whereas the information from another sensor may only be able to distinguish classes of objects. The Bayesian approach, in contrast, would not be able to fuse the information from both sensors. Dempster-Shafer evidential reasoning is an extension to the Bayesian approach that makes explicit any lack of information concerning a proposition’s probability by separating firm belief for the proposition from just its plausibility. In the Bayesian approach all propositions (e.g. objects in the environment) for which there is no information are assigned an equal a priori probability. When additional information from a sensor becomes available and the number of unknown propositions is large relative to the number of known propositions, an intuitively unsatisfying result of the Bayesian approach is that the probabilities of known propositions become unstable. In the Dempster-Shafer approach this is avoided by not assigning unknown propositions an a priori probability (unknown propositions are assigned instead to “ignorance”) [122].
2.2 Environment Mapping and Self-Localization

2.2.7 Modeling the Uncertainty of Ultrasonic Measurements

The radial and angular uncertainty of ultrasonic measurements can be modeled by statistical distributions. There is a general conformity in literature that the radial uncertainty may be represented by a Gaussian distribution (normal distribution), which is particularly applied with the grid-based mapping approach. However, differing propositions exist concerning the representation of the angular uncertainty. In a number of mapping algorithms the angular uncertainty is neglected and the range measurements are assumed to originate from the acoustic axis of the sensor (e.g. [36, 37, 41, 45, 51]), which affects the quality of resulting environment models, especially if the beam width of the employed sensors is large. In other research work the application of a normal distribution is proposed (e.g. [42, 43, 46, 47, 48]) or the usage of a uniform distribution is suggested (e.g. [32, 35, 56, 63, 64, 71, 72]) for modeling the angular uncertainty within the sensor cone.

Within this thesis, the assumption of a uniform distribution is advocated, which is justified as follows: Most ultrasonic sensor systems record the detection of an object if a received echo signal exceeds a certain threshold. In order to compensate for the decrease in intensity of a propagating sound pulse due to atmospheric attenuation and spherical divergence, either a time variable threshold or a fixed threshold together with a time variable gain amplifier is employed. Consequently, a given object can be detected with uniform probability over an angular range as wide as the echo signal is strong enough to exceed the threshold. Although the angular detectability range varies in width depending on the reflectivity and directivity of the observed object, the reverse assumption of an unknown object being located with higher probability along the acoustic axis of the sensor is not substantiated.

2.2.8 Review of Related Work

Crowley [41] at Carnegie-Mellon University described in 1985 techniques for constructing line segments from range scans obtained with a rotating ultrasonic sensor, for integrating such line segment descriptions to build a composite local model of the environment, and for correcting the estimated location of the robot. Line segments are created using a variation of the recursive line splitting algorithm which is often used to find edges in images. The correspondence between constructed line segments and line segments in the composite local model is used to update the composite local model and to correct errors in the estimated robot location. Consequently, for
each constructed line segment a corresponding line segment in the composite local model is determined. Then, the average difference in angle between these segments is computed and the constructed line segments are rotated around the position of the robot according to the average error in angle. Next, the average difference in position between corresponding segments is computed and the constructed line segments are translated according to the average error in $x$- and $y$-position. Subsequently, segments in the composite local model are updated whenever a partial overlap with newly constructed line segments exists. The average errors in rotation and translation are finally used to correct the estimated orientation and position of the robot.

At I.N.P. Grenoble (Institut National Polytechnique de Grenoble), Crowley presented in 1989 techniques for modeling static [42] and dynamic [43] environments with range data obtained from a belt of ultrasonic sensors, and for correcting estimated robot locations. Line segments are extracted from the range data if at least three consecutive depth readings align within a tolerance, and successive points are included in counterclockwise order until a point fails the criteria for inclusion in a segment. When a new line segment is obtained from the ultrasound data it is matched to the composite local model by comparing it with all existing segments in order to detect similarities in orientation, collinearity, and overlap. Any uncertainty in the vehicle’s estimated location is included in the uncertainty of the extracted line segment before matching commences. Each match of an observed line segment provides an one-dimensional constraint on position and position uncertainty as well as a constraint on orientation and orientation uncertainty of the robot. These constraints are applied in a Kalman filter update formula to correct the estimated robot location. In a similar manner, the observed line segment serves to correct orientation and orientation uncertainty as well as position and position uncertainty of the corresponding segment in the composite local model. Furthermore, the observed line segment contributes to define the spatial extent of the corrected line segment.

Schiele and Crowley [65] reported in 1994 on experiments for estimating robot locations using occupancy grids. In this framework, the environment is modeled with a local and a global occupancy grid. A technique is described for extracting line segments from occupancy grids based on a Hough transform. The Hough transform enters for a point representing the center of an occupied grid cell all possible straight lines passing that point into parameter space. Thus, local maxima in parameter space represent straight lines existing in the grid (cp. [101]). Four methods are compared for matching a local grid to a global grid, which are summarized as matching “segment to segment”, “grid to segment”, “segment to grid”, and “grid
to grid”. The best results are obtained by extracting line segment descriptions from the two grids and matching these descriptions. The position and orientation at which the local model best matches the global model provides an innovation vector for updating the estimated location of the robot with an extended Kalman filter. Finally, an updating process integrates the data from the local grid into the global grid.

Moravec and Elfes [61] at Carnegie-Mellon University described in 1985 the integration of sonar range measurements from multiple sensors and multiple views into a grid map. The sonar readings provide information about empty and occupied volumes within the cone of a sensor. This information is modeled by probability profiles and projected onto the map. A created map consists of probably empty, probably occupied, and unknown regions. The paper also describes an algorithm to match two grid maps in order to derive the displacement and rotation angle between them for identifying robot locations.

In 1987, Elfes and Matthies [46] demonstrated the utility of the grid-based representation in combining range measurements from different sensor systems by integrating sonar and stereo vision range data. Each cell in the so-called occupancy grid contains a probabilistic estimate of its state. These estimates are obtained from individual sensor models that describe the uncertainty in the range data. A Bayesian estimation scheme is applied to update the current map using successive range readings from each sensor.

Moravec [62] described in 1988 the Bayesian statistical foundation for sensor fusion in certainty grids, which allows the map to be incrementally updated in a uniform way from various sensor sources. The approach can correctly model the fuzziness of each reading, while at the same time combining multiple measurements to produce sharper map features, and it can deal correctly with uncertainties in the robot position.

A comprehensive description of the occupancy grid approach is provided in the Ph.D. thesis by Elfes published in 1989 [47], and a concise version of the final framework is contained e.g. in an anthology edited by Abidi and Gonzales in 1992 [48]. The author describes the underlying stochastic formulation and discusses the applications to a variety of robotic tasks. These include range-based mapping, multi-sensor integration, path-planning and obstacle avoidance, handling of robot position uncertainty, incorporation of precompiled maps, recovery of geometric representations, and other related problems.

Brown [38] from AT&T Bell Laboratories described in 1985 a technique for determining range and direction to a planar surface using ultra-
sonic range measurements from three general (i.e. not necessarily collinear) sensor positions. A direct solution of the vector equation is discussed to illustrate the solution complexity in direct form. Then, a simplifying linear transformation applied to the direct form and a further simplifying sensor configuration are described which reduces the solution complexity considerably. Next, an alternative formulation of the problem is presented from which a geometrical insight can be obtained. In this formulation, the three-dimensional problem of locating the planar surface is decomposed into two simpler two-dimensional problems.

Smith and Cheeseman [66] at SRI International and NASA Ames Research Center described in 1986 a general method for estimating the relationships between coordinate frames representing the relative locations of objects. As a robot moves from one place to another, the uncertainty about its location with respect to its initial location grows. Each relative move is represented by an uncertain transformation. Relationships measured with external sensors are also represented as uncertain transformations. The information from the external sensors is used to improve the knowledge about the global location of the robot. Even if the robot is unable to sense the initial reference objects, its location can be determined if an accurately sensed relationship exists between the initial reference objects and currently observable objects. Thus, a multiply connected network of uncertain transformations allows the robot to locate itself within the environment.

In 1990, Smith et al. [67] at General Motors Research Laboratories, the University of California at Berkeley, and NASA Ames Research Center proposed a stochastic map as representation for spatial information. The map contains the estimates of relationships among objects and their uncertainties. State estimation theory is applied for estimating parameters of an entire spatial configuration of objects.

Drumheller [45] at Massachusetts Institute of Technology described in 1987 a method of absolute robot localization within a known room using data from a sonar range finder. The room is modeled by segments indicating the locations of walls. The localization process extracts straight line segments from the sonar range data, correlates extracted and modeled segments, and eliminates implausible configurations. The objective is to obtain a match in position and orientation between the sonar contour and the room outline yielding the robots absolute location. The algorithm for extracting straight line segments from the sonar contour is an iterative endpoint fit (cp. [101]), which creates a set of longest subcontours containing a minimum number of points with a maximum distance from a line through the endpoints of the subcontour.
Kanayama and Noguchi [53] from the University of California at Santa Barbara described in 1989 a method to extract linear features of the environment with ultrasonic sensors by translational and rotational scanning. Least squares fitting is used for extracting longest possible linear segments from the data. The environment is assumed to be orthogonal, i.e. there is a global coordinate system such that all polygonal obstacles are bounded by edges which are parallel to the $x$- or $y$-axis. For translational scanning, a parallel alignment of the robot to the environment features must be ascertained, since this operation requires a scan-path along the obstacle surfaces. During translation of the robot along a scan-path, a sensor gathers data points of objects with a beam direction perpendicular to the translational motion. Since the robot follows a linear scan-path, the observation positions of the sensor can be assumed to lie on the $x$-axis of a local coordinate system. In contrast, radial range data is gathered by a sensor from a single observation position during rotation of the robot on the spot.

Beckerman and Oblow [34] at Oak Ridge National Laboratory presented in 1990 a methodology for the treatment of systematic errors in grid maps by utilizing overlapping measurement views. The systematic errors manifest themselves as conflicts between interpretations of overlapping data. A four-valued labeling scheme and a simple logic for label combination are introduced. The four labels, occupied, empty, unknown, and conflict are applied to mark the cells of navigation maps. The conflict label is used to denote cells that have been identified to be occupied in one measurement and empty in another measurement. The quaternary character of the approach enables spatial pattern analysis and imposes simple consistent labeling conditions to remove the conflicts once they are identified. Thus, the systematic errors are treated by resolving conflicts and re-labeling conflicting cell assignments to achieve a consistent interpretation of the data.

Cox [39, 40] at AT&T Bell Laboratories and NEC Research Institute described in 1990 and 1991 a matching algorithm for estimating the location of a robot vehicle within known environments based on data from an optical range finder and odometry. A two-dimensional a priori map of the environment is represented as a collection of discrete line segments. The matching algorithm determines the congruence between the range finder data and the a priori map to correct errors in the odometry estimate. Assuming that the displacement between image and model is small, for each point in the image the nearest line segment in the model is likely to be the corresponding line segment. Congruence between the image and the model is achieved in an iterative procedure by minimizing the total squared distance between the image points and their nearest line segments. When the
procedure converges, the rotation and translation parameters describing the congruence are fixed. The approach is not suitable for a general matching problem, in which the assumption of small displacement between image and model is not necessarily valid.

Borenstein and Koren [36, 37] from the University of Michigan introduced in 1991 a fast map building method for real-time obstacle avoidance based on a two-dimensional Cartesian histogram grid as a world model. The world model is updated continuously with range data from ultrasonic sensors. In a first stage, the histogram grid is reduced to a one-dimensional polar histogram that is constructed around the location of the robot. Each sector in the polar histogram contains a value representing the polar obstacle density in that direction. In a second stage, the algorithm selects the most suitable sector from among all polar histogram sectors with a low polar obstacle density. The steering of the robot is aligned with that direction while simultaneously guiding the mobile robot towards the target.

Leonard and Durrant-Whyte [56] from the University of Oxford reported in 1991 on the application of an extended Kalman filter for model-based mobile robot localization that relies on naturally occurring environment features (geometric beacons). A geometric beacon is a special type of target that can be reliably observed in successive sensor measurements. The localization algorithm is formalized as a tracking problem, utilizing the extended Kalman filter to match a number of observed geometric beacons to an a priori map of beacon positions. The Kalman filter relies on a plant model and on a measurement model. The plant model describes how the location of the vehicle changes in response to control inputs, with an estimate of dead-reckoning errors. The measurement model expresses sensor observations in terms of vehicle locations and observed beacon positions, with an estimate of measurement noise. The localization algorithm employs steps of prediction, observation, matching, and estimation.

In another paper in 1991, Leonard and Durrant-Whyte [57] at NEC Research Institute and the University of Oxford discussed the problem of simultaneous map building and localization. Using ultrasonic sensors, geometric features in the environment can be precisely learned and subsequently tracked for accurate localization. Building a map requires the interpretation of sensor information to estimate the positions of geometric features in a global reference frame. Localization is the task of using geometric feature position estimates contained in the map to determine the robot location. The sensing strategy aims to eliminate the vehicle location uncertainty associated with a new set of measurements before using the measurements to update the map.
Leonard et al. [58] at Oxford University and NEC Research Institute presented in 1992 a unified approach to sonar-based navigation combining dynamic map building and localization in a common multi-target tracking framework. Geometric features (targets, beacons) are represented by a covariance matrix and a credibility measure. A priori known features are provided to the map with minimum covariance and maximum credibility. Sonar measurements obtained with a rotating sensor form regions of constant depth corresponding to observed targets. During each location update cycle, predicted measurements are generated for the geometric features in the map and compared with actual sensor observations. Successful matches cause the credibility of a feature to be increased, unpredicted observations are used to initialize new geometric features, and unobserved predictions result in the credibility of a feature being decreased. Estimation of the robot location is performed using only matched predictions and observations that correspond to targets (geometric beacons) with high belief measures.


Forsberg et al. [49] from the University of Luleå described in 1993 the combination of odometry data and laser scanner data to estimate the pose (position and orientation) of a mobile robot using an extended Kalman filter. Given estimates of the relative position and orientation of the robot, a range weighted Hough transform and a least squares method are applied to extract distance and orientation parameters of straight surface elements (walls) from the laser scanner data. Range measurements inside parameter boundaries are first determined by the Hough transform, and improved estimates of the surface parameters are then calculated from the samples within the parameter boundaries using the least squares method.

McKerrow [60] at the University of Wollongong presented in 1993 an algorithm to calculate outline segments of objects in the environment of a mobile robot from ultrasonic range data. The sensors are modeled by an arc model, presupposing that the object causing the reflection lies on an arc with a radius equal to the range measurement and an angle equal to the beam width. Two arcs can be obtained with a single ultrasonic sensor by sensing a surface from one position, moving the robot straight in a direction roughly parallel to the object, and sensing the surface again from another position. Considering the motion of the robot and the orientation of the beam axis, it can be verified by geometric criteria whether a common
tangent exists to the two arcs. If the two range measurements stem from a planar object, the location of the surface is obtained by calculating the common tangent. To derive an outline segment from a common tangent, the points of intersection between the two arcs and the tangent are determined.

Kröse et al. [54] from the University of Amsterdam described in 1993 the representation of an environment by sample points derived from ultrasonic sensor data and by straight lines formed from collinear (or almost collinear) sample points. Initially, a sample point is modeled along the beam axis of the sensor. Under certain conditions, this model is corrected to reduce the angular uncertainty in obstacle direction. If a new measurement falls within a defined distance to a line segment, the measurement is considered to belong to the line and the estimated parameters (position and orientation) of the line are updated utilizing a Kalman filter. If the new measurement does not belong to a line, it is added to the set of sample points. If within the set of sample points a subset of collinear (or almost collinear) points is formed, a new line segment is added to the environment model replacing the involved points.

Weiß et al. [69] from the University of Kaiserslautern presented in 1994 an algorithm for tracking position and orientation of a mobile system by correlating infrared range finder scans. Properties of the environmental structure are exploited for matching two scans obtained at different locations. Derivatives of the range finder scans are calculated and used to determine the translational and rotational displacement of the mobile system by cross-correlation. Presupposition for a successful match is that significant data exists with structural correspondence between the scans.

Lu [59] addressed in a Ph.D. thesis at the University of Toronto in 1995 the problem of mobile robot pose estimation in unknown environments by shape registration (shape alignment) of laser range scans. Two iterative scan matching algorithms are proposed, which do not require feature extraction or segmentation. The first algorithm matches data points using tangent directions in each scan, and the second algorithm establishes point to point correspondences between the scans. Based on the results of aligning scans in pairs, the registration and integration of multiple range scans is studied for environment mapping.

Curran and Kyriakopoulos [44] at Rensselaer Polytechnic Institute presented in 1995 algorithms to localize and to control a mobile robot in partially known environments. The proposed a priori representation of the en-
2.2 Environment Mapping and Self-Localization

Environment is a set of line segments. An extended Kalman filter is used to combine dead-reckoning, ultrasonic, and infrared sensor data with the a priori map information to estimate the actual position and orientation of the robot.

Vestli [68] reported in a Ph.D. thesis at the Swiss Federal Institute of Technology in 1995 on mobile robot localization using a world reference model and laser range finder data. It is assumed that structures like walls, corners, and cylinders are present in the environment and that the positions of these geometric features are registered in the world model. Algorithms were developed to extract the same structures from the sensor data. With the knowledge of the positions of geometric features in the world coordinate system and in the robot coordinate system, the modeled structures and the sensed structures can be matched and a system of constraint equations for the robot location can be solved. Parameters for potential positions of walls and cylinders are calculated by least squares fitting of a geometric model (straight line or circle, respectively) to consecutive subsets of the sensor data. The subsets are generated by shifting a window over the complete data set. Parameters for potential positions of corners are obtained by calculating intersections of straight lines. Subsequent clustering in the solution space of the previous step yields the positions of geometric features in the robot coordinate system. The applied clustering technique is based on determining expected cluster positions in order to assign the previously calculated parameters to respective clusters. The determination of the expected cluster positions relies on the odometric location estimate of the robot in the world coordinate system as well as on the registered structures in the world model. Consequently, only modeled structures can be extracted. With the aid of a method for clustering the data without relying on a priori information the process could be reversed to construct a world model.

Pagac et al. [64] from the University of Sydney described in 1996 the construction and maintenance of a grid-based map by modeling ultrasonic sensor information and utilizing probabilistic reasoning. The ultrasonic measurements are integrated into the map by considering the uncertainties of the sensor readings and using the Dempster-Shafer inference rule. The Dempster-Shafer method supports probabilities

\[ P_{\text{empty}}(\text{cell}) + P_{\text{occupied}}(\text{cell}) + P_{\text{empty, occupied}}(\text{cell}) = 1 \]

for the state of a cell in contrast to the Bayes approach which requires probabilities

\[ P_{\text{empty}}(\text{cell}) + P_{\text{occupied}}(\text{cell}) = 1. \]

The Dempster-Shafer theory of evidence is characterized by a frame of discernment, basic probability assignment, belief and plausibility functions, as well as the Dempster’s rule of combination (cp. [117]).
Berler and Shimony [35] at Ben Gurion University of the Negev proposed in 1997 a probabilistic approach for grid-based mapping with sonar sensors, which relaxes the independence assumption made by Elfes that the cell states can be estimated as independent random variables. Dependencies introduced between readings are represented by the topology of a Bayes network, which also represents the conditional distributions between elements of the model (readings and spatial regions).

Hoppenot and Colle [51] from the University of Evry addressed in 1998 the real-time localization of a low-cost mobile robot with a limited set of ultrasonic measurements in a known environment. For each ultrasonic measurement, a pinpoint along the beam axis of the sensor is calculated, according to the odometric location of the robot. Two localization algorithms are presented. The first algorithm matches each pinpoint with the nearest segment in a model of the known environment. Minimizing the sum of the squared distances between the pinpoints and the modeled segments yields an corrected estimate of the robot location. The second algorithm matches segments computed from the pinpoints by linear regression with associated segments in the environment model. In this case, the estimated robot location is corrected by the average difference in orientation between the computed segments and the modeled segments, and subsequently by the average differences in $x$- and $y$-position between the midpoints of the computed segments and the modeled segments.

Wijk et al. [70] at the Royal Institute of Technology presented in 1998 a scheme of triangulation-based fusion for filtering ultrasonic sensor data. The fusion scheme relies on triangulation of sonar measurements recorded from different robot locations for estimating feature positions in the environment. Thus, intersections of arcs (triangulation points) define positions of vertical edge targets. A modified version of an occupancy grid is used for environment mapping with triangulation points. Sonar measurements that are likely to have hit a mutual vertical edge of an object are grouped together by a voting scheme in order to register reliably detectable targets in the map.

In 2000, Wijk and Christensen [71] suggested an application of the triangulation-based fusion scheme to robot pose tracking. Since the fusion scheme extracts triangulation points from vertical edges of door posts, shelves, table legs, etc. in the environment as stable natural landmarks, the detected landmarks can be matched to features in a prerecorded reference map in order to estimate the robot location. The pose tracking is implemented as a classical extended Kalman filter, which uses odometry read-
ings for predictions and triangulation point observations for measurement updates.

A comprehensive description of triangulation-based fusion with application to mapping and localization tasks including thorough experimental results is provided in the Ph.D. thesis submitted by Wijk in 2001 [72].

Nagatani et al. [63] at Carnegie Mellon University described in 1999 a method for fusing sonar data from adjacent sensing positions by intersecting sonar arcs which represent possible object locations in order to determine the actual location of an object. Only the median of all transversal intersections with an arc is considered to provide an accurate position estimate. The method is suitable for mapping environments with narrow passageways.

Anousaki and Kyriakopoulos [32] at the National Technical University of Athens reported in 1999 on map building and localization algorithms for mobile robot navigation using dead-reckoning and ultrasonic range data. Local world models are created based on the concept of occupancy grids, and a global world model is built by extracting line segments from the occupancy grids. The radial and the angular uncertainty within the sensor cone are represented by a combined probability distribution function, and the cell values of the occupancy grid are raised according to the obtained profile. An accumulator matrix is employed with dimensions corresponding to the number of unknown parameters of the line segments described in polar coordinates. Parameters of every possible straight line passing through an occupied grid cell are determined and the corresponding elements of the accumulator matrix are increased (cp. [101]). When the possible straight lines for all occupied grid cells are entered, the accumulator matrix is searched for peaks, which indicate the parameters of dominant lines existing in the occupancy grid. The number of dominant lines may be reduced by centroid linkage clustering. Subsequently, the cells of the occupancy grid are grouped into sets according to the dominant lines. A set of grid cells belonging to a dominant line can be subdivided into smaller sets by single linkage clustering in order to represent separate segments of one dominant line. Finally, a least squares method is applied to extract the parameters of straight lines from the sets of grid cells.

Bailey et al. [33] from the Australian Centre for Field Robotics presented in 2000 a graph theoretic method for robot localization, which is applicable to data association problems where the features are observed via a batch process (e.g. laser scanning, radar, video). Considering two laser scans obtained from different unknown locations, the largest subsets of
features common to both scans are matched to each other in order to derive the relative change in the vehicle pose. The data association problem of matching common features between two observations can be transformed to the graph theoretic problem of finding the maximum common subgraph between two graphs. Batch observations are represented in a graph by assigning extracted feature types (points or lines) to graph vertices and invariant geometric relationships between features (distances or angles) to graph edges.

Zhang and Ghosh [73] at Washington University described in 2000 map building and localization techniques for a mobile robot equipped with a laser range finder. Line segments are used as the basic elements for map building as well as for local and global localization. The line segments are derived from the range data by recursive line splitting. For local localization, newly extracted line segments are matched to a previously built global map. To find pairs of corresponding line segments, the extracted line segments are transformed into global coordinates based on odometric pose estimation. A correspondence check is performed for each extracted line segment against all line segments in the global map, and local localization is achieved by matching the corresponding line segments. Based on pose estimation from local localization, the global map is updated by region expansion and line segment fusion. The parameters (direction and midpoint) of a global line segment are updated as a weighted mean of the parameters of the contributing local and global line segments. For global localization, a search algorithm is proposed to find as many correspondences as possible between local and global line segments using geometric relations which are preserved at coordinate transformations.

Gutmann [50] described in a Ph.D. thesis at the University of Freiburg in 2000 various methods based on laser range finder data for scan matching, self-localization, environment mapping, and path planning of autonomous mobile systems. Interesting with regard to this research are the algorithms for feature extraction (lines, corners, edges) from laser scans. Recursive line fitting is applied for line extraction.

Jeon and Kim [52] at Korea Advanced Institute of Science and Technology presented in 2001 an algorithm for feature-based probabilistic map building, which utilizes time of flight and amplitude of signal information from sonar sensors. A set of hypothesized targets (plane, corner, edge) originating from a single measurement is refined by subsequent measurements using an extended Kalman filter and Bayesian conditional probability. The map building is regarded as matching and tracking of unknown
static targets. According to the matching results, the location and the existence probability of targets are updated.

2.3 Fault Detection, Diagnosis, and Recovery

While many applications of fault detection and diagnosis already exist for monitoring the behavior of various dynamic systems (e.g. machines, airplanes, spaceships), approaches in the field of autonomous robotics are gradually evolving. One reason for this might lie in the “autonomy” of these systems itself. Their high complexity and the diversity of their tasks and working environments make a coverage of possible faults very difficult. Another reason might be that many functional problems regarding the usability of autonomous robots in human environments are still unsolved and research primarily concentrates on solving these problems. This order of priority is logically consistent in view of realizing system dependability since the individual functionality of a prospective product determines the framework for implementing fault tolerant behavior. Nevertheless, fault tolerance should be addressed in research, since dependability issues will finally be a decisive factor whether a new generation of robots will become products.

An advantage of autonomous systems in comparison with conventional dynamic systems is their high degree of adaptability to exceptional situations, which allows them to compensate for faults by recovery strategies. For this reason, fault detection and recovery are often treated in a common context. Although automatic fault recovery is desirable, in critical cases an operator can manually guide the system to safe conditions (e.g. by remote control with the aid of a camera).

Fault tolerant behavior refers to the autonomous detection and identification of faults as well as the ability to continue functioning after a fault has occurred [90]. Safety, reliability, efficiency, and productivity – generally spoken the achievement of high performance – are reasons for fault tolerant behavior of mobile robots.

Remote, dangerous, or harsh environments (e.g. planets, nuclear reactors, or oceans) limit the ability of humans to perform supervisory or corrective tasks. In such environments it is vital to the success of a mission that the robot continues functioning effectively despite a fault having occurred. However, other autonomous systems should be fault tolerant as well in order to be of high practical benefit, to perceive dangerous situations, and not to represent a safety risk themselves.
2.3.1 Terminology

The terminology in the field of supervision, fault detection, and diagnosis is not consistent. E.g., the terms fault, failure, malfunction, and error are used synonymously by different authors. The IFAC SAFEPROCESS Technical Committee therefore tried to find commonly accepted definitions. Some of the committee’s suggestions are quoted below [84]:

- **Fault**: A deviation of at least one characteristic property or parameter of the system from the acceptable / usual / standard condition.

- **Failure**: A permanent interruption of a system’s ability to perform a required function under specified operating conditions.

- **Malfunction**: An intermittent irregularity in the fulfillment of a system’s desired function.

- **Error**: A deviation between a measured or computed value (of an output variable) and the true, specified, or theoretically correct value.

- **Fault detection**: Determination of the faults present in a system and the time of detection.

- **Fault isolation**: Determination of the kind, location, and time of detection of a fault. Fault isolation follows fault detection.

- **Fault identification**: Determination of the size and time-variant behavior of a fault. Fault identification follows fault isolation.

- **Fault diagnosis**: Determination of the kind, size, location, and time of detection of a fault. Fault diagnosis follows fault detection and includes fault isolation as well as fault identification.

- **Monitoring**: A continuous real-time task of determining the conditions of a physical system, by recording information as well as recognizing and indicating anomalies in the behavior.

- **Supervision**: Monitoring a physical system and taking appropriate actions to maintain the operation in the case of faults.
2.3 Fault Detection, Diagnosis, and Recovery

2.3.2 Potential Faults

Potential faults occurring with mobile robots:
- operation execution failures (obstacles on the path, closed doors, displaced objects, ...)
- sensor faults (ultrasonic sensors, infrared sensors, laser sensors, gyroscopes, wheel encoders, ...)
  - permanent faults (e.g. “blind” sensors)
  - sporadic faults (e.g. “hallucinating” sensors)
  - calibration errors (e.g. “squinting” sensors)
- actuator faults (motors, joints, ...)
- hardware faults (electronic components, local processors, ...)
- uncertainty in sensing, control, and geometric models of the robot or the environment
- specific fault conditions (low battery level, flat tire, ...)
- individual faults and combinations of faults (concurrent faults, accumulative faults)
- other fault conditions (e.g. unexpected and unanticipated faults)

2.3.3 Fault Detection and Diagnosis

Methods of fault detection and diagnosis for mobile robots:
- monitoring sensor readings and comparing them against preprogrammed thresholds, which change as the task changes
- monitoring verification flags to confirm completed operation execution
- indicating specific fault conditions by different fault flags (e.g. battery level flag)
- exploiting the context provided by the time history of the motion (i.e., if the sensor reflects a plausible motion, then the robot has some confidence that the sensor is working)
- exploiting the context provided by complementary sensors (i.e., if reliable complementary sensors agree with the sensor in question, then the system has more confidence in that sensor)
- actuator faults can be inferred through appearing concurrent faults of sensors the behavior of which depends on that actuator working
- local processor faults appear as a common fault of all sensors the values of which are computed by that processor
- building fault trees designed to identify faults
- identification of dual faults by the use of hierarchical structures
• sensor assessment through sensor self-tests by turning the robot on the spot or sensor online monitoring during normal operation
• employing redundancy for detection of faults by using sensors with different physical properties

An important class of fault detection and diagnosis methods are the model-based approaches. In this context, observer-based schemes using mathematical process models, multiple model adaptive estimation (MMAE) using Kalman filters, and dynamic neural network models are often applied.

2.3.4 Fault Recovery

Fault recovery strategies for mobile robots:
• replication of hardware (multiple sensors and actuators provide complementary capabilities)
• redundant control behaviors (the controller is designed with redundant strategies to perform a task)
• robust virtual sensors (these are sensors that remain reliable despite actual sensor faults)
• retrying (addresses transient faults)
• dynamic calibration (addresses sensor drifts)
• reconfiguration (addresses permanent failures)
  • disable damaged components (sensors, actuators, local processors, ...)
  • adopt a plan that is stable without the use of the damaged components
  • reintegrate repaired components

Treatment of unidentifiable, unexpected, or unanticipated faults remains a difficult problem. Recovery from failures by automatically altering the control structure presents another challenge in designing intelligent robotic systems.

2.3.5 Review of Related Work

This subsection is organized as follows: Firstly, an overview of fault detection and diagnosis for dynamic systems is provided in order to present the basic concepts in this field. Fault detection, diagnosis, and recovery for autonomous systems can be divided into robotic manipulators and mobile robots since they require individual approaches. Solutions for robotic manipulators are reviewed secondly and for mobile robots thirdly. For an ad-
ditional literature review concerning fault detection, diagnosis, and recovery the interested reader be referred to [80].

**Dynamic Systems**

Willsky [100] at Massachusetts Institute of Technology surveyed in 1976 design methods for failure detection in stochastic dynamic systems. The survey comprises failure sensitive filters, voting systems, multiple hypotheses filter techniques, jump process formulations, and innovations-based detection systems. Willsky concludes that the choice of an appropriate design heavily depends on the particular application.

Gertler [81] from George Mason University presented in 1988 a survey of model-based techniques to detect and isolate failures in complex processes. The methods are based on analytical redundancy provided by a mathematical model of the system. The main components of such techniques are residual generation using the model, signature generation via statistical testing, and signature analysis.


Ayoubi [75] from the Technical University of Darmstadt introduced in 1994 a fault diagnosis scheme for technical processes by nonlinear dynamic systems identification. A dynamic multi-layer perceptron neural network is applied to identify black box models of a process. The identified models are used for fault diagnosis similar to observer-based schemes. The residuals between the measured process output and the outputs estimated by the bank of models are used as numerical symptoms for the fault detection and diagnosis. The fault diagnosis scheme was applied to monitor the turbine state of a turbosupercharger.

Lane and Maybeck [85] at Air Force Institute of Technology presented in 1994 a failure detection and identification concept for an unmanned research airplane based on multiple model adaptive estimation (MMAE) using fifteen elemental Kalman filters (six for flight control actuator failures, eight for sensor failures, and one for the fully functional aircraft). The algorithm is extended for the identification of dual failures through the use of a hierarchical structure of filter banks. Menke and Maybeck [86] reported in 1995 the application of multiple model adaptive estimation
(MMAE) to the VISTA (Variable In-flight Stability Test Aircraft) F-16 flight control system. Single actuator and sensor failures are addressed first, followed by dual actuator and sensor failures. The system is evaluated for complete (“hard”) failures, partial (“soft”) failures, and combinations of hard and soft actuator and sensor failures. Residual monitoring is discussed for single and dual failure scenarios.

Abdallah et al. [74] at the University of New Mexico surveyed in 1996 traditional approaches as well as applications of neural networks for fault detection. They also reported implementation results from a feasibility study about the usage of neural networks for detecting faults in the viscous damper bearing of a helicopter drive shaft.

Isermann and Ballé [84] from the Technical University of Darmstadt presented in 1997 a historical overview as well as some trends of model-based fault detection and diagnosis for technical processes. Based on discussions within the IFAC SAFEPROCESS Technical Committee, some proposals for the terminology in the field of supervision, fault detection, and diagnosis are also provided.

**Robotic Manipulators**

Moya and Davidson [87] from Sandia National Laboratories described in 1986 a sensor driven, fault tolerant control of a remote maintenance robot system surrounding a pulsed nuclear reactor. The recovery procedure relies on operator assistance and guarantees that the moving components of the system do not collide with the reactor during recovery.

Donald [77] at Cornell University described in 1990 a theory of planning multi-step error detection and recovery strategies for compliant motion assemblies. His work addresses motion planning with uncertainty in sensing, control, and the geometric models of the robot and the environment.

The “Robots for Hazardous Environments Group” at Rice University focuses on fault tolerance and reliability for robots to be used in space and nuclear applications, where repairs are often difficult and failures are potentially catastrophic. Cavallaro and Walker [76] surveyed in 1994 the NASA and Military standards on fault tolerance and reliability applied to robotics. Hamilton and Visinsky [83] described in 1994 fault tolerant algorithms and architectures for robotics. They concentrated on fault tolerance in the robot controller, and highlighted the importance and potential of
multiprocessor control architectures from the fault tolerance perspective. Visinsky et al. presented in 1994 an expert system framework for fault detection and fault tolerance in robotics [98]. Through the use of an expert system, different strategies for failure mode analysis, fault detection, tolerance, reconfiguration, and repair can be linked into one framework. The key to the approach is the integration of fault trees into the expert system. Visinsky et al. also published in 1994 a survey on robotic fault detection and fault tolerance [99].

Vemuri et al. [97] from the University of Cincinnati presented in 1998 a learning architecture with neural networks for online approximation of an off-nominal system behavior. The idea is applied to monitor a rigid-link robotic manipulator for changes in the system dynamics due to faults.

**Mobile Robots**

Noreils [88] from LAAS-CNRS proposed in 1990 an approach of integrating error recovery in a mobile robot control system. The architecture includes detection of external and internal faults. Error recovery is based on the development of a failure tree using an error knowledge base.

Payton et al. [89] from Hughes Artificial Intelligence Center and Hughes Undersea Weapons Systems Division described in 1992 a highly distributed fault tolerant control system, which must be able to compensate for faults even when their cause cannot be explicitly identified and to deal with events that cannot be anticipated at design time. The control system was developed for an autonomous underwater vehicle that has to remain operational for several weeks without human intervention. If the control system is unable to diagnose and correct specific faults, it handles such events by attempting to “do whatever works”. Consequently, the challenge is not to identify all things that can possibly happen, but rather to identify all possible things the vehicle can do.

Ferell [78] at Massachusetts Institute of Technology described in 1994 failure recognition and fault tolerance capabilities of a six-legged robot. The system recognizes and compensates sensor and actuator failures. It tolerates a variety of sensor failures such as decalibration, erroneous readings, and permanent failures. It also tolerates combinations of failures such as concurrent failures and accumulative failures. Possible fault tolerance strategies are (a) replication of hardware (multiple sensors and actuators provide complementary capabilities), (b) redundant control behaviors (the controller is designed with redundant strategies to perform a task), and (c)
robust virtual sensors (the controller uses robust virtual sensors to confine hardware failures to low-level control).

Soika [94, 95] from Siemens Corporate Technology presented in 1997 a probabilistic framework for grid-based failure detection and calibration of external sensors on mobile robots. Statements about the robot’s environment provided by sensor measurements are represented in an occupancy grid. For all grid cells consistency measures are derived by evaluating the information stored in them. Readings of faulty sensors lead to inconsistencies within the robot’s model of the environment. A probabilistic formulation of the sensor model and the statements about the environment allows for a quantitative examination of consistencies. By combining a sensor’s consistency measures over several statements, the robot is able to deliberate about the condition of the sensor. This research work is one of a very few approaches dedicated to fault detection for external sensors (i.e. environment sensors) on mobile robots.

Scheding et al. [92, 93] at the University of Sydney provided in 1997 and 1998 an analysis of Kalman filter based systems with respect to fault detection in navigation systems by a frequency domain approach. The analysis is applicable to any linear system the state of which is estimated by a Kalman filter. It is shown that redundancy must be employed and that diverse sensors should be used. In [92], the technique is applied to data from a laser range finder and a sonar sensor. Using sensors based on different physical principals, the fault modes are different and faults due to sensor modeling errors are detectable in the innovations of the filter. In [93], the frequency domain approach is applied to data from a gyroscope and a laser sensor. By combining the high frequency characteristics of the gyroscope and the low frequency behavior of the laser sensor using a Kalman filter, the whole spectrum of platform rotation maneuvers can be tracked.

Roumeliotis et al. [90, 91] from the University of Southern California introduced in 1998 an application of multiple model adaptive estimation (MMAE) for fault detection and identification on mobile robots, where models of the nominal system behavior and the behavior under various types of faults are embedded in parallel estimators. Each of these estimators is a Kalman filter, which is tuned to a particular fault and predicts values for the sensor readings. A fault detection and identification module is responsible for processing the residuals (the differences between the predicted and actual sensor readings) to decide which fault has occurred. Mechanical faults (e.g. flat tires) [90] as well as faults of internal sensors (e.g.
gyroscope or wheel encoders) [91] are examined. Goel et al. [82] proposed in 2000 a modification of the previous work by applying a backpropagation neural network for processing the set of filter residuals as a pattern in order to decide which fault has occurred, i.e. which filter is better tuned to the correct state of the robot.

Fernández and Simmons [79] at Carnegie Mellon University presented in 1998 a deterministic approach to robust execution monitoring, which uses a hierarchy of monitors, structured in layers of different specificity. The upper, general monitors detect large classes of exceptions, like unexpected and unanticipated situations. The lower, specific monitors try to obtain more information about the underlying problem that causes the exceptions, for use in recovery. Fernández extended the deterministic approach in a Ph.D. thesis [80] at the University of Vigo in 2000 by a stochastic model that takes into account the uncertainty about the state of the robot. Using the monitors already developed as observers, a model based on Partially Observable Markov Decision Processes (POMDP) has been built.
3 A Novel Ultrasonic Sensing System

This chapter presents a newly developed ultrasonic sensing system for autonomous mobile systems. It is also described how wide-angled ultrasonic transducers can be used to obtain substantial information of the environment. This can be achieved by exploiting the overlapping of detection cones from neighbor sensors and by receiving cross echoes between them. The novel ultrasonic sensing system allows the detection of multiple echoes from different echo paths for each sensor. In this way, a significantly higher number of echoes can be obtained in comparison with conventional ultrasonic sensing systems for mobile robots.

An introduction to the chapter is provided in Section 3.1. A specification of the newly developed ultrasonic sensing system is presented in Section 3.2. The employed ultrasonic sensor is described in Section 3.3. The hardware and the software of the ultrasonic sensing system are addressed in Section 3.4 and Section 3.5, respectively. The advantages of wide-angled environment coverage with ultrasonic transducers are demonstrated in Section 3.6. The contributions of this chapter are summarized in Section 3.7.

3.1 Introduction

Ultrasonic transducers are preferably used to obtain three-dimensional information of the environment. The most commonly used sonar device for mobile robots is the well known Polaroid ultrasonic ranging system [125]. Due to the relatively narrow detection angle of the Polaroid sensors as well as the angular and spatial displacement of the sensors on a robot, cross echoes from neighbor sensors rather arise from multiple reflections than from an object being located within the beam width overlapping of the sensors. The standard Polaroid system is configured to send out a short ultrasonic pulse and to measure the “time of flight” (TOF) until a first echo from an object can be detected by the same sensor. To enhance robot perception, the strategy proposed in this dissertation is to use wide-angled ultrasonic transducers and to detect cross echoes from neighbor sensors as
well as multiple echoes per sensor. For this purpose, a novel ultrasonic sensing system has been designed [129, 130, 133] and algorithms have been developed to process the sensor data in order to obtain improved environmental information. Investigations have shown that cross echoes generally provide highly valuable data. This is not necessarily the case with multiple echoes. Although up to four consecutive echoes per sensor can be received with the new sensing system, the first arriving echoes normally contain the most useful data, since in narrow environments later arriving echoes are often affected by multiple reflections due to their longer TOF.

The ultrasonic sensing system has been deployed on the intelligent wheelchair (MAid), and the experimental robot Nomad XR4000 serving as a platform to simulate an automatically guided hospital bed (AutoBed). When semi-autonomously or autonomously transporting people through private and public surroundings, a high perception of the robot is extremely important due to safety reasons. Within this chapter, the circular sensor arrangement on the experimental robot is described, and the behavior of the ultrasonic sensing system is demonstrated in a typical indoor environment (see Section 1.5).

In order to completely understand the motivation for a novel ultrasonic sensing system, the reader be referred to the review of related work in Subsection 2.1.7, where the advantages of binaural and triaural sensing in comparison with monaural sensing are described. Different paradigms of ultrasonic sensor arrangements are apparent in mobile robotics and in automotive applications. In one paradigm, narrow-angled sensors are distributed in small distances along the periphery of a mobile robot, and in the other paradigm, wide-angled sensors are distributed in large distances along the bumper of a car. In both cases, the sensors cover respective sectors of the environment exclusively, i.e. the number of employed sensors is dependent on the beam width. Within this dissertation, it is proposed to arrange wide-angled sensors with small angular and spatial displacements on autonomous mobile systems. In this way, considerable beam width overlapping with a number of neighbor sensors is obtained, and multiple cross echoes may be received. Consequently, the novel ultrasonic sensing system can be defined as a multi-aural measurement system. Since the triangulation-based algorithms described in Subsection 2.1.7 are not very efficient if the sensor distances are small relative to the obstacle distances, new algorithms of parameter space clustering (PSC) and tangential regression (TR) have been developed to obtain increased information of the environment with such sensor configurations on mobile robots (see Section 4.3).
3.2 Specification of the Ultrasonic Sensing System

The ultrasonic sensing system is explained for the circular sensor arrangement on the experimental robot Nomad XR4000. The Nomad XR4000 at FAW has been additionally equipped with two antipodal Laser Ranging Systems from SICK Electro-Optics.

The new ultrasonic sensing system consists of a ring of 24 sensors, spaced with an angular displacement of 15° and mounted on top of the robot. In contrast to the most commonly applied ultrasonic range finders from Polaroid Corporation [125] with a detection cone of up to 30°, wide-angled sensors from Robert Bosch GmbH [124] are employed with an elliptic detection cone of 120° in one axis and 60° in the other axis. The latter sensors are primarily manufactured for automotive use, e.g. for parking pilots.

On the one hand, wide-angled ultrasonic transducers can detect objects within a broader domain, but on the other hand, they make it more difficult to estimate within this broader domain the actual location of the object causing the echo. In order to gain a substantial advantage of the broader detection angle, exploitation of the overlapping of the detection cones from neighbor sensors is proposed. This is feasible when the sensors are placed close enough to each other and the angular displacement is not larger than the detection angle.

In the case of the new sonar sensing system for the Nomad XR4000, the 24 Bosch sensors are arranged exactly above the upper ring of 24 Polaroid sensors which this robot is originally equipped with (Figure 3.1). This facilitates comparison of the two sonar sensing systems. The two laser range finders are also mounted on top of the robot in order to scan just above the sonar rings, which allows comparison between sonar readings and laser readings.
The ultrasonic sensors are orientated with the 60° detection angle to a horizontal alignment and with the 120° detection angle for vertical object detection. With this kind of sensor arrangement, beam width overlapping up to the third neighbor sensor on each side is attained. The sensors could also be orientated with the 120° detection angle to a horizontal alignment, and beam width overlapping with even more neighbor sensors would thus be achieved.

The direct and cross echoes as well as the associated detection angles are defined as follows: Direct echoes exist if the transmitting sensor also receives the signal, and cross echoes exist if one of the neighbor sensors receives it. With the chosen sensor orientation, the horizontal detection angle for direct echoes is 60° and for cross echoes approximately 45°, 30°, or 15°, depending on whether the first, second, or third neighbor sensor is involved, since the angular displacement of the sensors is 15°. Consequently, 24 direct echo paths and $2 \times 3 \times 24 = 144$ cross echo paths exist. Up to 4 echoes can be received via each echo path, which results in $4 \times (24 + 144) = 672$ detectable echoes per measurement cycle. Isometric lines of direct echo paths can be depicted as circular arcs of 60°, and isometric lines of cross echo paths as elliptic arcs of about 45°, 30°, or 15°.
If the beam width overlaps up to the third neighbor sensor on each side, every eighth sensor in the ring can be fired simultaneously without mutual interference. The other sensors are in a listening mode during this time. The sensor in the middle of seven passive sensors between two firing sensors will normally not receive any echoes, though this might occur due to multiple reflections. After the first three sensors have been fired and the ultrasound energy within the environment has fallen beneath a detectable level, the next three sensors can be fired. Since the employed sensors have an attainable distance detection range of about 2 m, which corresponds to a TOF of about 12 ms for an ultrasonic pulse, a measurement period of this duration is defined. Subsequently, it is waited for additional 8 ms to allow the ultrasound energy in the surrounding to decline further. Thus, a basic measurement cycle time of 20 ms is obtained. Firing all 24 sensors in groups of three sensors successively, results in a cumulative measurement cycle time of 160 ms. In addition to this, 90 ms are provided for transferring the collected data with a transfer rate of 115200 Bit/s via serial interface to the robot PC. Consequently, the overall system cycle time is 250 ms, which corresponds to a sampling rate of 4 Hz. This sampling rate could be increased by several optimization measures.

Detailed development records about the hardware and the software of the novel ultrasonic sensing system exist as internal documents.

### 3.3 Ultrasonic Sensor

The employed sensing devices are piezoceramic ultrasonic sensors of type 2.1 from Robert Bosch GmbH (Figure 3.2), which are primarily manufactured for automotive use, e.g. for parking pilots. The sensors have the following features [124]:

- specified distance detection range: 0.25 m - 1.2 m
- attainable distance detection range: 0.2 m - 2.0 m
- detection cone (elliptic): ±60° × ±30°
- frequency range: 42 kHz – 45 kHz
- typical pulse duration: 300 µs
- resonator decay time: 900 µs
- digital bidirectional data transfer
- integrated signal conditioning
The sensors use the same piezoceramic disc to transmit pulses and to receive echoes. During an excitation time of typically 300 \( \mu s \) the acoustic transducer sends out a pulse which can be received after a resonator decay time of 900 \( \mu s \). Consequently, a dead time of 1.2 ms exists for the transmitting sensor to receive echoes. This dead time corresponds to a minimal attainable distance value of circa 0.2 m. The presence of nearer objects can be recognized by the transmitting sensor without obtaining an absolute distance value. However, the dead time does not apply to idle neighbor sensors, which are able to receive cross echoes from the transmitting sensor instantly and thus are able to measure the distances of closer objects. For this reason, only sensors with non-overlapping detection angles should be fired at the same time.

The digital bidirectional signal line of the sensor is used for transmitting pulses and for detecting echoes. The idle potential of the signal line is High. In order to transmit a pulse, the signal line can be drawn to Low for a period of e.g. 300 \( \mu s \) by the electronic control unit. Subsequently, the signal line will remain Low for a period of 900 \( \mu s \) in order to allow the resonator to decay. Thereafter the signal line switches to High and the sensor is ready to detect echoes. The received echo signals are pre-processed within the sensor by an integrated circuit for signal conditioning (band...
pass filtering and threshold-logic). For each detected echo the sensor draws the signal line to zero for as long as the return signal exceeds the threshold, i.e. the sensor outputs a binary signal that reflects the time instants of arriving echoes. The TOF of the ultrasonic pulse can be determined by measuring the elapsed time between firing the sensor and detecting a High-Low-edge resulting from an echo. A typical timing diagram of a sensor’s signal line is shown in Figure 3.3.

![Timing diagram of a sensor's signal line](image)

**Figure 3.3:** Timing diagram of a sensor’s signal line

By using a single sensor, only the distance between the sensor and an object can be determined. Hence an object can be located anywhere in the respective distance on a circular arc within the detection cone of the sensor (Figure 3.4). If two sensors with overlapping detection cones are used and an object is located within the overlapping domain, a direct echo exists between each sensor and the object as well as a cross echo from each sensor via the object to the other sensor. In such cases the exact location of the object can be calculated geometrically (Figure 3.5).
3.3 Ultrasonic Sensor

**Figure 3.4:** Distance detection of a cylindrical object with a single sensor

**Figure 3.5:** Location detection of a cylindrical object with two sensors by geometric means
Ultrasonic transducers do not emit energy homogeneously over the radiation range, and the received energy depends on the angle of incidence of the echo at the sensor. For this reason, the maximal attainable distance value varies with the radiation angle. The signal attenuation as a function of the radiation angle is represented by the radiation characteristic of the ultrasonic sensor (Figure 3.6).

![Radiation characteristic of the ultrasonic sensor](image)

**Figure 3.6:** Radiation characteristic of the ultrasonic sensor

If the surface of an object has a perpendicular falling within the detection cone of a sensor and if the return energy exceeds the sensor’s threshold, the object can be detected (Figure 3.7). If the normal to the surface does not fall within the detection cone, the ultrasound will be specularly reflected by the surface and no echo will be received by the sensor (Figure 3.8).
Figure 3.7: Object surface having a perpendicular falling inside the detection cone of a sensor

Figure 3.8: Object surface having a perpendicular falling outside the detection cone of a sensor
3.4 Hardware

The hardware of the novel ultrasonic sensing system (Figure 3.9) consists of a commercially available digital signal processor card (DSP-Board) as well as a self-developed interface card (Daughter-Board). The DSP-Board is programmed via parallel port, and the sensor data is transferred via serial port. Up to 32 ultrasonic sensors can be connected to the electronic control unit. The supply voltages can be derived from the switching power supply of a PC.

![Figure 3.9: Hardware of the novel ultrasonic sensing system](image)

3.4.1 DSP-Board

As device for data acquisition, data pre-processing, and data transmission via serial interface to the robot PC, a DSP-Board from Texas Instruments is utilized. The DSP-Board consists of a 32-Bit Floating-Point Digital Signal Processor TMS320C6711, 16 MB External SDRAM, 128 KB External Flash ROM, a Power Management Device, an Expansion Daughter Card Interface, and a PC Parallel Port Interface. The DSP operates at a clock
rate of 150 MHz with a performance of up to 900 MFLOPS (million floating-point operations per second).

Up to 32 ultrasonic sensors can be connected to the 32-Bit data bus of the Expansion Memory Interface of the DSP-Board. The acquired sensor data is transferred to the robot PC via one of the two multi-channel buffered serial ports provided on the Expansion Peripheral Interface of the DSP-Board.

### 3.4.2 Daughter-Board

The self-developed Daughter-Board provides an interface between the DSP-Board and the peripherals. On the one hand, it comprises the signal conditioning between the data lines of the ultrasonic sensors and the external data lines of the Expansion Memory Interface. On the other hand, it includes a serial interface to the robot PC by connecting the two multi-channel buffered serial ports of the Expansion Peripheral Interface via a multi-channel RS-232 driver/receiver-device. The Daughter-Board additionally provides the supply voltages for the DSP-Board and for the ultrasonic sensors.

### 3.5 Software

For software development based on the TMS320C6711 DSP, the Code Composer Studio from Texas Instruments has been used.

The developed DSP software comprises a module for data acquisition and data pre-processing as well as a module for sending the collected data to one of the multi-channel buffered serial ports. Additionally, the External Flash ROM has been programmed in order to boot the DSP-Board with the developed software.

The LINUX software for the robot PC includes a server program and a client program. The server program receives the collected data via a serial port and writes it into shared memory. The client program reads the data from shared memory and provides it to the robot control software.
3.6 Advantages of Wide-Angled Environment Coverage

If two narrow-angled sensors with non-overlapping detection cones are employed and a circular object is located within the detection angle of one sensor, a direct echo \(d_1\) exists between the sensor and the object (Figure 3.10). In such cases, only the distance but not the exact location of the object can be determined, since the object can be placed anywhere in the respective distance on a circular arc within the detection angle of this sensor. In contrast, if two wide-angled sensors with overlapping detection cones are utilized and a circular object is located within the overlapping detection domain, direct echoes \(d_1, d_2\) exist between each sensor and the object as well as cross echoes \(c_{12}, c_{21}\) from each sensor via the object to the other sensor (Figure 3.11). In such cases, the distance and the exact location of the object can be determined by exploiting the complementary sensor information.

**Figure 3.10:** Distance determination of a cylindrical object with narrow-angled sensors
Generally, an object can be detected by an ultrasonic sensor if the surface of the object has a perpendicular reflection line falling within the detection angle of the sensor. This condition be defined as “detectability condition”. If an object is located within the detection cone of a sensor but the surface of the object has no perpendicular reflection line falling within the detection angle of the sensor, the transmitted ultrasound will be reflected away from the sensor. Figure 3.12 shows a situation with a plane object and narrow-angled sensors, where no echoes exist, since the detectability condition is not satisfied. Figure 3.13 shows a situation with the same plane object and wide-angled sensors, where direct echoes ($d_1$, $d_2$) as well as cross echoes ($c_{12}$, $c_{21}$) exist, since the detectability condition is complied with.
**Figure 3.12:** Object surface having a perpendicular falling outside the detection cones of narrow-angled sensors.

**Figure 3.13:** Object surface having a perpendicular falling inside the detection cones of wide-angled sensors.
The reliable detection of plane objects with smooth surfaces represents a challenge for ultrasonic sensing. For instance, the detection of mirrors and panes is very important, since these objects cannot be recognized by laser range finders. However, the detection of other materials can be of similar concern, since laser range finders also cannot recognize objects above and below the scanning level. In order to improve three-dimensional robot perception with wide-angled sensors, algorithms for detecting plane objects have been developed, which will be outlined in Section 4.3.

Distance readings have been acquired within the evaluation environment described in Section 1.5 with both ultrasonic sensing systems as well as with the laser range finders. For comparison between sonar readings and laser readings, a polygonal environment model is extracted from laser range finder data as will be explained in Section 4.2. Figure 3.14 shows the distance readings obtained from the Polaroid ultrasonic sensing system together with the polygonal environment model. Isometric lines for direct echoes obtained from the narrow-angled sensors are depicted as horizontal circular arcs of 30°. Figure 3.15 shows the first received distance readings obtained from the novel ultrasonic sensing system together with the polygonal environment model. Isometric lines for direct and cross echoes obtained from the wide-angled sensors are depicted as horizontal circular arcs of 60° and elliptic arcs of about 45°, 30°, or 15°, respectively. Echo paths with no detectable echo within the operating range of the sensors are represented by arcs corresponding to the maximum distance detection range \( d_{\text{max}} = 2 \text{ m} \).
Figure 3.14: Distance readings from the Polaroid ultrasonic sensing system

Figure 3.15: Distance readings from the novel ultrasonic sensing system
By comparing Figure 3.14 and Figure 3.15, the significantly higher number of echo arcs obtained with the novel ultrasonic sensing system in comparison with the Polaroid ultrasonic sensing system can be realized. In both figures it can be observed that echo arcs appear at \((x, y) = (1.3\ m, 0.0\ m)\) and \((x, y) = (0.8\ m, -0.7\ m)\) which do not correspond to the polygonal environment model. The differing distance readings arise from a table edge and from a chair back-rest. Due to the lower height of these objects they could not be recognized by the laser range finders and they are consequently not contained in the polygonal environment model extracted from laser range finder data. By examining the polygonal environment model exactly, it can also be noticed that the cardboard on the desk between the two computer tables (see Figure 1.7, Figure 1.8, and Figure 1.9) was not present during this experiment. Another cardboard extending one of the movable walls at \((x, y) = (0.0\ m, -0.5\ m)\) for test purposes (see Figure 1.7) was attached with the upper edge on a height exactly coinciding with the scanning level of the laser range finders. Therefore, this cardboard could be properly detected by the laser range finders only in some experiments. In the experimental results presented in Figure 3.14 and Figure 3.15, the cardboard was detected by both ultrasonic sensing systems, but not detected by the laser range finders. A typical phenomenon when a laser range finder beam strikes an object edge is that the reflection point appears to be further distant than the actual object edge. This effect will be observable in subsequent figures.

3.7 Contributions

In this chapter, a novel ultrasonic sensing system for autonomous mobile systems has been presented. The benefits of the newly developed ultrasonic sensing system and the concept of high-resolution ultrasonic sensing are manifold. The application of sensors with a broad beam width supports complete environmental coverage. Additionally, the proposed multi-aural sensor configuration with widely overlapping detection cones yields a high angular resolution due to exploitation of cross echoes between neighbor sensors. Moreover, the acquisition of multiple echoes per sensor further improves object perception. As a result, by a given number of sensors a significantly higher number of echoes can be utilized in comparison with conventional ultrasonic sensing systems for mobile robots. Consequently, it becomes unnecessary to rotate the whole robot or a sensor array in small angular steps around the vertical axis for scanning the environment. Thus, mechanical wear is prevented, battery energy is saved, the measurement
rate is significantly increased, and the navigation safety is improved. Up to 672 echoes can be received with 24 sensors per measurement cycle. The sampling rate of currently 4 measurement cycles per second can still be increased. Ultrasonic sensors designed for automotive applications are employed which are robust, reliable, and cheap. Consequently, they are suitable as well for usage on service robots. The proposed ultrasonic sensing system has gained industrial interest to be used on cleaning robots and automated guided vehicles (AGVs) for navigation control, and on electric wheelchairs for collision avoidance (e.g. with furniture or door-posts) during manual operation.
4 Local Environment Modeling

Within this chapter, it is described how local environment models can be created from laser range finder data and from ultrasonic sensor data. Two novel approaches of high-resolution ultrasonic imaging by the fusion of sensor data from multiple wide-angled ultrasonic transducers with overlapping detection cones are introduced. The novel fusion approaches, defined as “parameter space clustering” (PSC) and “tangential regression” (TR), consider the physical properties of sonar sensors as well as the reflection properties of ultrasound. This allows local environment modeling by reliable determination of straight line segments describing the boundary of objects. In Chapter 6, it will be demonstrated how local environment models created at different robot locations can be merged for building a global environment map.

An introduction to local environment modeling is given in Section 4.1. Environment modeling with laser range finder data is explained in Section 4.2, and environment modeling with ultrasonic sensor data by parameter space clustering and tangential regression is described in Section 4.3. Experimental results from various robot locations are presented in Section 4.4. The contributions of the chapter are reviewed in Section 4.5.

4.1 Introduction

Ultrasonic measurements may be accompanied by the problem of identifying the respective object to which the distance from a transducer has been measured. This problem seems irrelevant in applications such as sonar depth finding for ships. But even there, ambiguities occasionally arise between the sound being reflected either by the soil of the sea or by a dense swarm of fish. Ambiguities are far greater and more frequent in office and home environments, where service robots are supposed to operate. To encounter the ambiguity problem, arrays of sensors instead of single sensors can be used for determining the boundary of objects. In medical solutions, ultrasound pulses are phase modulated by applying so-called phased ar-
rays, which could possibly further increase angular resolution in ultrasonic environment modeling as well.

Within this thesis, high-resolution ultrasonic sensing is realized in a twofold sense. On the one hand, multiple wide-angled ultrasonic transducers are employed so that signals from several sensors may hit the same object and reflections may be received by the emitting sensors (direct echoes) as well as by passive neighbor sensors (cross echoes). On the other hand, data evaluation relies on fitting approaches that combine geometric models (geometric primitives) with reflection properties of ultrasound.

The geometric primitives under consideration are straight line segments, which can be spatially integrated for modeling complex environments. Information about the number and the length of straight line segments in a scene is not assumed to be available.

The object boundaries in the three-dimensional environment are assumed to be orthogonal to the horizontal ground plane in which the robot moves, and all sensors are arranged in a plane parallel to the ground plane. This allows two-dimensional modeling of the environment.

The detection angle of an ultrasonic sensor is defined by a main lobe which can be represented by a bounded detection cone. In case of a direct echo, the sensor measurement is associated with a reflecting object having a perpendicular reflection line falling within the detection angle of the sensor. In case of a cross echo, the special condition of orthogonal reflection is replaced by the general condition of the reflection angle being equal to the incidence angle, while cross echoes may occur within the overlapping detection domain of the involved sensors.

Sensors providing direct echoes will in the following be denoted as real sensors, and combinations of sensors delivering cross echoes will be referred to as virtual sensors. The imaginary location of a virtual sensor is in the middle of the two real sensors constituting the cross echo path.

It is distinguished between “reflecting lines” and “reflection lines” in a sense that reflecting lines represent reflecting surfaces which cause reflections, while reflection lines symbolize direct echo propagations to physical locations of real sensors or cross echo propagations to imaginary locations of virtual sensors.

Straight line extraction from ultrasonic sensor data has represented a research topic for many years. The methods described in the review of related work in Subsection 2.2.8 show deficiencies when they are used for fitting lines to sensor data acquired from a single robot location, owing to the reflection properties of ultrasound. Moreover, in most cases, the robot must be turned around its own axis in small angular steps in order to obtain a dense ultrasonic environment scan as a basis for straight line extraction.
Some related approaches utilize triangulation to determine the position of reflecting objects (see Subsection 2.1.7). Since triangulation-based algorithms are not very efficient if the sensor distances are small relative to the obstacle distances, the algorithms of parameter space clustering and tangential regression have been developed. By applying these algorithms, extraction of straight line segments becomes possible from a single robot location and without turning the robot around its own axis for data acquisition.

A few approaches related to the parameter space clustering algorithm also extract straight lines by applying a Hough transform [32, 49, 65]. However, these methods do not completely exploit the reflection properties of ultrasound, particularly the requirement of a perpendicular reflection line from a reflecting surface to a sensor.

In an approach closely related to the tangential regression algorithm [60], a common tangent is calculated for two circular arcs, obtained from consecutive range readings of the same sensor at different locations, by taking into account the orientation of the beam axis and the motion of the robot. A closed-form solution is developed for this geometry. Consequently, an over-determined situation of fitting a common tangent to more than two circular arcs is not considered.

The operation of the developed algorithms will be described in detail for the initial robot location within the office room introduced in Section 1.5, and final results will be presented for all robot locations marked by numbered crosses in Figure 1.9.

4.2 Environment Modeling with Laser Range Finder Data

As a basis to systematically compare sonar readings with laser readings, a polygonal model of the environment is created from the laser range finder data [128, 133]. Figure 4.1 displays the distance readings measured with the two laser scanners. The scanners are usually configured to carry out a 360°-scan with a resolution of 0.5°, consequently delivering up to 722 samples for a complete scan around the robot. The first laser scanner was orientated in positive y-direction and the second laser scanner in negative y-direction. Hence, the distance readings depicted as crosses (y > 0 m) result from the first scanner and the distance readings depicted as circles (y < 0 m) result from the second scanner. It might be observed that the resolution of the distance readings of scanner 1 is lower than that of scanner 2, since scanner 1 is a PLS scanner with 50 mm distance resolution and scanner 2 is a LMS scanner with 10 mm distance resolution.
To extract a polygonal model by orthogonal regression which fits to the data of both laser range finders, the following method is applied:

Firstly, the laser range finder samples are transformed from the polar coordinate systems of the scanners into the rectangular coordinate system of the robot. Then, the samples from both scanners are combined to a single list of data points. After that, a line is drawn through the first and the last data point within this list, i.e. the first sample of scanner 1 at 0° and the last sample of scanner 2 at 360°. Subsequently, the distances of all intermediate data points to this line are calculated, and the data point with the largest distance to the line is determined. Now, the list of data points is divided at this data point, and a reduced list is created containing the samples up to the dividing data point. Thereafter, a regression line is fitted to the data points of the reduced list, and the root mean square error \( RMSE \) is calculated. If the root mean square error is lower than a predefined threshold (e.g. \( RMSE < 0.02 \) m), the obtained regression line will be the first element of the polygonal model to be created. If the root mean square error is higher than the threshold, a new line is drawn through the first and the last data point within the reduced list. Again, the distances of all intermediate data points to this line are calculated, and the data point with the largest distance to the line is determined, which will be the next dividing point for creating a further reduced list. Now, a regression line is fitted to the data
points of the further reduced list, and again the root mean square error is calculated.

These steps are continued recursively until a regression line with a root mean square error lower than the predefined threshold will have been found. The final dividing point will then be the starting point for determining the next element of the polygonal model. For this purpose, a list of data points is created containing the samples from the final dividing point up to the last data point within the original list. Next, a regression line is fitted to the data points of this list, and the root mean square error is calculated. If the root mean square error is lower than the predefined threshold, the obtained regression line will be the second element of the polygonal model to be created. If the root mean square error is higher than the threshold, a line is drawn through the first and the last data point within this list, and it is continued in the same manner as described above until regression lines will have been fitted to all data points within the original list.

Now, the polygonal model still has to be closed between the last and the first data point. Therefore, the data points belonging to the last regression line and those appertaining to the first regression line are merged to a new list. Then, a regression line is fitted to the data points of the new list, and it is checked whether the root mean square error is still lower than the predefined threshold. If this is the case, the former two regression lines are replaced by the new one. If it is not the case, the new list of data points is divided according to the explained scheme in order to create two or more regression lines, which then replace the former two regression lines.

For related literature on regression line fitting to laser scanner data see [50, 59, 68].

The described method of extracting a polygonal model from laser range finder data is a typical “divide and conquer” algorithm with a time complexity of $O(m \log m)$ in the average case and $O(m^2)$ in the worst case, where $m$ is the number of samples in the complete scan around the robot.

Regression lines are described using Hesse’s normal form of the equation of a line with the perpendicular distance $\rho$ from the origin in the coordinate system and the angle $\theta$ between the normal of the line and the $x$-axis:

$$x \cos \theta + y \sin \theta = \rho$$

(4.1)
Given a subset $Z$ of $n$ data point vectors $z_i$:

$$Z = \{z_i\} = \left\{ \begin{bmatrix} x_i \\ y_i \end{bmatrix} \right\}, \quad i = 1, \ldots, n$$  \hfill (4.2)

In order to fit a regression line to the subset $Z$ of $n$ data point vectors $z_i$, the following objective function is minimized:

$$F(\rho, \theta) = \sum_{i=1}^{n} (x_i \cos \theta + y_i \sin \theta - \rho)^2$$  \hfill (4.3)

The objective function describes a typical case of orthogonal regression with uncertainties in both coordinates, i.e. with data point offsets perpendicular to the regression line. A closed-form solution for calculating the regression line parameters $\rho$ and $\theta$ as well as an equation for the associated sum squared error $SSE$ are e.g. given in [59]:

$$\rho = \pi \cos \theta + \gamma \sin \theta$$  \hfill (4.4)

$$\theta = \frac{1}{2} \arctan \left( \frac{-2S_{xy}}{S_{yy} - S_{xx}} \right)$$  \hfill (4.5)

$$SSE = \min_{\rho \in \mathbb{R}, \theta \in [0, 2\pi]} F(\rho, \theta) = \frac{1}{2} \left( S_{xx} + S_{yy} - \sqrt{4S_{xy}^2 + (S_{xy} - S_{xx})^2} \right)$$  \hfill (4.6)

with

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$  \hfill (4.7)

$$\bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i$$  \hfill (4.8)
The root mean square error $RMSE$ of a subset of $n$ data points to the fitted regression line is:

$$ RMSE = \sqrt{\frac{SSE}{n}} $$  

(4.12)

When all straight line elements of the polygonal model have been obtained, the intersections between consecutive regression lines can be calculated, and straight line segments of defined length can be determined. Figure 4.2 presents the polygonal model fitted to the data of both laser range finders.
Figure 4.2: Polygonal model fitted to the data of both laser range finders

Regarding the polygonal model, it is desired to distinguish between real line segments which describe existent features of the environment and pseudo line segments which arise due to the observation perspective of the laser range finders. The existence of a pseudo line segment is assumed if $\rho < 0.4 \text{ m}$. This threshold has been heuristically ascertained. To determine pseudo line segments at the transition points between both laser range finders, straight line segments having different signs in the $y$-coordinates of their endpoints are checked whether these endpoints coincide with the minimum and maximum detection angles of the scanners. Figure 4.3 shows the polygonal environment model created from laser range finder data, including all real line segments, pseudo line segments, and their intersections, which are illustrated as solid lines, dashed lines, and points, respectively.
The steps of the algorithm for creating an environment model from laser range finder data by orthogonal regression are summarized in a flow chart in Figure 4.4. The main procedure calls subroutines for extracting regression lines and for data fusion between the last and the first samples. The subroutines are illustrated in separate flow charts in Figure 4.5 and Figure 4.6.
Figure 4.4: Main procedure for creating an environment model from laser range finder data by orthogonal regression
4.2 Environment Modeling with Laser Range Finder Data

Figure 4.5: Subroutine for extracting regression lines
4.3 Environment Modeling with Ultrasonic Sensor Data

In order to benefit from the increased sensor information obtained with the novel ultrasonic sensing system, algorithms for adequate data processing are required. For this reason, the approaches of parameter space clustering (PSC) and tangential regression (TR) have been developed. This section describes both algorithms and their application for creating local environment models from ultrasonic sensor data.

Figure 4.6: Subroutine for data fusion between the last and the first samples
4.3.1 Parameter Space Clustering (PSC)

The approach for straight line extraction presented within this subsection is the algorithm of parameter space clustering [133]. The idea of parameter space clustering in brevity is to discretize all circular and elliptic echo arcs by potential reflecting lines of infinitesimal length, to Hough transform the potential reflecting lines, to cluster them in parameter space, and to calculate the cluster centers that represent extracted straight line elements.

**Process of Parameter Space Clustering**

An environment model can be created from ultrasonic sensor data by parameter space clustering as follows:

Firstly, all circular echo arcs of real sensors and elliptic echo arcs of virtual sensors within the attainable distance detection range are represented by a predefined number \( n_p \) of equidistant points on the arc. Then, it is assumed that an echo could originate from the surface of an object being located at each discrete point on the echo arc and having a perpendicular reflection line to the real or virtual sensor. Potential reflecting surfaces are assumed to be vertical, and they are represented by reflecting lines on the level of the sensors, thus implying horizontal echo paths between the sensors and the objects. It is recommendable to discretize all echo arcs by the same number \( n_p \) of points, regardless of their length. This accommodates the fact that shorter echo arcs, emerging either due to shorter distance readings or due to cross echoes, yield a more constricted location information on the arc. Figure 4.7 shows the discretization of an echo arc by e.g. \( n_p = 6 \) equidistant points as well as associated straight line elements representing potential reflecting surfaces. In this figure, the dashed lines indicate perpendicular reflection lines for ultrasound from potential reflecting surfaces to a real or virtual sensor, and the dashed-dotted lines indicate the \( \rho/\theta \)-representation of the potential reflecting lines with regard to the origin and the \( x \)-axis in the coordinate system.
By using Hesse’s normal form of the equation of a line, the potential reflecting lines are Hough transformed into parameter space, and can be illustrated as points in a rectangular $\rho/\theta$-diagram. Now, a $100 \times 100$ occupancy grid $A$ is deployed in parameter space, with a cell size of 0.03 m in $\rho$-dimension and a cell size of 0.0628 rad in $\theta$-dimension. Then, the number of points in each cell is accumulated. Thereafter, a two-dimensional low-pass filter is applied to the occupancy grid $A$ in order to obtain a filtered occupancy grid $B$ by smoothing between neighbor cells. The implemented filter is an 11-by-11 zero phase FIR (finite impulse response) filter, having its cutoff at:

$$\frac{f_c}{f_N} = 0.2$$  \hspace{1cm} (4.13)

where $f_c$ is the cutoff frequency and $f_N$ is the Nyquist frequency. The Nyquist frequency $f_N$ is equal to half the sampling frequency $f_s$. The zero phase property maintains the integrity of the filtered data by preventing phase distortion. The filter has been designed by the window method using a Hamming window. The 11-by-11 filter matrix $h$ is visualized in Figure 4.8.
Filtering an occupancy grid $A$ with the filter matrix $h$ is equivalent to convolving $A$ with $h$. Thus, the two-dimensional filter equation for computing the filtered occupancy grid $B$ is:

$$B(n_1, n_2) = A(n_1, n_2) * h(n_1, n_2) = \sum_{i_1} \sum_{i_2} A(i_1, i_2) \cdot h(n_1 - i_1, n_2 - i_2) \quad (4.14)$$

Next, it is searched for highly occupied grid cells in the filtered occupancy grid $B$ by comparing the occupancy values with a predefined threshold. According to this, all points belonging to cells with an occupancy value beneath the threshold are discarded and all points belonging to cells with an occupancy value beyond the threshold are used for cluster formation. The clusters consist of points belonging to regions of cohesive highly occupied grid cells, that are determined by a connected components analysis with 8-neighborhood connectivity. The proposed approach via connected components analysis allows cluster formation without requiring the number of expected clusters to be predefined. Subsequently, the cen-
ters of gravity for the clusters are calculated, which represent the parameters $\rho$ and $\theta$ of extracted straight line elements:

\[
\rho = \bar{\rho} = \frac{1}{n} \sum_{i=1}^{n} \rho_i
\]  

(4.15)  

\[
\theta = \bar{\theta} = \frac{1}{n} \sum_{i=1}^{n} \theta_i
\]  

(4.16)

where $n$ is the number of points within a cluster, $(\rho_i, \theta_i)^T$ are the coordinates of the points belonging to a cluster, and $(\bar{\rho}, \bar{\theta})^T$ are the coordinates of the respective cluster center.

The covariance matrix $R$ representing the uncertainties associated with the parameters $\rho$ and $\theta$ of extracted straight line elements is estimated as follows:

\[
R = \begin{bmatrix}
\sigma^2_{\rho} & \sigma_{\rho\theta} \\
\sigma_{\rho\theta} & \sigma^2_{\theta}
\end{bmatrix}
\]

\[
\approx \begin{bmatrix}
\frac{1}{n-1} \sum_{i=1}^{n} (\rho_i - \bar{\rho})^2 & \frac{1}{n-1} \sum_{i=1}^{n} (\rho_i - \bar{\rho})(\theta_i - \bar{\theta}) \\
\frac{1}{n-1} \sum_{i=1}^{n} (\rho_i - \bar{\rho})(\theta_i - \bar{\theta}) & \frac{1}{n-1} \sum_{i=1}^{n} (\theta_i - \bar{\theta})^2
\end{bmatrix}
\]  

(4.17)

where $\sigma_\rho$ is the standard deviation (uncertainty) in the perpendicular distance $\rho$ from the origin in the coordinate system, $\sigma_\theta$ is the standard deviation (uncertainty) in the angle $\theta$ between the normal of the line and the $x$-axis, and $\sigma_{\rho\theta}$ is the covariance between $\rho$ and $\theta$. The parameters $\rho$ and $\theta$ are uncorrelated (i.e., $\sigma_{\rho\theta} \approx 0$), as can be verified by empirical analyses.

Finally, the individual extent of the extracted straight line elements can be determined in order to derive straight line segments of defined length. For this purpose, all points belonging to regions of cohesive highly occupied grid cells are regarded, and the real and virtual sensors they stem from are identified. This is in consequence of the fact that a real or virtual sensor
contributed to the constitution of a straight line element if a Hough transformed potential reflecting line associated with a discrete point on the echo arc of the respective sensor is involved in the calculation of the cluster center representing the extracted straight line element in parameter space. For each extracted straight line element, a list of the real and virtual sensors that contributed to the respective cluster is created. Then, perpendicular reflection lines are drawn from the extracted straight line elements to the contributive real and virtual sensors. Next, the two outermost perpendicular reflection lines are determined for each extracted straight line element. The points of intersection between the extracted straight line elements and their outermost perpendicular reflection lines define the endpoints and the length of straight line segments.

The parameters of the algorithm are the number \( n_p \) of equidistant points per echo arc, the cell size as well as the number of cells in the occupancy grid, the cutoff \( f_c / f_N \) of the two-dimensional low-pass filter, and the occupancy threshold for defining highly occupied grid cells.

In the described way, straight line segments can be extracted at different robot locations and can be spatially integrated in order to obtain a detailed environment model. Figure 4.9 illustrates the steps of the algorithm for creating an environment model from ultrasonic sensor data by parameter space clustering in a flow chart. The main procedure calls a subroutine for determining straight line segments of defined length. The subroutine is illustrated in a separate flow chart in Figure 4.10.
Figure 4.9: Main procedure for creating an environment model from ultrasonic sensor data by parameter space clustering
4.3 Environment Modeling with Ultrasonic Sensor Data

Subroutine
“determine straight line segments of defined length”

For all points belonging to regions of cohesive highly occupied grid cells, identify the real and virtual sensors they stem from.

For each extracted straight line element, create a list of the real and virtual sensors that contributed to the respective cluster.

Draw perpendicular reflection lines from the extracted straight line elements to the contributive real and virtual sensors.

Determine the two outermost perpendicular reflection lines for each extracted straight line element.

Calculate the intersections between the extracted straight line elements and their outermost perpendicular reflection lines.

The points of intersection define the endpoints and the length of straight line segments.

Return to main procedure

Figure 4.10: Subroutine for determining straight line segments of defined length

Example of Parameter Space Clustering

Figure 4.11 shows the distance readings from the ultrasonic sensing system together with the polygonal environment model created from laser range finder data. Figure 4.12 displays the discretization of all ultrasonic echo arcs by \( n_p = 11 \) equidistant points. Figure 4.13 illustrates the Hough transformed line elements that represent potential reflecting surfaces as points in a rectangular \( \rho/\theta \)-diagram. Figure 4.14 and Figure 4.15 show the occupancy grid in parameter space before and after applying a two-dimensional low-pass filter. Figure 4.16 displays regions of cohesive highly occupied grid cells remaining after thresholding. Figure 4.17 illustrates the points belonging to grid cells with an occupancy value beyond the threshold forming clusters as well as the calculated cluster centers representing extracted straight line elements in parameter space. Figure 4.18 shows the extracted straight line elements in parameter space. Figure 4.19 displays one extracted straight line element in detailed representation. Figure 4.20 illustrates how the points of intersection between the extracted straight line elements and their outermost perpendicular reflection lines define the endpoints and the length of straight line segments.
Figure 4.11: Distance readings from the ultrasonic sensing system

Figure 4.12: Discretization of the ultrasonic echo arcs by equidistant points
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Figure 4.13: Hough transformed line elements in parameter space

Figure 4.14: Occupancy grid in parameter space
Figure 4.15: Occupancy grid in parameter space after applying a two-dimensional low-pass filter

Figure 4.16: Regions of cohesive highly occupied grid cells remaining after thresholding
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Figure 4.17: Cluster centers representing extracted straight line elements in parameter space

Figure 4.18: Extracted straight line elements in workspace (bold lines)
Figure 4.19: Detailed representation of one extracted straight line element (bold line)

Figure 4.20: Determining straight line segments of defined length
It can be noted that the three straight line segments extracted from ultrasonic sensor data by parameter space clustering correspond to straight line segments in the polygonal environment model created from laser range finder data. The maximum deviation between corresponding lines is $\Delta \rho = 0.032$ m in distance and $\Delta \theta = 7.415^\circ$ in orientation.

### 4.3.2 Tangential Regression (TR)

The approach for straight line extraction presented within this subsection is the algorithm of tangential regression [136]. The idea of tangential regression is to calculate the parameters of a tangential regression line so that it represents the best fit to multiple circular or elliptic echo arcs of distance measurements from different sensor positions. The implicated particularity is that the distance measurements indicate potential regression points on curves instead of fixed regression points. This attribute is required because of the broad detection domain of wide-angled ultrasonic sensors.

Using ultrasonic measurements for geometric model alignments entails the difficulty that the number of measurements may be lower or higher than the number of parameters of the geometric primitives under consideration, leading to under-constrained or over-constrained alignment equations. Over-constrained alignment problems are formulated as fitting problems.

As already described, ultrasonic measurements of a planar surface by real or virtual sensors lead in a two-dimensional setting to circular or elliptic arcs of potential reflecting points associated with tangents to the curves (reflecting lines) representing potential reflecting surfaces. The objective of tangential regression for direct and cross echoes is to fit a common tangent to a number of circular and elliptic arcs obtained by measurements from adjacent sensors. Evidently, the positions of geometric primitives can be determined when several measurements are available.

Minimizing the Euclidean distance between points and simple curves is aimed in orthogonal regression (see e.g. [118]), and in constructing fairly general curves such as so-called principal curves (see e.g. [121]). Like in standard least squares regression, the data points for orthogonal regression as well as for principal curve approximation are given explicitly and remain fixed.

An essential feature of tangential regression for geometric model fitting to ultrasonic sensor data is that the assumed positions of the reflecting points vary along isometric lines (circular or elliptic arcs) due to the low angular resolution of ultrasonic sensor measurements, and according to the assumed position of a geometric primitive in order to assure sound propa-
gation obeying the condition of the reflection angle being equal to the incidence angle. This is in contrast to orthogonal regression for geometric model fitting to laser range finder data, where the positions of the reflecting points are considered to remain fixed justified by the high angular resolution of laser range finder measurements, and where the condition of the reflection angle being equal to the incidence angle is irrelevant due to the reflection properties of sound and light being different (see Subsection 2.1.1).

**Direct Echoes**

Within this paragraph, tangential regression is formally established for direct echoes. In the case of a direct echo, the distance between the sensor and the reflecting point equals half the distance that the ultrasound wave has traveled. The reflecting point is located on a circular arc around the sensor position. Restrictions due to the bounded detection cone of the ultrasonic sensors will be considered.

The non-trivial fitting cases for a straight line are that of \( n \geq 3 \) measurements. Tangential Regression will be derived firstly for the special case of one-dimensional sensor arrangement and secondly for the general case of two-dimensional sensor arrangement. The fitting problem is visualized for one-dimensional sensor arrangement in Figure 4.21 and for two-dimensional sensor arrangement in Figure 4.22.
Figure 4.21: Fitting problem for one-dimensional sensor arrangement

Figure 4.22: Fitting problem for two-dimensional sensor arrangement
The following denotations are introduced together with the figures:

- sensor positions $s_i$
- distance measurements $m_i$
- potential reflecting points on the echo arcs $p_i$
- tangential regression line $H$
- assumed reflecting points on the tangential regression line $q_i$

A number $n$ of sensors be located at known sensor positions $s_i = (s_{i,x}, s_{i,y}) \in \mathbb{R}^2, \ i = 1, \ldots, n$, which refer to a Cartesian coordinate system. In case the sensors are mounted on a mobile platform, the coordinate system moves with the platform so that the sensor coordinates remain fixed. Thus, all geometry is at this stage expressed in the robot coordinate system. The sensors obtain measurements $m_i \in \mathbb{R}_0^+, \ i = 1, \ldots, n$, which are presumed to stem from reflections by the same straight line.

The objective is to calculate the parameters of a tangential regression line $H = \{ x \mid n^T x = \rho \}$ such that the sum of squared distances between potential reflecting points on the echo arcs $p_i$ and assumed reflecting points on the tangential regression line $q_i$ becomes minimal:

$$\min \sum_{i=1}^n \| p_i - q_i \|^2$$

(4.18)

where $\| p_i - q_i \|$ denotes the Euclidean distance between a potential reflecting point $p_i$ and an assumed reflecting point $q_i$.

Two conditions must be fulfilled by an obtained regression line in order to be valid:

- The root mean square error $RMSE$ of a subset of $n$ potential reflecting points on the echo arcs $p_i$ to the fitted regression line must be lower than a predefined threshold.
- The fitted regression line must have a perpendicular reflection line falling within the detection range of all sensors involved.

Ideally, the tangential regression line $H$ becomes a common tangent to all circular arcs with radii $m_i$ around sensor positions $s_i$ by intersecting the measurement circles at the potential reflecting points $p_i$. 
The problem for \( n = 3 \) measurements is loosely related to the Apollonius Tenth Problem, where for any three given circles a general circle is sought that is tangential to the given circles. A general circle can either be a proper circle or a straight line. However, the Apollonius Tenth Problem describes a perfect circular fit instead of a linear regression.

Hesse’s normal form of the equation of a line \( H \) with the perpendicular distance \( \rho \) from the origin in the coordinate system and the angle \( \theta \) between the unit normal vector \( \mathbf{n} \) of the line and the \( x \)-axis is given by:

\[
x \cos \theta + y \sin \theta - \rho = 0 \tag{4.19}
\]

\[
\mathbf{n}^T \mathbf{x} - \rho = 0 \tag{4.20}
\]

with

\[
\mathbf{n} = \begin{bmatrix} n_x \\ n_y \end{bmatrix} = \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix} \tag{4.21}
\]

\[
\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix} \tag{4.22}
\]

The distance \( d \) of an arbitrary point \( \mathbf{p} = (p_x, p_y) \in \mathbb{R}^2 \) to the line \( H \) is:

\[
d = p_x \cos \theta + p_y \sin \theta - \rho \tag{4.23}
\]

\[
d = \mathbf{n}^T \mathbf{p} - \rho \tag{4.24}
\]

Inserting the assumed reflecting points on the tangential regression line \( \mathbf{q}_i = (q_{i,x}, q_{i,y}) \in \mathbb{R}^2 \), \( i = 1, \ldots, n \), into Hesse’s normal form results in:

\[
q_{i,x} \cos \theta + q_{i,y} \sin \theta - \rho = 0, \quad i = 1, \ldots, n \tag{4.25}
\]
Correspondingly, inserting the potential reflecting points on the echo arcs \( \mathbf{p}_i = (p_{i,x}, p_{i,y})^T \in \mathbb{R}^2 \), \( i = 1, \ldots, n \), into Hesse’s normal form results in:

\[
d_i = \| \mathbf{p}_i - \mathbf{q}_i \| = p_{i,x} \cos \theta + p_{i,y} \sin \theta - \rho, \quad i = 1, \ldots, n
\] (4.26)

The coordinates of the potential reflecting points on the echo arcs \( \mathbf{p}_i = (p_{i,x}, p_{i,y})^T \) can be expressed as:

\[
p_{i,x} = s_{i,x} + m_i \cos \theta, \quad i = 1, \ldots, n
\] (4.27)

\[
p_{i,y} = s_{i,y} + m_i \sin \theta, \quad i = 1, \ldots, n
\] (4.28)

Inserting Eq. 4.27 and Eq. 4.28 into Eq. 4.26 yields:

\[
\| \mathbf{p}_i - \mathbf{q}_i \| = p_{i,x} \cos \theta + p_{i,y} \sin \theta - \rho
\]
\[
= (s_{i,x} + m_i \cos \theta) \cos \theta + (s_{i,y} + m_i \sin \theta) \sin \theta - \rho
\]
\[
= s_{i,x} \cos \theta + s_{i,y} \sin \theta + m_i (\sin^2 \theta + \cos^2 \theta) - \rho
\]
\[
= s_{i,x} \cos \theta + s_{i,y} \sin \theta + m_i - \rho, \quad i = 1, \ldots, n
\] (4.29)

Eq. 4.29 is valid for \( n \mathbf{s}_i - \rho < 0, \quad i = 1, \ldots, n \), which means that all sensors are located in the negative half plane of the tangential regression line \( H \). The condition of all sensors lying on one side of the tangential regression line \( H \) must be agreeable with the geometric setting of the problem, i.e. it must be guaranteed from the measurement constellation that no reflecting surface may pass between the sensors. This is typically the case when the sensors are arranged along the periphery of a mobile platform.

Thus, the trigonometric formulation of the objective function is:

\[
F(\rho, \theta) = \sum_{i=1}^{n} \| \mathbf{p}_i - \mathbf{q}_i \|^2 = \sum_{i=1}^{n} (s_{i,x} \cos \theta + s_{i,y} \sin \theta + m_i - \rho)^2
\] (4.30)
A) A closed-form solution of tangential regression for one-dimensional sensor arrangement \((s_{i,y} = 0, \ i = 1, \ldots, n)\) can be derived by minimizing the trigonometric formulation of the objective function in Eq. 4.30:

\[
\min_{\rho \in \mathbb{R}_{+}, \theta \in [0, 2\pi]} F(\rho, \theta) = \min_{\rho \in \mathbb{R}_{+}, \theta \in [0, 2\pi]} \sum_{i=1}^{n} (s_{i,x} \cos \theta + s_{i,y} \sin \theta + m_i - \rho)^2
\]

\[
= \min_{\rho \in \mathbb{R}_{+}, \theta \in [0, 2\pi]} \sum_{i=1}^{n} (s_{i,x} \cos \theta + 0 \cdot \sin \theta + m_i - \rho)^2
\]

\[
= \min_{\rho \in \mathbb{R}_{+}, \theta \in [0, 2\pi]} \sum_{i=1}^{n} (s_{i,x} \cos \theta + m_i - \rho)^2
\]  

(4.31)

Necessary condition for a minimum is:

\[
\nabla F(\rho, \theta) = 0
\]  

(4.32)

The partial derivatives are:

\[
\frac{\partial F}{\partial \rho} = \sum_{i=1}^{n} 2 \cdot (s_{i,x} \cos \theta + m_i - \rho) \cdot (-1) = 0
\]  

(4.33)

\[
\frac{\partial F}{\partial \theta} = \sum_{i=1}^{n} 2 \cdot (s_{i,x} \cos \theta + m_i - \rho) \cdot (-s_{i,x} \sin \theta) = 0
\]  

(4.34)

Solving Eq. 4.33 for the variable \(\rho\) results in:

\[
\rho = \frac{1}{n} \sum_{i=1}^{n} s_{i,x} \cos \theta + \frac{1}{n} \sum_{i=1}^{n} m_i
\]

\[
= \bar{s}_{i,x} \cos \theta + \bar{m}_i
\]  

(4.35)
Inserting the expression for $\rho$ into Eq. 4.34 and then solving Eq. 4.34 for $\theta$ yields two solutions:

$$\theta_{1,2} = \pm \arccos \left( \frac{\sum_{i=1}^{n} m_i s_{i,x} - n \cdot \bar{m}_i \bar{s}_{i,x}}{\sqrt{\sum_{i=1}^{n} s_{i,x}^2 - n \cdot \bar{s}_{i,x}^2}} \right)$$  \hspace{1cm} (4.36)

Whenever all sensors are arranged on a straight line, the solution is not unique. Reflecting an optimal location of the tangential regression line $H$ along the straight line through the sensor positions leads to another optimal location. However, only one solution for $\theta$ may fulfill both conditions for a valid regression line (see Figure 4.21).

B) A closed-form solution of tangential regression for two-dimensional sensor arrangement ($s_{i,y} \in \mathbb{R}, i = 1, \ldots, n$) by minimizing the trigonometric formulation of the objective function in Eq. 4.30 appears to be not derivable. For this reason, Eq. 4.30 is transformed into an algebraic formulation of the objective function:

$$F(\rho, n_x, n_y) = \sum_{i=1}^{n} \|p_i - q_i\|^2 = \sum_{i=1}^{n} \left( n_x s_{i,x} + n_y s_{i,y} + m_i - \rho \right)^2$$  \hspace{1cm} (4.37)

where the unity condition for the normal vector $\mathbf{n} = (n_x, n_y)^T$ must be satisfied:

$$\|\mathbf{n}\| = \sqrt{n_x^2 + n_y^2} = \sqrt{\cos^2 \theta + \sin^2 \theta} = 1$$  \hspace{1cm} (4.38)

$$n_y = \pm \sqrt{1 - n_x^2}$$  \hspace{1cm} (4.39)
C) An approximate solution for two-dimensional sensor arrangement can be obtained by minimizing the algebraic formulation of the objective function in Eq. 4.37:

\[
\min_{\rho \in \mathbb{R}^n, n_x, n_y \in [-1,1], \alpha, \beta \in [-1,1]} F(\rho, n_x, n_y)
\]

\[
= \min_{\rho \in \mathbb{R}^n, n_x, n_y \in [-1,1], \alpha, \beta \in [-1,1]} \sum_{i=1}^{n} (n_x s_{i,x} + n_y s_{i,y} + m_i - \rho)^2
\]

(4.40)

The approximate solution involves first neglecting the unity condition for the normal vector \( n \) of the line \( H \) and later normalizing the solution vector \( n^0 \) of the relaxed problem.

Necessary condition for a minimum is:

\[
\nabla F(\rho, n_x, n_y) = \mathbf{0}
\]

(4.41)

The partial derivatives are:

\[
\frac{\partial F}{\partial \rho} = \sum_{i=1}^{n} 2 \cdot (n_x s_{i,x} + n_y s_{i,y} + m_i - \rho) \cdot (-1) = 0
\]

(4.42)

\[
\frac{\partial F}{\partial n_x} = \sum_{i=1}^{n} 2 \cdot (n_x s_{i,x} + n_y s_{i,y} + m_i - \rho) \cdot s_{i,x} = 0
\]

(4.43)

\[
\frac{\partial F}{\partial n_y} = \sum_{i=1}^{n} 2 \cdot (n_x s_{i,x} + n_y s_{i,y} + m_i - \rho) \cdot s_{i,y} = 0
\]

(4.44)
Solving Eq. 4.42 for the variable \( \rho \) results in:

\[
\rho = n_x \cdot \frac{1}{n} \sum_{i=1}^{n} s_{i,x} + n_y \cdot \frac{1}{n} \sum_{i=1}^{n} s_{i,y} + \frac{1}{n} \sum_{i=1}^{n} m_i
\]

\[
= n_x \cdot \bar{s}_{i,x} + n_y \cdot \bar{s}_{i,y} + \bar{m}_i
\]

(4.45)

Inserting the expression for \( \rho \) into Eq. 4.43 and Eq. 4.44 leads to the following two-equation linear system:

\[
\begin{bmatrix}
\sum_{i=1}^{n} s_{i,x}^2 - n \cdot \bar{s}_{i,x}^2 & \sum_{i=1}^{n} s_{i,x} s_{i,y} - n \cdot \bar{s}_{i,x} \bar{s}_{i,y} \\
\sum_{i=1}^{n} s_{i,x} s_{i,y} - n \cdot \bar{s}_{i,x} \bar{s}_{i,y} & \sum_{i=1}^{n} s_{i,y}^2 - n \cdot \bar{s}_{i,y}^2
\end{bmatrix}
\begin{bmatrix}
 n_x \\
n_y
\end{bmatrix}
= \begin{bmatrix}
-\sum_{i=1}^{n} m_i s_{i,x} + n \cdot \bar{m}_i \bar{s}_{i,x} \\
-\sum_{i=1}^{n} m_i s_{i,y} + n \cdot \bar{m}_i \bar{s}_{i,y}
\end{bmatrix}
\]

(4.46)

The solution vector \( \mathbf{n}^0 = (n_x^0, n_y^0)^T \) of the two-equation linear system in Eq. 4.46 is:

\[
n_x^0 = \frac{\left( \sum_{i=1}^{n} s_{i,x} s_{i,y} - n \cdot \bar{s}_{i,x} \bar{s}_{i,y} \right) \left( \sum_{i=1}^{n} m_i s_{i,y} - n \cdot \bar{m}_i \bar{s}_{i,y} \right) - \left( \sum_{i=1}^{n} s_{i,x} s_{i,y} - n \cdot \bar{s}_{i,x} \bar{s}_{i,y} \right) \left( \sum_{i=1}^{n} m_i s_{i,x} - n \cdot \bar{m}_i \bar{s}_{i,x} \right)}{\left( \sum_{i=1}^{n} s_{i,x}^2 - n \cdot \bar{s}_{i,x}^2 \right) \left( \sum_{i=1}^{n} s_{i,y}^2 - n \cdot \bar{s}_{i,y}^2 \right) - \left( \sum_{i=1}^{n} s_{i,x} s_{i,y} - n \cdot \bar{s}_{i,x} \bar{s}_{i,y} \right) \left( \sum_{i=1}^{n} m_i s_{i,x} - n \cdot \bar{m}_i \bar{s}_{i,x} \right)}
\]

(4.47)
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\[
n^0_y = \frac{\sum_{i=1}^{n} s_{i,x} s_{i,y} - n \cdot \bar{s}_{i,x} \bar{s}_{i,y}}{\left( \sum_{i=1}^{n} s_{i,x}^2 - n \cdot \bar{s}_{i,x}^2 \right) \left( \sum_{i=1}^{n} s_{i,y}^2 - n \cdot \bar{s}_{i,y}^2 \right)} \left( \sum_{i=1}^{n} m_i s_{i,x} - n \cdot \bar{m}_i \bar{s}_{i,x} \right) \left( \sum_{i=1}^{n} m_i s_{i,y} - n \cdot \bar{m}_i \bar{s}_{i,y} \right)^2
\]

(4.48)

If the solution vector \( n^0 = (n^0_x, n^0_y) \) is a unit normal vector, then it also is an exact solution of the tangential regression problem. Otherwise, the solution vector \( n^0 \) can be normalized by \( \|n^0\| \) to become an approximate solution of the tangential regression problem:

\[
n^0_x \rightarrow \frac{n^0_x}{\|n^0\|}
\]

(4.49)

\[
n^0_y \rightarrow \frac{n^0_y}{\|n^0\|}
\]

(4.50)

D) Without affecting the geometry and the measurements, the sensor positions can be translated such that their center of gravity falls into the origin of the coordinate system. Denoting the new sensor coordinates by \( s'_i = (s'_{i,x}, s'_{i,y}) \) leads to:

\[
\frac{1}{n} \sum_{i=1}^{n} s'_{i,x} = \bar{s}'_{i,x} = 0
\]

(4.51)

\[
\frac{1}{n} \sum_{i=1}^{n} s'_{i,y} = \bar{s}'_{i,y} = 0
\]

(4.52)
The two-equation linear system in Eq. 4.46 then simplifies to:

\[
\begin{bmatrix}
\sum_{i=1}^{n} s_{i,x}^2 & \sum_{i=1}^{n} s_{i,x} s_{i,y} \\
\sum_{i=1}^{n} s_{i,x} s_{i,y} & \sum_{i=1}^{n} s_{i,y}^2
\end{bmatrix}
\begin{bmatrix}
 n_x \\
n_y
\end{bmatrix}
= 
\begin{bmatrix}
 -\sum_{i=1}^{n} m_i s_{i,x}' \\
 -\sum_{i=1}^{n} m_i s_{i,y}'
\end{bmatrix}
\]  

(4.53)

The solution vector \( \mathbf{n}^0 = (n_x^0, n_y^0)^T \) of the simplified two-equation linear system in Eq. 4.53 becomes:

\[
n_x^0 = \frac{\sum_{i=1}^{n} s_{i,x} s_{i,y}' \cdot \sum_{i=1}^{n} m_i s_{i,y}' - \sum_{i=1}^{n} s_{i,y}^2 \cdot \sum_{i=1}^{n} m_i s_{i,x}'}{\sum_{i=1}^{n} s_{i,x}'^2 \cdot \sum_{i=1}^{n} s_{i,y}'^2 - \left( \sum_{i=1}^{n} s_{i,x}' s_{i,y}' \right)^2} 
\]

\[
n_y^0 = \frac{\sum_{i=1}^{n} s_{i,x} s_{i,y}' \cdot \sum_{i=1}^{n} m_i s_{i,y}' - \sum_{i=1}^{n} s_{i,y}^2 \cdot \sum_{i=1}^{n} m_i s_{i,x}'}{\sum_{i=1}^{n} s_{i,x}'^2 \cdot \sum_{i=1}^{n} s_{i,y}'^2 - \left( \sum_{i=1}^{n} s_{i,x}' s_{i,y}' \right)^2} 
\]

(4.54), (4.55)

As described, the solution vector \( \mathbf{n}^0 \) may require normalization to unity.

According to the Cauchy Schwartz inequality (cp. [103]) the determinant of the two-equation linear system in Eq. 4.53 is always greater or equal to zero. Equality to zero is given if one of the following conditions is fulfilled:

\[
s_{i,x}' = \alpha_1 \cdot s_{i,y}', \quad \forall i, \alpha_1 \in \mathbb{R} 
\]

(4.56)

\[
s_{i,y}' = \alpha_2 \cdot s_{i,x}', \quad \forall i, \alpha_2 \in \mathbb{R} 
\]

(4.57)
The conditions in Eq. 4.56 and Eq. 4.57 imply that all sensors are arranged on a straight line through the origin of the coordinate system. Thus, whenever this is not the case, the determinant is different from zero and the solution of the linear system is unique.

E) An exact solution for two-dimensional sensor arrangement can be derived by a Lagrangian approach, where the algebraic formulation of the objective function and the side condition for the normal vector \( \mathbf{n} \) yield the Lagrange function. Assuming again for simplification that the center of gravity of the sensor positions falls into the origin of the coordinate system, the Lagrange function becomes:

\[
L(\rho', n_x, n_y, \lambda) = \sum_{i=1}^{n} \| \mathbf{p}_i - \mathbf{q}_i \|^2
\]

\[
= \sum_{i=1}^{n} \left( n_i s_{i,x} + n_i s_{i,y} + m_i - \rho' \right)^2 + \lambda (n_x^2 + n_y^2 - 1)
\]

The critical point conditions for the Lagrange function are:

\[
\frac{\partial L}{\partial \rho'} = \sum_{i=1}^{n} 2 \left( n_i s_{i,x} + n_i s_{i,y} + m_i - \rho' \right) \cdot (-1) = 0
\]  \hspace{1cm} (4.59)

\[
\frac{\partial L}{\partial n_x} = \sum_{i=1}^{n} 2 \left( n_i s_{i,x} + n_i s_{i,y} + m_i - \rho' \right) \cdot s_{i,x}' + 2 \cdot \lambda \cdot n_x = 0
\]  \hspace{1cm} (4.60)

\[
\frac{\partial L}{\partial n_y} = \sum_{i=1}^{n} 2 \left( n_i s_{i,x} + n_i s_{i,y} + m_i - \rho' \right) \cdot s_{i,y}' + 2 \cdot \lambda \cdot n_y = 0
\]  \hspace{1cm} (4.61)

\[
\frac{\partial L}{\partial \lambda} = n_x^2 + n_y^2 - 1 = 0
\]  \hspace{1cm} (4.62)
Solving Eq. 4.59 for the variable $\rho'$ results in:

$$\rho' = n_x \cdot \frac{1}{n} \sum_{i=1}^{n} s_{i,x}' + n_y \cdot \frac{1}{n} \sum_{i=1}^{n} s_{i,y}' + \frac{1}{n} \sum_{i=1}^{n} m_i \quad (4.63)$$

Inserting the expression for $\rho'$ into Eq. 4.60 and Eq. 4.61 leads to the remaining three-equation system:

$$\begin{align*}
\begin{bmatrix}
\sum_{i=1}^{n} s_{i,x}'^2 + \lambda & \sum_{i=1}^{n} s_{i,x}' s_{i,y}' \\
\sum_{i=1}^{n} s_{i,x}' s_{i,y}' & \sum_{i=1}^{n} s_{i,y}'^2 + \lambda
\end{bmatrix}
\begin{bmatrix}
 n_x \\
n_y
\end{bmatrix}
&=
\begin{bmatrix}
- \sum_{i=1}^{n} m_i s_{i,x}' \\
- \sum_{i=1}^{n} m_i s_{i,y}'
\end{bmatrix}
\quad (4.64)
\end{align*}$$

Thus, the solution of the three-equation system in Eq. 4.64 is a common point of intersection between two straight lines (first and second equation) and a unit circle (third equation).

The normal vector $\mathbf{n} = (n_x, n_y)^T$ can be expressed as a function of the Lagrange multiplier $\lambda$ by calculating the coordinates of the point of intersection between the two straight lines given through the first and second equation in Eq. 4.64:

$$n_x = \frac{\sum_{i=1}^{n} s_{i,x}' s_{i,y}' - \sum_{i=1}^{n} m_i s_{i,x}' \sum_{i=1}^{n} s_{i,y}'}{\sum_{i=1}^{n} s_{i,x}'^2 + \lambda \sum_{i=1}^{n} s_{i,y}'^2 + \lambda}$$

$$\quad (4.65)$$
By inserting the expressions for \( n_x \) and \( n_y \) into the third equation in Eq. 4.64, the equation of the unit circle becomes a conditional equation for the Lagrange multiplier \( \lambda \):

\[
\sum_{i=1}^{n} m_i s_i^* x_i \sum_{i=1}^{n} s_i^2 + \lambda
\]

\[
\sum_{i=1}^{n} m_i s_i^* y_i \sum_{i=1}^{n} s_i^2 s_i^* y_i
\]

\[
\sum_{i=1}^{n} s_i^2 + \lambda \sum_{i=1}^{n} s_i^* s_i^* y_i
\]

\[
\sum_{i=1}^{n} s_i^2 s_i^* y_i \sum_{i=1}^{n} s_i^2 + \lambda
\]

\[
(4.66)
\]

\[
\left( \sum_{i=1}^{n} s_i^{*2} + \lambda \sum_{i=1}^{n} s_i^{*2} s_i^{*2} y_i \right) + \left( \sum_{i=1}^{n} s_i^{*2} + \lambda \sum_{i=1}^{n} s_i^{*2} s_i^{*2} y_i \right) = 1
\]

\[
(4.67)
\]
Converting Eq. 4.67 yields a quartic equation in $\lambda$:

\[
\lambda^4 + A^4 + 2 \sum_{i=1}^{n} s_i^2 + 2 \sum_{i=1}^{n} s_{i,y}^2 \\
+ A^2 \left[ + 4 \sum_{i=1}^{n} s_{i,x}^2 \sum_{i=1}^{n} s_{i,y}^2 + \left( \sum_{i=1}^{n} s_{i,x}^2 \right)^2 + \left( \sum_{i=1}^{n} s_{i,y}^2 \right)^2 \\
- 2 \left( \sum_{i=1}^{n} s_{i,x} s_{i,y}^2 \right)^2 - \left( \sum_{i=1}^{n} m_s s_{i,x}^2 \right)^2 - \left( \sum_{i=1}^{n} m_s s_{i,y}^2 \right)^2 \right] \\
+ A \left[ + 2 \sum_{i=1}^{n} s_{i,x} s_{i,y}^2 \sum_{i=1}^{n} m_s s_{i,x}^2 + \sum_{i=1}^{n} m_s s_{i,y}^2 \\
+ 2 \sum_{i=1}^{n} s_{i,x} s_{i,y}^2 \left( \sum_{i=1}^{n} s_{i,x}^2 \right)^2 - \left( \sum_{i=1}^{n} s_{i,x} s_{i,y}^2 \right)^2 - \left( \sum_{i=1}^{n} m_s s_{i,x}^2 \right)^2 \right] \\
+ \left[ - \left( \sum_{i=1}^{n} s_{i,x} s_{i,y}^2 \right)^2 \left( \sum_{i=1}^{n} m_s s_{i,x}^2 \right)^2 - \left( \sum_{i=1}^{n} s_{i,x} s_{i,y}^2 \right)^2 \left( \sum_{i=1}^{n} m_s s_{i,y}^2 \right)^2 \right] \\
+ 2 \sum_{i=1}^{n} s_{i,x} s_{i,y}^2 \sum_{i=1}^{n} m_s s_{i,x}^2 \sum_{i=1}^{n} s_{i,y}^2 - \left( \sum_{i=1}^{n} s_{i,x} s_{i,y}^2 \right)^2 \left( \sum_{i=1}^{n} m_s s_{i,x}^2 \right)^2 \\
- \left( \sum_{i=1}^{n} s_{i,x}^2 \right)^2 \left( \sum_{i=1}^{n} m_s s_{i,y}^2 \right)^2 - \left( \sum_{i=1}^{n} s_{i,x} s_{i,y}^2 \right)^2 \left( \sum_{i=1}^{n} m_s s_{i,x}^2 \right)^2 \\
+ \left( \sum_{i=1}^{n} s_{i,x}^2 \right)^2 \left( \sum_{i=1}^{n} s_{i,y}^2 \right)^2 \left( \sum_{i=1}^{n} m_s s_{i,x}^2 \right)^2 \right] \\
- 2 \sum_{i=1}^{n} s_{i,x} s_{i,y}^2 \left( \sum_{i=1}^{n} s_{i,x} s_{i,y}^2 \right)^2 \right] \right] = 0
\]
The quartic equation can be solved by either algebraic or numeric means. (Polynomial equations of up to fourth order are solvable using only rational operations and finite root extractions. General polynomial equations of fifth and higher order cannot be solved rationally with finite root extractions, as was proved by Abel and Galois using group theory (Abel’s impossibility theorem).)

Inserting the four solutions for the Lagrange multiplier $\lambda$ into Eq. 4.65 and Eq. 4.66 results in four candidate vectors $\mathbf{n} = (n_x, n_y)$ for local extreme values of the Lagrange function in Eq. 4.58. The candidate vector that yields a minimum objective value is the solution vector $\mathbf{n}^0 = (n_x^0, n_y^0)^T$.

The solution value $\rho^0$ is derived by inserting the solution vector $\mathbf{n}^0 = (n_x^0, n_y^0)^T$ into the following equation:

$$\rho = n_x \cdot \bar{s}_{i,x} + n_y \cdot \bar{s}_{i,y} + \rho'$$

(4.69)

F) An alternative exact solution for two-dimensional sensor arrangement can be obtained by inserting Eq. 4.39 into the objective function in Eq. 4.37:

$$F(\rho, n_x) = \sum_{j=1}^{n} \| p_j - q_j \|^2 = \sum_{j=1}^{n} \left( n_x s_{i,x} + \left( \pm \sqrt{1-n_x^2} \right) s_{i,y} + m_i - \rho \right)^2$$

(4.70)

Thus, the problem results in minimizing without side condition the objective function in Eq. 4.70:

$$\min_{\rho \in \mathbb{R}_+^n, n_x \in [1,1]} F(\rho, n_x)$$

$$= \min_{\rho \in \mathbb{R}_+^n, n_x \in [1,1]} \sum_{i=1}^{n} \left( n_x s_{i,x} + \left( \pm \sqrt{1-n_x^2} \right) s_{i,y} + m_i - \rho \right)^2$$

(4.71)
Necessary condition for a minimum is:

$$\nabla F(\rho, n_i) = 0$$  \hspace{1cm} (4.72)

The partial derivatives are:

$$\frac{\partial F}{\partial \rho} = \sum_{i=1}^{n} 2 \cdot \left( n_i s_{i,x} + \left( \pm \sqrt{1 - n_i^2} \right) s_{i,y} + m_i - \rho \right) (-1)$$

$$= 0$$  \hspace{1cm} (4.73)

$$\frac{\partial F}{\partial n_i} = \sum_{i=1}^{n} 2 \cdot \left( n_i s_{i,x} + \left( \pm \sqrt{1 - n_i^2} \right) s_{i,y} + m_i - \rho \right) \left( s_{i,x} - \frac{n_i s_{i,y}}{\pm \sqrt{1 - n_i^2}} \right)$$

$$= 0$$  \hspace{1cm} (4.74)

Solving Eq. 4.73 for the variable $\rho$ results in:

$$\rho = n_x \cdot \frac{1}{n} \sum_{i=1}^{n} s_{i,x} + \left( \pm \sqrt{1 - n_x^2} \right) \frac{1}{n} \sum_{i=1}^{n} s_{i,y} + \frac{1}{n} \sum_{i=1}^{n} m_i$$

$$= n_x \cdot \bar{s}_{i,x} + \left( \pm \sqrt{1 - n_x^2} \right) \bar{s}_{i,y} + \bar{m}_i$$  \hspace{1cm} (4.75)
Inserting the expression for $\rho$ into Eq. 4.74 and then converting Eq. 4.74 yields a fourth order polynomial in $n_x$:

$$n_x^4 \left( 4 \left( \sum_{i=1}^{n} s_{ix,y}^x - n \cdot \bar{s}_{ix,y} \right)^2 + \left( \sum_{i=1}^{n} s_{ix,y}^2 - n \cdot \bar{s}_{ix,y}^2 - \sum_{i=1}^{n} s_{ix}^2 + \sum_{i=1}^{n} \bar{s}_{ix,y}^2 \right)^2 \right) + n_x^2 \left( 4 \left( \sum_{i=1}^{n} s_{ix,y} - n \cdot \bar{s}_{ix,y} \right) \left( \sum_{i=1}^{n} m_{ix} - n \cdot \bar{m}_{ix} \right) \right)
+ 2 \left( \sum_{i=1}^{n} s_{ix,y}^2 - n \cdot \bar{s}_{ix,y}^2 - \sum_{i=1}^{n} s_{ix}^2 + \sum_{i=1}^{n} \bar{s}_{ix,y}^2 \right) \left( \sum_{i=1}^{n} m_{ix} - n \cdot \bar{m}_{ix} \right) \right) + \left( \sum_{i=1}^{n} m_{ix} - n \cdot \bar{m}_{ix} \right)^2 \right)^2 \right) = 0 \tag{4.76}
$$

Inserting all roots of the fourth order polynomial into the expression for $\rho$ (Eq. 4.75) results in four candidate vectors $(\rho, n_x^0)^T$ for local extreme values of the objective function in Eq. 4.70. The candidate vector that yields a minimum objective value is the solution vector $(\rho^0, n_x^0)^T$. 
Cross Echoes

Within this paragraph, tangential regression is formally established for cross echoes. In the case of a cross echo, it is unknown how the distance that the ultrasound wave has traveled splits up between emitting sensor, reflecting point, and receiving sensor. This causes the reflecting point being located on an elliptic arc around the sensor positions.

The sensor positions \( s \) define the focal points of the measurement ellipse. A reflecting line, which is represented by a tangent to the measurement ellipse, has the property of reflecting a beam from one focal point to the other focal point. The measurement ellipse is visualized in Figure 4.23.

Figure 4.23: Measurement ellipse

For the sake of simplicity, it is assumed that the location of the tangential regression line \( H \) is to be estimated from the direct and cross echoes of only \( n = 2 \) sensors. Two direct and two cross echoes exist for \( n = 2 \) sensors, where the two cross echoes should ideally provide identical distance measurements.
The coordinates of the two sensors are:

\[
s_1 = (s_{1,x}, 0)^T = (-s_{2,x}, 0)^T
\]

(4.77)

\[
s_2 = (s_{2,x}, 0)^T = (-s_{1,x}, 0)^T
\]

(4.78)

The wave of a cross echo travels the total distance \(m_{1,2} = m_{2,1}\) from one sensor via a reflecting point to another sensor. The reflecting point is located on a measurement ellipse \(E_m\) with major axis \(a\) and minor axis \(b\) (cp. [108]):

\[
E_m = \left\{ x : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \right\}
\]

(4.79)

\[
a = \frac{m_{1,2}}{2}
\]

(4.80)

\[
b = \sqrt{\left(\frac{m_{1,2}}{2}\right)^2 - s_{1,x}^2}
\]

(4.81)

It is further assumed that:

\[
\left(\frac{m_{1,2}}{2}\right)^2 - s_{1,x}^2 > 0
\]

(4.82)

Setting up tangential regression requires to compute the coordinates of the potential reflecting points on the echo arcs \(p_i\) as a function of the sensor positions \(s_i\), the measurements \(m_i\), and the orientation of the tangential regression line \(H\). This can be achieved by tangents \(T\) through the points on the echo arcs \(p_i\) being parallel to the tangential regression line \(H\). Since for each measurement ellipse \(E_m\) there are two tangents \(T\) parallel to the tangential regression line \(H\), the consideration is restricted to the upper half space of the coordinate system \((y > 0)\). This restriction makes a tangent \(T\) unique whenever the overlapping detection domain of both sensors falls within the half space above the abscissa, which is practically justified.
The tangent $T$ through a point $x_0 = (x_0, y_0)^T$ on the measurement ellipse $E_m$ is given by the equation (cp. [108]):

$$\begin{align*}
T &= \left\{ x \mid \frac{x}{a^2} + \frac{y}{b^2} = 1 \right\}
\end{align*}$$

(4.83)

The tangential regression line $H$ is given by the equation:

$$\begin{align*}
H &= \left\{ x \mid n_s x + n_n y = \rho \right\}
\end{align*}$$

(4.84)

It is assumed again that all sensors are located in the negative half plane of the tangential regression line $H$. If the sensor positions $s_1$ and $s_2$ lie on the $x$-axis and their center of gravity falls into the origin of the coordinate system this yields:

$$\begin{align*}
n_s s_{1,x} + n_s s_{1,y} - \rho &< 0 \\
n_s s_{1,x} - \rho &< 0 \\
n_s (-s_{2,x}) - \rho &< 0 \\
n_s s_{2,x} + n_s s_{2,y} - \rho &< 0 \\
n_s s_{2,x} - \rho &< 0 \\
n_s (-s_{1,x}) - \rho &< 0
\end{align*}$$

(4.85) (4.86)

Adding up the two inequalities leads to:

$$\begin{align*}
-2 \rho &< 0 \\
\rho &> 0
\end{align*}$$

(4.87)
4.3 Environment Modeling with Ultrasonic Sensor Data

A point of intersection \((0, y^0)\) between the tangential regression line \(H\) and the y-axis of the coordinate system with \(y^0 > 0\) leads to:

\[
\begin{align*}
n_x, 0 + n_y y^0 - \rho &= 0 \\
n_x, 0 + n_y y^0 &> 0 \\
n_y y^0 &> 0 \\
n_y &> 0
\end{align*}
\] (4.88)

The tangent \(T\) becomes parallel to the tangential regression line \(H\) if their normal vectors are proportional:

\[
\begin{align*}
\frac{x_0}{a^2} &= \alpha \cdot n_x \\
x_0 &= \alpha \cdot n_x \cdot a^2, \quad \alpha \in \mathbb{R}
\end{align*}
\] (4.89)

\[
\begin{align*}
\frac{y_0}{b^2} &= \alpha \cdot n_y \\
y_0 &= \alpha \cdot n_y \cdot b^2, \quad \alpha \in \mathbb{R}
\end{align*}
\] (4.90)

Inserting the coordinates of the point \(x_0 = (x_0, y_0)^T\) into the equation of the measurement ellipse \(E_m\) (Eq. 4.79) leads to:

\[
\alpha = \pm \sqrt{\frac{1}{n_x^2 \cdot a^2 + n_y^2 \cdot b^2}}
\] (4.91)

The choice of the positive of these two values for \(\alpha\) leads to the tangent \(T\) which intersects the measurement ellipse \(E_m\) above the abscissa \((y_0 > 0)\) because of \(b^2 > 0\) and \(n_y > 0\).
The joint objective function for the two direct echoes and the cross echo thus becomes for $\|a\| = 1$:

$$F(\rho, n_x, n_y) = (p_1 - q_1)^2 + (p_2 - q_1)^2 + (p_{1,2} - q_{1,2})^2$$

$$= (n^T p_1 - \rho)^2 + (n^T p_2 - \rho)^2 + (n^T p_{1,2} - \rho)^2$$

$$= (n^T s_1 + m_1 - \rho)^2 + (n^T s_2 + m_2 - \rho)^2 + (n^T x_0 - \rho)^2$$

$$= (n_x s_{1,x} + n_y s_{1,y} + m_1 - \rho)^2$$

$$+ (n_x s_{2,x} + n_y s_{2,y} + m_2 - \rho)^2$$

$$+ \left( n^T \cdot \alpha^T \cdot \left[ \begin{array}{c} n_x \cdot a^2 \\ n_y \cdot b^2 \end{array} \right] \cdot \rho \right)^2$$

$$= (n_x s_{1,x} + m_1 - \rho)^2$$

$$+ (n_x s_{2,x} + m_2 - \rho)^2$$

$$+ \left( n^T \cdot \alpha^T \cdot \left[ \begin{array}{c} n_x \cdot a^2 \\ n_y \cdot b^2 \end{array} \right] - \rho \right)^2$$

$$= (n_x s_{1,x} + m_1 - \rho)^2$$

$$+ (n_x s_{2,x} + m_2 - \rho)^2$$

$$+ \left( n^T \cdot \alpha^T \cdot \left[ \begin{array}{c} n_x \cdot a^2 \\ n_y \cdot b^2 \end{array} \right] - \rho \right)^2$$

(4.92)

Tangential regression amounts to minimizing this function over all unit vectors $\mathbf{n} = (n_x, n_y)^T$ with $n_y \geq 0$ and over all real values $\rho \geq 0$.

Due to the obvious difficulties regarding this minimization, only approximations will be considered in the continuation. In order to simplify the minimization problem, measurement ellipses are approximated by circles, which is practically justified if the displacement between the involved sensors is relatively low in comparison with the total distance $m_{1,2}$ that the ultrasound wave has traveled:

$$2s_{1,x} \ll m_{1,2}$$

(4.93)
The approximation of a measurement ellipse by a circle is shown in Figure 4.24.

![Diagram](image)

**Figure 4.24:** Approximation of a measurement ellipse by a circle

This approximation permits the utilization of the objective function derived for direct echoes also for cross echoes. As the overlapping detection domain of two sensors due to their limited beam width falls within the half space above the abscissa and extends on both sides of the positive ordinate, the elliptic arc representing the beam width overlapping of the sensors can be approximated by a circular arc of corresponding length with radius $r$:

$$r = b = \sqrt{\frac{m_{1,2}^2}{2}} - s_{1,x}^2$$

(4.94)
In Figure 4.25, a virtual sensor is indicated at the origin of the coordinate system. If an object is located with a reflecting point along the acoustic axis of the virtual sensor, i.e. on the positive ordinate, the approximation error is zero. If the reflecting point deviates from the acoustic axis, the approximation error steadily increases towards the boundaries of the overlapping detection domain. As the circular arc diverges from the elliptic arc, a distance error regarding the reflecting point as well as an orientation error regarding the reflecting line arises, which in Figure 4.25 becomes visually apparent outside the overlapping detection domain.

**Figure 4.25:** Approximation of the overlapping detection domain of two real sensors by the virtual detection cone of one virtual sensor

**Process of Tangential Regression**

An environment model can be created from ultrasonic sensor data by tangential regression as follows:

Firstly, a minimum number of required samples for representing a valid regression line is heuristically defined to \( n = 6 \). Thereafter, all sensors and virtual sensors are arranged in a list in an ascending order according to the orientation angle of their acoustic axis. Beginning with the first sensor in the list, an initial regression line is fitted to the distance readings of \( n \) consecutive sensors, and the root mean square error \( RMSE \) is calculated. Then,
it is verified if the root mean square error is lower than a predefined threshold (e.g. \( \text{RMSE} < 0.02 \text{ m} \)), and if the obtained regression line has a perpendicular reflection line falling within the detection angle of all sensors involved. If one of these conditions is not fulfilled, the fitted regression line will be discarded. In this case, proceeding with the second sensor in the list, a new regression line will be fitted to the distance readings of \( n \) consecutive sensors and the root mean square error will be calculated. If one of these conditions is not fulfilled, the fitted regression line will be discarded. In this case, proceeding with the second sensor in the list, a new regression line will be fitted to the distance readings of \( n \) consecutive sensors and the root mean square error will be calculated. Again, it is verified for the obtained regression line if both conditions are fulfilled. Additional sensors are included recursively until a regression line with a root mean square error higher than the predefined threshold results or until the obtained regression line does not have a perpendicular reflection line falling within the detection range of all sensors involved. In this case, the current regression line is discarded, and the last regression line meeting both conditions is adopted as a valid straight line element. A valid regression line is the best fit of a common tangent to a maximum number of circles or approximated circles representing isometric lines of direct echo paths or cross echo paths. Now, it is proceeded with the following sensor in the list not being involved in the last valid straight line element for extracting the next valid straight line element. These steps are continued and all valid straight line elements are gathered until the distance reading of the last sensor in the list has been reached. Subsequently, it is checked if a valid straight line element exists involving distance readings of sensors at the end of the list and distance readings of sensors at the beginning of the list. Therefore, the distance readings from the beginning of the last valid straight line element up to the end of the first valid straight line element are merged to a new list. Then, it is searched for new valid straight line elements within this list as has been described for the primary list. As a result, one, two, or three new valid straight line elements will be obtained for replacing the former two. If two valid straight line elements will be obtained again, they can slightly differ from the former two, since additional distance readings from the beginning or the end of the original list might be fitted. One valid straight line element will be obtained if the former first and last valid straight line element merge to a single one. Three valid straight line elements will be obtained if an additional valid straight line element emerges between the former two. Finally, the individual extent of the extracted straight line elements can be determined in order to derive straight line segments of defined length. For this purpose, the real and virtual sensors that contributed echo arcs to
the calculation of the tangential regression lines are identified. For each extracted straight line element, a list of the contributive real and virtual sensors is created. Then, perpendicular reflection lines are drawn from the extracted straight line elements to the contributive real and virtual sensors. Next, the two outermost perpendicular reflection lines are determined for each extracted straight line element. The points of intersection between the extracted straight line elements and their outermost perpendicular reflection lines define the endpoints and the length of straight line segments.

The parameters of the algorithm are the minimum number $n$ of required samples for representing a valid regression line, and the threshold for the root mean square error $RMSE$ of valid regression lines.

In the described way, straight line segments can be extracted at different robot locations and can be spatially integrated in order to obtain a detailed environment model. Figure 4.26 illustrates the steps of the algorithm for creating an environment model from ultrasonic sensor data by tangential regression in a flow chart. The main procedure calls subroutines for searching valid straight line elements, for data fusion between the last and the first sensors, and for determining straight line segments of defined length. The subroutines are illustrated in separate flow charts in Figure 4.27, Figure 4.28, and Figure 4.29.
4.3 Environment Modeling with Ultrasonic Sensor Data

Figure 4.26: Main procedure for creating an environment model from ultrasonic sensor data by tangential regression
Figure 4.27: Subroutine for searching valid straight line elements
Figure 4.28: Subroutine for data fusion between the last and the first sensors
Figure 4.29: Subroutine for determining straight line segments of defined length

**Example of Tangential Regression**

Figure 4.30 shows the robot (large circle) together with the positions of the real and virtual ultrasonic sensors (small circles), the circular and elliptic arcs depicting the direct and cross echo paths, the reflection lines (dashed lines), and three straight line elements (bold lines) extracted from ultrasonic sensor data by tangential regression. As a reference, the figure also includes the polygonal environment model created from laser range finder data (polygon composed of thin solid and dashed lines). Figure 4.31 displays one extracted straight line element (bold line) in detailed representation. Figure 4.32 illustrates how the points of intersection between the extracted straight line elements and their outermost perpendicular reflection lines define the endpoints and the length of straight line segments.
Figure 4.30: Extracted straight line elements (bold lines)

Figure 4.31: Detailed representation of one extracted straight line element (bold line)
It can be noted that the three straight line segments extracted from ultrasonic sensor data by tangential regression correspond to straight line segments in the polygonal environment model created from laser range finder data. The maximum deviation between corresponding lines is $\Delta \rho = 0.036 \text{ m}$ in distance and $\Delta \theta = 5.089^\circ$ in orientation.

### 4.3.3 Comparison between PSC and TR

By applying parameter space clustering and tangential regression, local environment models of comparable quality have been derived from a single ultrasonic scan around the robot at a fixed location. Both approaches are robust with regard to parameter adjustments. Although a chair back-rest at location $(x, y) \approx (1.0 \text{ m}, -0.7 \text{ m})$ could be detected by some ultrasonic sensors, the number of received echoes was insufficient to extract a straight line segment from this robot location. A very few laser scanner samples appear at the location of the chair back-rest as well. However, because of the fact that the scanning level of the laser range finders just coincides with the top edge of the chair back-rest, these samples are of sparse, sporadic, and inaccurate occurrence. The inaccuracies are due to a phenomenon already mentioned in Section 3.6 that when a laser range finder beam strikes an object edge the reflection point appears to be further dis-
tant than the actual object edge. The chair back-rest could thus not be properly modeled in the polygonal environment model created from laser range finder data.

The PSC algorithm should be utilized when impaired sensor information is to be expected (e.g. due to sensor faults), since this approach is robust against severely outlying data points. In contrast, the TR algorithm would fail in such cases, since the extraction of a straight line element would be interrupted at strong outliers due to an increasing root mean square error.

A merit of the TR algorithm is that it guarantees for extracted straight line elements the existence of perpendicular reflection lines within the detection angles of all real and virtual sensors which contributed to the calculation of the respective regression lines. This also implies that the determination of straight line segments of defined length is more restrictive. However, since the detection angles of ultrasonic sensors under various measurement conditions are not exactly known, the angular flexibility of the PSC algorithm may be in some situations considered as an advantage.

4.4 Experimental Results from Various Robot Locations

Figure 4.33 to Figure 4.66 present experimental results obtained within the evaluation environment introduced in Section 1.5 at the 17 robot locations marked by numbered crosses in Figure 1.9.

![Figure 4.33: Robot Location 1 – PSC](image1)

![Figure 4.34: Robot Location 1 – TR](image2)
Figure 4.35: Robot Location 2 – PSC

Figure 4.36: Robot Location 2 – TR

Figure 4.37: Robot Location 3 – PSC

Figure 4.38: Robot Location 3 – TR

Figure 4.39: Robot Location 4 – PSC

Figure 4.40: Robot Location 4 – TR
4.4 Experimental Results from Various Robot Locations

- **Figure 4.41**: Robot Location 5 – PSC
- **Figure 4.42**: Robot Location 5 – TR
- **Figure 4.43**: Robot Location 6 – PSC
- **Figure 4.44**: Robot Location 6 – TR
- **Figure 4.45**: Robot Location 7 – PSC
- **Figure 4.46**: Robot Location 7 – TR
Figure 4.47: Robot Location 8 – PSC
Figure 4.48: Robot Location 8 – TR
Figure 4.49: Robot Location 9 – PSC
Figure 4.50: Robot Location 9 – TR
Figure 4.51: Robot Location 10 – PSC
Figure 4.52: Robot Location 10 – TR
4.4 Experimental Results from Various Robot Locations

Figure 4.53: Robot Location 11 – PSC

Figure 4.54: Robot Location 11 – TR

Figure 4.55: Robot Location 12 – PSC

Figure 4.56: Robot Location 12 – TR

Figure 4.57: Robot Location 13 – PSC

Figure 4.58: Robot Location 13 – TR
Figure 4.59: Robot Location 14 – PSC

Figure 4.60: Robot Location 14 – TR

Figure 4.61: Robot Location 15 – PSC

Figure 4.62: Robot Location 15 – TR

Figure 4.63: Robot Location 16 – PSC

Figure 4.64: Robot Location 16 – TR
4.5 Contributions

In this chapter, local environment modeling by extracting straight line segments has been described. The main contributions of the chapter are two new algorithms, called parameter space clustering and tangential regression, which have been developed for deriving local environment models from ultrasonic sensor data. With the aid of these algorithms, a number of straight line segments can be extracted by a single ultrasonic scan around the robot from a fixed location and without turning the robot around its own axis for data acquisition. Another important advantage of both algorithms is that the reflection properties of ultrasound are properly exploited by considering the requirement of a perpendicular reflection line from a reflecting surface to a sensor.

Further potential applications of parameter space clustering and tangential regression are in diagnostic medical sonography, seismic investigations, radar imaging, ultrasonic underwater sensing, and wavefront approximation.

4.5.1 Parameter Space Clustering

The contribution presented in Subsection 4.3.1 is the algorithm of parameter space clustering (PSC). The Hough transform has been extensively used in computer vision for boundary detection in images [101] and has been previously applied in robotics for straight line extraction from sensor data [32, 49, 65]. The distinction of parameter space clustering from re-
lated work comprises the discretization of circular and elliptic echo arcs by potential reflecting lines of infinitesimal length, the Hough transform of all potential reflecting lines into an occupancy grid in parameter space, the two-dimensional low-pass filtering of the occupancy grid, the connected components analysis to determine regions of cohesive highly occupied grid cells for cluster formation without requiring the number of expected clusters to be predefined, and the calculation of cluster centers that represent extracted straight line elements in parameter space.

4.5.2 Tangential Regression

The contribution presented in Subsection 4.3.2 is the algorithm of tangential regression (TR). Standard regression as well as orthogonal regression have been widely employed in robotics for straight line extraction from sensor data [32, 38, 49, 50, 51, 53, 59, 68, 110]. In a closely related approach [60], a common tangent is calculated for two circular arcs by a closed-form solution. Consequently, an over-determined situation of fitting a common tangent to more than two circular arcs is not considered. The idea of tangential regression is to calculate the parameters of a tangential regression line so that it represents the best fit to multiple circular or elliptic echo arcs of distance measurements from different sensor positions. The difference to orthogonal regression is that the distance measurements indicate potential regression points on curves instead of fixed regression points.
5 Mobile Robot Self-Localization

Local environment models can be spatially integrated to build global environment maps while the robot moves. In order to accurately project newly extracted straight line segments into the world coordinate system, the robot location within the environment must be estimated at each time step as precisely as possible. Within this chapter, the application of an extended Kalman filter (EKF) to mobile robot self-localization based on straight line segments is explained.

In order to determine correspondences (matches) between newly extracted straight line segments and straight line segments in the global environment map, each newly extracted segment is compared with all segments in the global map that are supposed to be detectable according to the odometric robot location. Matching can be performed by comparison of line parameters.

Using straight line segments in the global environment map as reference features (targets), the robot location can be estimated by multi-target tracking. Each match of a newly extracted segment with a segment in the global environment map provides a one-dimensional constraint on the robot position as well as a constraint on the robot orientation. These constraints can be exploited by adapting the extended Kalman filter framework in order to consider distance information as well as angular information for calculating the updated robot location estimate.

An introduction concerning the application of an extended Kalman filter to mobile robot self-localization is presented in Section 5.1. The extended Kalman filter equations are provided in Section 5.2. Self-localization by multi-target tracking is explained in Section 5.3. Simultaneous localization and mapping (SLAM) is addressed in Section 5.4. The contributions of this chapter are recapitulated in Section 5.5.

5.1 Introduction

A Kalman filter is an optimal recursive data processing algorithm that incorporates all available measurements to estimate the current value of the
variables of interest in such a manner that the error is minimized statistically, by using (a) knowledge of the system and measurement device dynamics, (b) the statistical description of the system noises, measurement errors, and uncertainties in the dynamic models, and (c) any available information about initial conditions of the variables of interest. The Kalman filter performs a conditional probability density propagation for problems in which the system can be described through a linear model and in which system and measurement noises are white and Gaussian. Under these restrictions, the Kalman filter can be shown to be the best filter of any conceivable form [113].

Although the Kalman filter assumes linear systems, it is also widely applied to nonlinear systems. Based on knowledge of an approximate solution, the deviations from the reference can be described by linear equations. Thus, an approximate linear model provides the foundation for utilizing the Kalman filter. Commonly, such applications are successful, but occasionally, results are unsatisfactory due to the occurrence of divergence, which is a direct consequence of the errors introduced by the linear approximation. To reduce the approximation errors, an extended Kalman filter can be applied. In this case, the nonlinear system is linearized by employing the best estimates of the state vector as the reference values used at each stage for the linearization. However, the extended Kalman filter does not absolutely ensure the elimination of the divergence problem [123].

The Kalman filter has been applied for mobile robot environment mapping and self-localization by many researchers (e.g. [42, 43, 44, 49, 52, 54, 56, 57, 58, 65, 71, 72, 112]). The description of the extended Kalman filter in this thesis follows the notations by Bar-Shalom and Fortmann [102]. The application of the extended Kalman filter to self-localization by multitarget tracking is related to the approach by Leonard and Durrant-Whyte [112].

5.2 The Extended Kalman Filter Equations

The Kalman filter for recursive calculation of the sufficient statistic consisting of the mean and variance in the linear Gaussian case is the simplest possible state estimation filter. In the case of a linear system with non-Gaussian random variables, the same simple recursion yields an approximate mean and variance, which is the best linear estimate. The extended Kalman filter provides a similar framework for nonlinear systems. Such an estimator can be obtained by a series expansion of the nonlinear dynamics and measurement equations [102].
A first- or second-order extended Kalman filter provides a first- or second-order approximation to deal with nonlinear dynamics and measurement equations. State prediction is accomplished using the nonlinear plant equation. The plant model \( f[k, \dot{x}(k), u(k)] \) is linearized about the estimated state vector \( \dot{x}(k|k) \) by the Jacobian \( f_x(k) \) of the nonlinear plant model. The state prediction covariance matrix is computed utilizing the plant Jacobian \( f_x(k) \). Measurement prediction is performed using the nonlinear measurement equation. The measurement model \( h[k+1, \dot{x}(k+1)] \) is linearized about the predicted state vector \( \dot{x}(k+1|k) \) by the Jacobian \( h_x(k+1) \) of the nonlinear measurement model. The measurement prediction covariance matrix is computed utilizing the measurement Jacobian \( h_x(k+1) \). The measurement Jacobian \( h_x(k+1) \) is also used for calculating the filter gain matrix \( W(k+1) \).

The inputs to the Kalman filter are the system control input and the measurements. The a priori information are the system dynamics and noise properties of system and measurements. The outputs of the Kalman filter are the innovations (i.e. the differences between predicted and observed measurements) as well as the updated state estimate of the system.

Considering the nonlinear plant equation of a discrete-time, dynamic system:

\[
x(k + 1) = f[k, x(k), u(k)] + v(k)
\]

\[
v(k) \sim N(0, Q(k))
\]

where \( x(k) \) is the state vector, \( u(k) \) is the control input vector, and \( v(k) \) describes a sequence of zero-mean, white, Gaussian process noise with covariance matrix \( Q(k) \).

The nonlinear measurement (observation) equation is:

\[
z(k + 1) = h[k + 1, x(k + 1)] + w(k + 1)
\]

\[
w(k + 1) \sim N(0, R(k + 1))
\]

where \( w(k+1) \) describes a sequence of zero-mean, white, Gaussian measurement noise with covariance matrix \( R(k+1) \).
Given the history $Z^k$ of measurement vectors $z$ up to time $k$:

$$Z^k = \{z(j)\}, \quad j = 1, \ldots, k$$  \hfill (5.5)

In the nonlinear case, the state estimate $\hat{x}(k|k)$ is an approximate conditional mean of the state $x(k)$:

$$\hat{x}(k|k) = E[x(k)|Z^k]$$  \hfill (5.6)

Since the state estimate $\hat{x}(k|k)$ is not the exact conditional mean of the state $x(k)$, the associated conditional state error covariance matrix $P(k|k)$ is strictly speaking an approximate mean square error:

$$P(k|k) = E \left\{ [x(k) - \hat{x}(k|k)][x(k) - \hat{x}(k|k)]^T | Z^k \right\}$$  \hfill (5.7)

To obtain the state prediction $\hat{x}(k+1|k)$, the nonlinear function in Eq. 5.1 is expanded in Taylor series around the state estimate $\hat{x}(k|k)$ with terms up to first or second order to yield the first- or second-order EKF, respectively. The expansion with first-order terms is:

$$x(k+1) = f[k, \hat{x}(k|k), u(k)] + f_x(k)[x(k) - \hat{x}(k|k)] + v(k)$$  \hfill (5.8)

$$f_x(k) = \left[ \nabla_x f^T(k, x, u) \right]_{x=\hat{x}(k|k), u=u(k)}$$  \hfill (5.9)

where $f_x(k)$ is the Jacobian of the vector $f$, evaluated at time $k$. 
The state prediction $\hat{x}(k+1|k)$ from time $k$ to $k+1$ is obtained by taking the expectation of Eq. 5.8 conditioned on $Z^k$:

$$\hat{x}(k + 1|k) = f[\hat{x}(k), \hat{x}(k|k), u(k)] \quad (5.10)$$

The first-order term in Eq. 5.8 is, in view of Eq. 5.6, approximately zero-mean and therefore does not appear in Eq. 5.10.

Subtracting Eq. 5.10 from Eq. 5.8 yields the state prediction error $\tilde{x}(k+1|k)$:

$$\tilde{x}(k + 1|k) = x(k + 1) - \hat{x}(k + 1|k) = f_x(k)\left[\hat{x}(k)-\hat{x}(k|k)\right] + v(k) \quad (5.11)$$

The state prediction covariance matrix $P(k+1|k)$ (approximate mean square error) is:

$$P(k + 1|k) = \mathbb{E}
[\tilde{x}(k + 1|k)\tilde{x}^T(k + 1|k)Z^k]
= f_x(k)P(k|k)f_x^T(k) + Q(k) \quad (5.12)$$

The measurement prediction $\hat{z}(k+1|k)$ for the first-order filter is:

$$\hat{z}(k + 1|k) = h[k + 1, \hat{x}(k + 1|k)] \quad (5.13)$$
Subtracting Eq. 5.13 from Eq. 5.3 yields the measurement prediction error \( \tilde{z}(k+1|k) \), which is the same as the measurement residual or innovation \( \nu(k+1) \):

\[
\nu(k+1) = \tilde{z}(k+1|k) = z(k+1) - \tilde{z}(k+1|k) = h[k+1,x(k+1)] + w(k+1) - h[k+1,\hat{x}(k+1|k)]
\]

(5.14)

The measurement prediction covariance matrix (also known as innovation covariance matrix) \( S(k+1) \) (approximate mean square error) is:

\[
S(k+1) = E\left[ (\tilde{z}(k+1|k) - \tilde{z}(k+1|k))^{T} (\tilde{z}(k+1|k) - \tilde{z}(k+1|k)) \right]
\]

\[
= h_{x}(k+1)P(k+1|k)h_{x}^{T}(k+1) + R(k+1)
\]

(5.15)

\[
h_{x}(k+1) = \left[ \nabla_{x}h^{T}(k+1,x) \right]_{x=x(k+1|k)}^{T}
\]

(5.16)

where \( h_{x}(k+1) \) is the Jacobian of the vector \( h \), evaluated at time \( k+1 \).

The filter gain matrix \( W(k+1) \) is:

\[
W(k+1) = P(k+1|k)h_{x}^{T}(k+1)S^{-1}(k+1)
\]

(5.17)

Thus, the updated state estimate \( \hat{x}(k+1|k+1) \) can be written as:

\[
\hat{x}(k+1|k+1) = \hat{x}(k+1|k) + W(k+1)\nu(k+1)
\]

(5.18)

Finally, the updated state covariance matrix \( P(k+1|k+1) \) is:

\[
P(k+1|k+1) = P(k+1|k) - W(k+1)S(k+1)W^{T}(k+1)
\]

(5.19)
5.3 Self-Localization by Multi-Target Tracking

Geometric features in the environment that are reliably observable with ultrasonic sensors from various robot locations (e.g. planar surfaces) can be utilized as targets for self-localization by multi-target tracking.

In the application of an extended Kalman filter to self-localization, the plant model describes the change in robot location in response to motor control inputs, with an estimate of dead-reckoning (odometry) errors. The measurement model allows a measurement prediction dependent on the predicted robot location and the locations of geometric features in the environment, with an estimate of measurement noise. The nonlinearities in the plant model and in the measurement model depend on the robot kinematics and on the measurement characteristics, respectively.

5.3.1 Feature Matching

The global environment map $M_W$ consists of straight line segments representing planar surfaces in a two-dimensional view. Straight line segments are described using Hesse’s normal form of the equation of a line with the perpendicular distance $\rho$ from the origin in the coordinate system and the angle $\theta$ between the normal of the line and the $x$-axis:

$$x \cos \theta + y \sin \theta - \rho = 0$$

(5.20)

All straight line segments in the global environment map $M_W$ can be considered as targets $t$, and the locations of the targets can be described in the parameters of Hesse’s normal form by target state vectors $t$:

$$t = \begin{bmatrix} \rho_i \\ \theta_i \end{bmatrix}$$

(5.21)

Consequently, the global environment map $M_W$ may be regarded as a set of $n_t$ target state vectors $t$:

$$M_W = \{t_i\} \quad i = 1, \ldots, n_t$$

(5.22)
If the global environment map $M^W$ is a priori provided as a floor plan, the targets can be considered stationary, and accurate knowledge of the target locations can be assumed. Thus, the target state vectors $t$ are not a function of time, and the modeled straight line segments need not to be represented with uncertainties ($\sigma_{\rho_t} = 0$, $\sigma_{\theta_t} = 0$).

The location (position and orientation) of the robot within the world coordinate system at time step $k$ is denoted by the state vector $x_r^W (k)$:

$$x_r^W (k) = \begin{bmatrix} x_r^W (k) \\ y_r^W (k) \\ \theta_r^W (k) \end{bmatrix}$$

(5.23)

The Kalman filter is represented in world coordinates, since the state $x_r^W (k)$ of the robot is defined in world coordinates.

According to odometry information, at time $k+1$ the robot is predicted to be at location $\tilde{x}_r^W (k+1|k)$. The predicted robot location is used to select $n_s$ targets in the global environment map $M^W$ from which straight line segments are potentially extractable. At the same time, $n_e$ straight line segments are actually extracted from the ultrasonic sensor data. Utilizing appropriate validation thresholds, the parameters of the $n_e$ extracted straight line segments are compared with the parameters of the $n_s$ selected targets in the global environment map $M^W$ in order to derive $n_c$ correspondences (matches).

The $n_c$ correspondences between measurement predictions $\hat{z}_j (k+1|k)$ and actual measurements (observations) $z_j (k+1)$ are used to calculate the innovations $v_j (k+1)$:

$$v_j (k+1) = z_j (k+1) - \hat{z}_j (k+1|k), \quad j = 1, \ldots, n_c$$

(5.24)
The predicted distance $\hat{\rho}_{rtj}(k+1|k)$ and the predicted angle $\hat{\theta}_{rtj}(k+1|k)$ from the predicted robot location $\hat{x}_r^W(k+1|k)$ to a stationary target location $t_j$ can be computed as follows:

\begin{align}
\hat{\rho}_{rtj}(k+1|k) &= \hat{x}_r^W(k+1|k) \cos \theta_{tj} + \hat{y}_r^W(k+1|k) \sin \theta_{tj} - \rho_{tj} \\
\hat{\theta}_{rtj}(k+1|k) &= \theta_{tj} - \hat{\theta}^W_r(k+1|k)
\end{align}

(5.25) \quad (5.26)

Each measurement prediction $\hat{z}_j(k+1|k)$ for a target is composed of a predicted distance $\hat{\rho}_{rtj}(k+1|k)$ and a predicted angle $\hat{\theta}_{rtj}(k+1|k)$:

$$\hat{z}_j(k+1|k) = \begin{bmatrix} \hat{\rho}_{rtj}(k+1|k) \\ \hat{\theta}_{rtj}(k+1|k) \end{bmatrix}$$

(5.27)

The measurement predictions $\hat{z}_j(k+1|k)$ are stacked into a composite vector $\hat{z}(k+1|k)$:

$$\hat{z}(k+1|k) = \begin{bmatrix} \hat{\rho}_{rt_1}(k+1|k) \\ \vdots \\ \hat{\rho}_{rt_v}(k+1|k) \\ \hat{\rho}_{rt_1}(k+1|k) \\ \vdots \\ \hat{\theta}_{rt_v}(k+1|k) \end{bmatrix}$$

(5.28)
Accordingly, each actual measurement (observation) \( z_j(k+1) \) is composed of a measured distance \( \rho_{r_{ij}}(k+1) \) and a measured angle \( \theta_{r_{ij}}(k+1) \):

\[
\begin{bmatrix}
\rho_{r_{ij}}(k+1) \\
\theta_{r_{ij}}(k+1)
\end{bmatrix}
\]  
(5.29)

The actual measurements \( z_j(k+1) \) are stacked into a composite vector \( z(k+1) \):

\[
\begin{bmatrix}
\rho_{r_{1i}}(k+1) \\
\vdots \\
\rho_{r_{ni}}(k+1) \\
\theta_{r_{1i}}(k+1) \\
\vdots \\
\theta_{r_{ni}}(k+1)
\end{bmatrix}
\]  
(5.30)

The composite innovation vector \( v(k+1) \) is:

\[
v(k+1) = z(k+1) - \hat{z}(k+1|k)
\]  
(5.31)
5.3.2 The Plant Model

The nonlinear plant equation describes how the state $\mathbf{x}^W_r(k)$ of the robot changes within one time step to the state $\mathbf{x}^W_r(k+1)$ in response to a control input $\mathbf{u}(k)$ and process noise $\mathbf{v}(k)$:

$$
\mathbf{x}^W_r(k + 1) = f[k, \mathbf{x}^W_r(k), \mathbf{u}(k)] + \mathbf{v}(k) 
$$

(5.32)

$$
\mathbf{v}(k) \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}(k))
$$

(5.33)

where $f[k, \mathbf{x}^W_r(k), \mathbf{u}(k)]$ is the nonlinear plant model.

The control input signal $\mathbf{u}(k)$ is based on wheel encoder (odometry) information about the robot motion. The odometric location of the robot is calculated by integration of wheel encoder signals. The process noise covariance matrix $\mathbf{Q}(k)$ is specified to model the odometry drift associated with the robot motion appropriately by the state prediction covariance matrix $\mathbf{P}(k+1|k)$.

The holonomic drive system of the Nomad XR4000 robot provides three unrestricted degrees of freedom ($x$, $y$, $\theta$), realized by four caster wheels with independently powered driving and steering axes. Consequently, this robot can be modeled by point kinematics. The control input $\mathbf{u}(k)$ consists of two translational components and one rotational component, defined with regard to the robot coordinate system:

$$
\mathbf{u}(k) = \begin{bmatrix}
\Delta x^R_r(k) \\
\Delta y^R_r(k) \\
\Delta \theta^R_r(k)
\end{bmatrix}
$$

(5.34)
Thus, the plant model or state transition function $f[k, x^w_r(k), u(k)]$ has the form:

$$
\begin{align*}
\mathbf{f}[k, x^w_r(k), u(k)] &= \\
&= \begin{bmatrix}
\Delta x^w_r(k) \cos \theta^w_r(k) - \Delta y^w_r(k) \sin \theta^w_r(k) \\
\Delta x^w_r(k) \sin \theta^w_r(k) + \Delta y^w_r(k) \cos \theta^w_r(k) \\
\theta^w_r(k) + \Delta \theta^w_r(k)
\end{bmatrix}
\end{align*}
$$

Concerning the intelligent wheelchair MAid, which is powered by a differential driving and steering system, point kinematics are not valid and a more complicated plant model is applied.

### 5.3.3 The Measurement Model

The nonlinear measurement equation describes how measurements (observations) $z(k+1)$ are related to the state $x^w_r(k+1)$ of the robot $r$ and the states $t$ of targets $t$ under presence of measurement noise $w(k+1)$:

$$
\begin{align*}
z(k + 1) &= \mathbf{h}\left[k + 1, x^w_r(k + 1), t\right] + w(k + 1) \\
w(k + 1) &\sim N(\mathbf{0}, \mathbf{R}(k + 1))
\end{align*}
$$

where $\mathbf{h}[k+1, x^w_r(k+1), t]$ is the nonlinear measurement model.

The measurement noise covariance matrix $\mathbf{R}(k+1)$ (cp. Section 4.3, Eq. 4.17) is specified to model the uncertainties $(\sigma_\rho, \sigma_\theta)$ associated with the parameters $\rho$ and $\theta$ of newly extracted straight line segments appropriately by the measurement prediction covariance matrix $\mathbf{S}(k+1|k)$. 
The measurement model \( h[k+1, \hat{x}_r^W(k+1\|k), t] \) for plane targets can be composed of functions for the predicted perpendicular distance \( \hat{\rho}_{\nu_i}(k+1\|k) \) from the center of the robot to a target line and functions for the predicted angle \( \hat{\theta}_{\nu_i}(k+1\|k) \) between the normal of a target line and the x-axis of the robot coordinate system:

\[
\begin{bmatrix}
\hat{\rho}_{\nu_i}(k+1\|k) \\
\vdots \\
\hat{\rho}_{\nu_v}(k+1\|k)
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\hat{x}_r^W(k+1\|k) \cos \theta_{\nu_i} + \hat{y}_r^W(k+1\|k) \sin \theta_{\nu_i} - \rho_{\nu_i} \\
\vdots \\
\hat{x}_r^W(k+1\|k) \cos \theta_{\nu_v} + \hat{y}_r^W(k+1\|k) \sin \theta_{\nu_v} - \rho_{\nu_v}
\end{bmatrix}
\]

\[
(5.38)
\]

\[
\begin{bmatrix}
\hat{\theta}_{\nu_i}(k+1\|k) \\
\vdots \\
\hat{\theta}_{\nu_v}(k+1\|k)
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\theta_{\nu_i} - \hat{\theta}_r^W(k+1\|k) \\
\vdots \\
\theta_{\nu_v} - \hat{\theta}_r^W(k+1\|k)
\end{bmatrix}
\]

\[
(5.39)
\]

\[
\begin{bmatrix}
\hat{x}_r^W(k+1\|k) \cos \theta_{\nu_i} + \hat{y}_r^W(k+1\|k) \sin \theta_{\nu_i} - \rho_{\nu_i} \\
\vdots \\
\hat{x}_r^W(k+1\|k) \cos \theta_{\nu_v} + \hat{y}_r^W(k+1\|k) \sin \theta_{\nu_v} - \rho_{\nu_v}
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\theta_{\nu_i} - \hat{\theta}_r^W(k+1\|k) \\
\vdots \\
\theta_{\nu_v} - \hat{\theta}_r^W(k+1\|k)
\end{bmatrix}
\]

\[
(5.40)
\]
5.3.4 Robot Location Prediction

Using the plant model and knowledge of the control input \( u(k) \), the robot location prediction \( \hat{x}_W^r(k+\|k) \) is as follows:

\[
\hat{x}_W^r(k + \|k) = \mathbf{f}[k, \hat{x}_W^r(k|k), u(k)]
\]  \hspace{1cm} (5.41)

The plant model \( \mathbf{f}[k, \hat{x}_W^r(k|k), u(k)] \) in Eq. 5.35 is linearized about the state estimate \( \tilde{x}_W^r(k|k) \) by the plant Jacobian \( \mathbf{f}_x^r(k) \):

\[
\mathbf{f}_x^r(k) = \left[ \nabla_{x^r} \mathbf{f}^T(k, x_W^r(u(k)|k)) \right]_{x_W^r = \tilde{x}_W^r(k|k), u = u(k)}^T
\]  \hspace{1cm} (5.42)

The robot location prediction covariance matrix \( P(k+\|k) \) is:

\[
P(k + \|k) = \mathbf{f}_x^r(k)P(k|k)\mathbf{f}_x^T(k) + Q(k)
\]  \hspace{1cm} (5.43)
5.3 Self-Localization by Multi-Target Tracking

5.3.5 Measurement Prediction

Using the robot location prediction $\hat{x}_r^W(k+1|k)$ and the stationary target locations $t$ in the global environment map $M^W$, the measurement predictions $\hat{z}(k+1|k)$ are as follows:

$$\hat{z}(k+1|k) = h[k+1, \hat{x}_r^W (k+1|k), t]$$ (5.44)

The measurement model $h[k+1, \hat{x}_r^W (k+1|k), t]$ in Eq. 5.40 is linearized about the state prediction $\hat{x}_r^W (k+1|k)$ by the measurement Jacobian $h_{x_r^W} (k+1)$:

$$h_{x_r^W} (k+1) = \nabla_{x_r^W} h^T (k+1, x_r^W)_{x_r^W = \hat{x}_r^W (k+1|k)}$$

$$= \begin{bmatrix}
\cos \theta_{t_1} & \sin \theta_{t_1} & 0 \\
\vdots & \vdots & \vdots \\
\cos \theta_{t_n} & \sin \theta_{t_n} & 0 \\
0 & 0 & -1 \\
\vdots & \vdots & \vdots \\
0 & 0 & -1
\end{bmatrix}$$ (5.45)

The measurement prediction covariance matrix $S(k+1)$ is:

$$S(k+1) = h_{x_r^W} (k+1)P(k+1|k)h_{x_r^W}^T (k+1) + R(k+1)$$ (5.46)
5.3.6 Robot Location Estimation

Robot location estimation is performed according to Eq. 5.17 - Eq. 5.19.

Filter gain matrix $W(k+1)$:

$$W(k + 1) = P(k + 1|k)h_{x_p}^T(k + 1)S(k + 1)$$  \hspace{2cm} (5.47)

Updated robot location estimate $\hat{x}_r^W(k+1|k+1)$:

$$\hat{x}_r^W(k + 1|k + 1) = \hat{x}_r^W(k + 1|k) + W(k + 1)v(k + 1)$$  \hspace{2cm} (5.48)

Updated robot location covariance matrix $P(k+1|k+1)$:

$$P(k + 1|k + 1) = P(k + 1|k) - W(k + 1)S(k + 1)W^T(k + 1)$$  \hspace{2cm} (5.49)

5.4 Simultaneous Localization and Mapping (SLAM)

If a global environment map $M^W(k)$ is constructed from sensor data by exploration, the targets cannot be considered stationary, and accurate knowledge of the target locations cannot be assumed. Thus, the target state vectors $t(k)$ are a function of time, and the modeled straight line segments need to be represented with uncertainties ($\sigma_{\rho} \geq 0$, $\sigma_{\theta} \geq 0$). The uncertainties of the segments determine the influence that the targets have on the updated robot location estimate.

A time-variable and uncertain global environment map $M^W(k)$ may be regarded as a set of $n_t$ time-variable target state vectors $t_i(k)$ with associated target state error covariance matrices $C_{t_i}(k)$:

$$M^W(k) = \{t_i(k), C_{t_i}(k)\} \quad i = 1, \ldots, n_t$$  \hspace{2cm} (5.50)

The robot location $x_r^W(k)$ defines the transformation between robot coordinates ($R$) and world coordinates ($W$). Newly extracted straight line
segments are projected into the world coordinate system based on the updated robot location estimate $\hat{x}_r^W(k+1|k+1)$. In the world coordinate system, the newly extracted straight line segments are then merged with the previously existing straight line segments in order to obtain an updated global environment map $M^W(k+1)$.

The system state vector $x_s^W(k)$ describing the robot in a time-variable environment $M^W(k)$ is composed of the robot state vector $x_r(k)$ and the target state vectors $t_i(k)$:

$$x_s^W(k) = [x_r^W(k), t_1(k), \ldots, t_n(k)]^T$$ (5.51)

Consequently, the system state error covariance matrix $P_s(k|k)$ becomes:

$$P_s(k|k) = \begin{bmatrix}
P_r(k|k) & C_{r,t_1}(k|k) & \cdots & C_{r,t_n}(k|k) \\
C_{r,t_1}(k|k) & C_{t_1}(k) & \cdots & C_{t_1,t_n}(k) \\
\vdots & \vdots & \ddots & \vdots \\
C_{r,t_n}(k|k) & C_{t_1,t_n}(k) & \cdots & C_{t_n}(k)
\end{bmatrix}$$ (5.52)

where $P_r(k|k)$ is the robot state error covariance matrix, $C_{r,t}(k|k)$ are robot to target cross-covariance matrices, $C_t(k)$ are target state error covariance matrices, and $C_{t,t}(k)$ are target to target cross-covariance matrices. The cross-covariance matrices $C_{r,t}(k|k)$ and $C_{t,t}(k)$ are non-zero, since the robot state estimate and the target state estimates are correlated. Thus, when the robot state estimate is updated, all robot to target covariance matrices must be recomputed, and when a target state estimate is updated, all cross-covariance matrices concerning this target must be recalculated [57, 112].

The system state error covariance matrix $P_s(k|k)$ can be decoupled by the following sensing strategy: At the initial robot location, the locations of detectable features are modeled precisely. After each move, observations of the previously learned features (targets) are utilized to determine the robot location accurately before the locations of newly detectable features are modeled based on the determined robot location. This approach reduces the cross-correlations so that individual robot and target state error covariance matrices can be used [57, 112].
5.5 Contributions

In this chapter, mobile robot self-localization based on straight line segments has been explained. Considering the application of an extended Kalman filter to self-localization by multi-target tracking as described by Leonard et al. [56, 57, 58, 112], because of the high angular uncertainty associated with sonar measurements forming regions of constant depth, only distance information (i.e. no orientation information) from the ultrasonic sensor data is used for computing the updated robot location estimate. In contrast, by extracting straight line segments from ultrasonic sensor data, in addition to the distance constraint an angle constraint is obtained. In order to be able to exploit both constraints for calculating the updated robot location estimate, the measurement model has been composed of functions considering distance information and functions considering angle information. Thus, the extended Kalman filter framework has been adapted for self-localization utilizing extracted straight line segments instead of distance measurements. The proposed method is also applicable to self-localization based on straight line segments derived from laser range finder data.
6 Global Environment Mapping

In Chapter 4, it has been described how local environment models can be created at different robot locations by extracting straight line segments from laser range finder data and from ultrasonic sensor data. In Chapter 5, mobile robot self-localization based on previously modeled and newly extracted straight line segments has been explained, in order to accurately project newly extracted straight line segments into the world coordinate system. Within this chapter, it is demonstrated how local environment models created at different robot locations can be merged for building a global environment map.

An introduction to global environment mapping is contained in Section 6.1. The coordinate transformations from robot coordinates into world coordinates are specified in Section 6.2. Forming regions of straight line segments in workspace is explained in Section 6.3. Straight line fusion by a clustering approach and by a regression approach is described in Section 6.4. Experimental results of global environment mapping with laser range finder data and ultrasonic sensor data are presented in Section 6.5. The contributions of the chapter are summarized in Section 6.6.

6.1 Introduction

In order to build global environment maps, the robot must move in space and observe the environment from various points of view. On the one hand, this allows to model further distant objects which could not be perceived from a stationary robot location by ultrasonic sensors due to the limited distance detection range, and on the other hand, it allows to model objects from various perspectives which is particularly useful concerning convex or wide objects. Figure 6.1 illustrates the modeling of geometric objects by straight line segments extracted at different robot locations.
Fusion methods have been developed for spatially integrating straight line segments extracted at different robot locations from laser range finder data and ultrasonic sensor data. The underlying idea is to aggregate locally extracted straight line segments in a global environment map, and to preferably replace them by merged straight line segments. Aggregation is achieved by transforming extracted straight line segments from robot coordinates into world coordinates, and straight line fusion can be performed by a clustering approach or by a regression approach.

The performance of the developed fusion methods will be demonstrated by spatially integrating the local environment models created as described in Chapter 4 from laser range finder data and from ultrasonic sensor data, i.e. the straight line segments extracted within the office room introduced in Section 1.5 at the 17 robot locations marked by numbered crosses in Figure 1.9.
6.2 Coordinate Transformations

Based on a control input $u(k) = (\Delta x^R(k), \Delta y^R(k), \Delta \theta^R(k))^T$ defined in Eq. 5.34 with regard to the robot coordinate system, and in conformity with the plant model $f[k, \hat{\mathbf{x}}^W(k, u(k))]$ in Eq. 5.35, the robot location prediction $\hat{\mathbf{x}}^W_r(k+1|k)$ in the world coordinate system according to Eq. 5.41 can be expressed as follows:

$$
\begin{bmatrix}
\hat{x}^W_r(k + 1|k) \\
\hat{y}^W_r(k + 1|k) \\
\hat{\theta}^W_r(k + 1|k)
\end{bmatrix}
= 
\begin{bmatrix}
\hat{x}^W_r(k|k) \\
\hat{y}^W_r(k|k) \\
\hat{\theta}^W_r(k|k)
\end{bmatrix}
+ 
\begin{bmatrix}
\cos \hat{\theta}^W_r(k|k) & -\sin \hat{\theta}^W_r(k|k) & 0 \\
\sin \hat{\theta}^W_r(k|k) & \cos \hat{\theta}^W_r(k|k) & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\Delta x^R(k) \\
\Delta y^R(k) \\
\Delta \theta^R(k)
\end{bmatrix}
$$

(6.1)

The updated robot location estimate $\mathbf{x}^W_r(k+1|k+1)$ in the world coordinate system can be obtained according to Eq. 5.48.

The endpoints $p$ of newly extracted straight line segments represented in a local environment map $M^R(k+1)$ in robot coordinates $\mathbf{x}^R_p(k+1)$ can be transformed into a global environment map $M^W(k+1)$ in world coordinates $\mathbf{x}^W_p(k+1)$ by the following coordinate transformation:

$$
\begin{bmatrix}
x^W_p(k+1) \\
y^W_p(k+1)
\end{bmatrix}
= 
\begin{bmatrix}
\hat{x}^W_p(k + 1|k + 1) \\
\hat{y}^W_p(k + 1|k + 1)
\end{bmatrix}
+ 
\begin{bmatrix}
\cos \hat{\theta}^W_p(k + 1|k + 1) & -\sin \hat{\theta}^W_p(k + 1|k + 1) \\
\sin \hat{\theta}^W_p(k + 1|k + 1) & \cos \hat{\theta}^W_p(k + 1|k + 1)
\end{bmatrix}
\begin{bmatrix}
x^R_p(k+1) \\
y^R_p(k+1)
\end{bmatrix}
$$

(6.2)
6.3 Forming Regions of Straight Line Segments

As a basis for straight line fusion, the aggregated straight line segments can be subdivided into neighborhood regions in workspace. For this purpose, straight line segments are grouped by their midpoints such that within each group all segments can be reached by a sequence of midpoints lying sufficiently close. The distance criterion for the connectivity of midpoints is deduced from the distance the robot has moved between two successive scan locations. Forming regions of straight line segments is realized by applying a Voronoi diagram and a Delaunay triangulation.

The Voronoi diagram is a net of convex polygonal cells named Voronoi polygons. The line segments of the diagram are called Voronoi edges, and the nodes are termed Voronoi vertices. All nearest neighbor points in a set of points cause a Voronoi edge between them. Each Voronoi vertex is the common intersection of three Voronoi edges. Equivalently, a Voronoi vertex is the center of a circle defined by the three points that cause the intersecting Voronoi edges [116].

The Euclidean distance between two points \( p \) and \( q \) be denoted by \( \text{dist}(p, q) \), and \( P = \{p_i\}, i = 1, \ldots, n \), be a set of \( n \) distinct points (sites) in the plane. Then, the Voronoi diagram of \( P \) is defined as the subdivision of the plane into \( n \) Voronoi polygons, one for each site in \( P \), with the property that a point \( q \) lies in the Voronoi polygon corresponding to a site \( p \) if and only if \( \text{dist}(p_i, q) < \text{dist}(p_j, q) \) for each \( p_j \in P \) with \( j \neq i \) [105].

The Voronoi diagram contains in a useful sense all the proximity information defined by the given set of distinct points. Problems involving the proximity of points in the plane can thus be solved efficiently by means of the Voronoi diagram. The Delaunay triangulation of a set of points is the straight line dual of the Voronoi diagram. This implies that the Voronoi diagram can be utilized to obtain the Delaunay triangulation. Computing a Voronoi diagram of a set of \( n \) points in the plane requires \( O(n \log n) \) operations in the worst case. The Delaunay triangulation can be calculated in linear time from the Voronoi diagram.

Given a set of data points (Delaunay nodes), the Delaunay triangulation produces a set of line segments (Delaunay edges) connecting every data point to its natural neighbors, i.e. all data points are linked by non-intersecting line segments so that the inside area of the convex hull is composed of triangles (see Figure 6.3, Figure 6.10, and Figure 6.13). Delaunay edges fulfilling the distance criterion for the connectivity of midpoints represent valid links between data points belonging to a common neighborhood region in workspace (see Figure 6.4, Figure 6.11, and Figure 6.14).
The approach of forming regions via Voronoi diagram and Delaunay triangulation corresponds to a single linkage clustering algorithm.

6.4 Straight Line Fusion

This section describes how global environment maps can be built by straight line fusion. A clustering approach and a regression approach are introduced for merging straight line segments extracted at different robot locations.

6.4.1 Clustering Approach

Beginning with the first neighborhood region in workspace, aggregated straight line segments belonging to the region are merged in parameter space. For this purpose, the straight line segments are extended to straight lines of infinite length and Hough transformed into parameter space. A $100 \times 100$ occupancy grid is deployed in parameter space, with a cell size of 0.10 m in $\rho$-dimension and a cell size of 0.0628 rad in $\theta$-dimension. Subsequently, it is proceeded as described in Subsection 4.3.1 to obtain cluster centers which represent merged straight lines in parameter space. In workspace, the endpoints of the aggregated straight line segments are projected onto the merged straight lines. The two outermost projection points limit the valid range of the merged straight lines, which yields merged straight line segments of defined length. The same steps are carried out for all neighborhood regions in workspace in order to replace all aggregated straight line segments by merged straight line segments.

6.4.2 Regression Approach

Beginning with the first neighborhood region in workspace, a regression line is calculated for the midpoints of the aggregated straight line segments belonging to the region. The sequence of appearance of the projected midpoints on this regression line yields a linear order of the segments belonging to the region. Aggregated straight line segments are considered for merging in the obtained order by cumulatively fitting new regression lines to pairs of endpoints of consecutive segments. Calculating a regression line as well as the root mean square error $RMSE$ is according to the equations given in Section 4.2 (Eq. 4.1 to Eq. 4.12). When the root mean square error of a new regression line exceeds a predefined threshold (e.g. $RMSE > 0.02$)
m), the previously calculated regression line with a root mean square error below this threshold is recalled to represent a merged straight line. Proceeding with the pair of endpoints of the segment which caused the root mean square error of the last calculated regression line to exceed the predefined threshold, the next merged straight line will be determined. Finally, the endpoints of all segments are projected onto their common regression lines, and the length of the merged straight line segments is defined by the outermost projection points. The same steps are carried out for all neighborhood regions in workspace in order to replace all aggregated straight line segments by merged straight line segments.

6.5 Experimental Results

This section presents experimental results of global environment mapping with laser range finder data and ultrasonic sensor data.

6.5.1 Environment Mapping with Laser Range Finder Data

Straight line segments extracted at different robot locations from laser range finder data (see Section 4.2) are aggregated in a global environment map as shown in Figure 6.2. The Delaunay triangulation of the midpoints of the aggregated straight line segments is visualized in Figure 6.3. Delaunay edges fulfilling the distance criterion for the connectivity of midpoints and thus representing valid links between data points belonging to a common neighborhood region in workspace are drawn separately in Figure 6.4.
Figure 6.2: Aggregated straight line segments derived from laser range finder data
Figure 6.3: Delaunay triangulation of the midpoints of the aggregated straight line segments.
In the case of the clustering approach, forming regions in workspace is not essential for obtaining good fusion results, but it may be advantageous in order to avoid incorrect associations in parameter space between spatially distant straight line segments. This is in contrast to the regression approach, where forming regions in workspace is required for deriving an appropriate order of straight line segments for cumulatively fitting new regression lines to pairs of endpoints of consecutive segments.
**Clustering Approach**

The fusion result obtained by the clustering approach without foregoing region formation is presented in Figure 6.5, and with preceding region formation is depicted in Figure 6.6.

*Figure 6.5: Merged straight line segments obtained by the clustering approach without foregoing region formation*
Figure 6.6: Merged straight line segments obtained by the clustering approach with preceding region formation
Regression Approach

The fusion result obtained by the regression approach without foregoing region formation is presented in Figure 6.7, and with preceding region formation is depicted in Figure 6.8.

Figure 6.7: Merged straight line segments obtained by the regression approach without foregouing region formation
6.5.2 Environment Mapping with Ultrasonic Sensor Data

Fusion results obtained by applying the clustering approach and the regression approach will be illustrated for straight line segments extracted at different robot locations from ultrasonic sensor data (see Section 4.3) by parameter space clustering and by tangential regression. The result of environment mapping with laser range finder data obtained by the clustering approach without foregoing region formation (see Figure 6.5) will serve as a reference for the results of environment mapping with ultrasonic sensor data. In the case of environment mapping with ultrasonic sensor data, only experimental results obtained with preceding region formation will be presented.
Figure 6.9 displays aggregated straight line segments derived from ultrasonic sensor data by parameter space clustering (see Subsection 4.3.1) together with a global laser range finder environment map. The Delaunay triangulation of the midpoints of the aggregated straight line segments is visualized in Figure 6.10. Delaunay edges fulfilling the distance criterion for the connectivity of midpoints are drawn separately in Figure 6.11.

*Figure 6.9:* Aggregated straight line segments derived from ultrasonic sensor data by parameter space clustering (bold lines)
Figure 6.10: Delaunay triangulation of the midpoints of the aggregated straight line segments
Figure 6.11: Delaunay edges fulfilling the distance criterion for the connectivity of midpoints.
Figure 6.12 displays aggregated straight line segments derived from ultrasonic sensor data by tangential regression (see Subsection 4.3.2) together with a global laser range finder environment map. The Delaunay triangulation of the midpoints of the aggregated straight line segments is visualized in Figure 6.13. Delaunay edges fulfilling the distance criterion for the connectivity of midpoints are drawn separately in Figure 6.14.

Figure 6.12: Aggregated straight line segments derived from ultrasonic sensor data by tangential regression (bold lines)
Figure 6.13: Delaunay triangulation of the midpoints of the aggregated straight line segments
Figure 6.14: Delaunay edges fulfilling the distance criterion for the connectivity of midpoints
**Clustering Approach**

Figure 6.15 and Figure 6.16 illustrate the replacement of aggregated straight line segments derived from ultrasonic sensor data by parameter space clustering and by tangential regression through merged straight line segments.

*Figure 6.15: Replacement of aggregated straight line segments derived from ultrasonic sensor data by parameter space clustering through merged straight line segments (bold lines)*
Figure 6.16: Replacement of aggregated straight line segments derived from ultrasonic sensor data by tangential regression through merged straight line segments (bold lines)
Regression Approach

Figure 6.17 and Figure 6.18 illustrate the replacement of aggregated straight line segments derived from ultrasonic sensor data by parameter space clustering and by tangential regression through merged straight line segments.
6.5 Experimental Results

6.5.3 Evaluation of the Clustering and Regression Approach

Application of the clustering approach and the regression approach for straight line fusion yields global environment maps of comparable quality.

When fusing straight line segments extracted from laser range finder data, straight line segments that cannot be merged are discarded, and only merged straight line segments are retained in the global environment map. In contrast, when fusing straight line segments extracted from ultrasonic sensor data, straight line segments that cannot be merged are retained as well.

Figure 6.18: Replacement of aggregated straight line segments derived from ultrasonic sensor data by tangential regression through merged straight line segments (bold lines)
From some few points, straight line segments could be extracted from ultrasonic sensor data by parameter space clustering (see Subsection 4.3.1) and tangential regression (see Subsection 4.3.2) for the chair back-rest at location \((x, y) = (1.0 \text{ m}, -0.7 \text{ m})\), which could thus also be represented in the global ultrasonic environment maps. The chair back-rest is not contained in the global laser range finder environment map, since it could not be properly modeled in the polygonal environment models created from laser range finder data at different robot locations due to its lower height (cp. Subsection 4.3.3). A few further inconsistencies appear between the ultrasonic and the laser range finder global environment maps because of varying object extents on different heights.

Considering that only 17 ultrasonic scans were taken at robot locations with relatively large distances of 0.25 m, 0.30 m, or 0.50 m to each other, more comprehensive global ultrasonic environment maps can be created by taking additional scans at closer robot locations.

Slightly modified versions of the described approaches allow incremental map building by integrating extracted straight line segments iteratively. In this mode of operation, the robot only maintains the global environment map and discards the local environment models after integration.

For simultaneous localization and mapping (SLAM), the robot location can be updated at each scanning instance by matching the local environment model with the global environment map as described in Chapter 5.

### 6.6 Contributions

In this chapter, global environment mapping by merging local environment models has been demonstrated. Fusion methods have been developed for spatially integrating straight line segments extracted at different robot locations from laser range finder data and ultrasonic sensor data in order to create detailed maps of complex environments. The underlying idea is to aggregate locally extracted straight line segments in a global environment map, and to preferably replace them by merged straight line segments. As a basis for straight line fusion, an approach of forming regions via Delaunay triangulation has been proposed for subdividing aggregated straight line segments into neighborhood regions in workspace. Straight line fusion has been realized by a clustering approach and by a regression approach. The performance of the developed algorithms has been proved in a typical office environment.
This chapter covers fault diagnosis for the perceptual system of mobile robots. As general basis for monitoring the state of environmental sensors, a fault detection model has been developed, which consists of sub-models for data from laser range finders and ultrasonic sensors. With the aid of the created models, different kinds of redundancy can be utilized, and consistency and plausibility checks can be executed. A two-step fault detection method compares in the first step real with simulated distance readings, and in the second step real distance readings among one another. The proposed fault detection method serves for failure recognition based on environment modeling and on hypotheses for expected sensor readings.

An introduction to fault diagnosis for the perceptual system of mobile robots is provided in Section 7.1. Deriving a simulation model of the ultrasonic sensing system is addressed in Section 7.2. Sensor evaluation by comparison of real and simulated ultrasonic sensor readings is explained in Section 7.3, and sensor evaluation by mutual comparison of real ultrasonic sensor readings is described in Section 7.4. Performing reliable overall sensor assessment is demonstrated in Section 7.5. The contributions of this chapter are reviewed in Section 7.6.

7.1 Introduction

With regard to safe navigation of autonomous mobile systems, failures or impairments of sensor systems represent an important challenge. For instance, it frequently occurs that individual sensors are covered or otherwise influenced (e.g. by interferences from other sensing systems or disturbing ultrasonic noise). Since collision-free navigation of autonomous mobile systems by increasing the active safety is required, faulty, maladjusted, covered, and otherwise impaired sensors should be recognized, and adequate measures for failure correction should be applied.

As already mentioned, the developed fault detection model for monitoring the state of environmental sensors consists of sub-models for data from laser range finders and ultrasonic sensors. The model for laser data relies
on straight line extraction for polygonal environments, whereas the model for sonar data depends on direct and cross echo evaluation. With the aid of the created models, physical, analytical, temporal, and spatial redundancy can be utilized, and consistency and plausibility checks can be executed.

In order to support specific fault recovery, failure classification can be performed. Measures for failure correction comprise sensor employment planning as well as context dependent (i.e. situation and environment dependent) sensor data fusion. Other important aspects of fault compensation to fulfill the mission with a reduced sensor system are the use of redundant hardware and alternative robot control strategies as well as virtual sensors and automatic sensor calibration.

The fault detection model has been adapted for the intelligent wheelchair MAid and the automatically guided hospital bed AutoBed, for which the proposed safety features are particularly important. Within this chapter, the created models are described for the circular sensor arrangement on the experimental robot Nomad XR4000, and their behavior is demonstrated in the introduced office environment (see Section 1.5).

In respect of related work concerning fault detection for ultrasonic sensors on mobile robots, the reader be referred to [94, 95]. In that research, fault detection is based on a grid representation of the environment. A grid map is constructed using a probabilistic sensor model similar to [47]. With the use of an occupancy grid it is possible to compare sensor readings made by different sensors, at different points of time, and from various positions and orientations in space. In contrast, the approach presented in this chapter does not inherently depend on temporal or spatial redundancy, i.e. it does not require a rotation of the robot on the spot or a movement in space in order to allow sensors to make redundant statements [128, 131, 135].

### 7.2 Simulation Model of the Ultrasonic Sensing System

The polygonal environment model created from laser range finder data as described in Section 4.2 (see Figure 4.3) serves as a basis to derive a simulation model of the ultrasonic sensing system. Concerning the polygonal environment model, it is distinguished between planes, corners, and edges (cp. [112]):

- A plane is represented by a line in the two-dimensional environment model. Lines are represented in Hesse’s normal form.
- A corner is a concave dihedral, and produces specular returns. Corners are represented as points in the two-dimensional model.
• An edge is a convex dihedral, and produces diffuse reflections. Like corners, edges are represented as points in the two-dimensional model.

Experiments with the employed ultrasonic sensors have confirmed that detectable echoes predominantly arise from planes and corners, since edges produce diffuse reflections. For this reason, in the simulation model of the ultrasonic sensing system possible reflections from edges are disregarded, and only returns from planes and corners are considered.

In order to explain the simulation model of the ultrasonic sensing system, the detection angles for the direct and cross echoes are reconsidered. With the chosen sensor orientation, the horizontal detection angle for direct echoes is 60° and for cross echoes approximately 45°, 30°, or 15°, depending whether the first, second, or third neighbor sensor is involved, since the angular displacement of the sensors is 15°. Consequently, \( n_{de} = 24 \) direct echo paths and \( n_{ce} = 3 \times 24 = 72 \) cross echo paths (in two transmit-receive directions) exist in the ultrasonic simulation model. Direct echo paths can be depicted as circular arcs of 60°, and cross echo paths as elliptic arcs of about 45°, 30°, or 15°.

To create the ultrasonic simulation model, all planes and corners of the polygonal environment model (pseudo features excepted) are checked whether they could cause detectable echoes, i.e. reflections which fall within the detection angle for direct or cross echoes of any sensor. Here, it has to be considered that planes (lines in the two-dimensional model) or corners (points in the two-dimensional model) can be occluded. A line is occluded if a perpendicular drawn from the line to the sensor intersects another element of the polygonal model, and a point is occluded if a line drawn from the point to the sensor intersects another element of the polygonal model. Finally, the shortest detectable distance is assigned to each direct and cross echo path, respectively.

Supposing that the polygonal environment model from laser range finder data is created separately, the time complexity of deriving the simulation model of the ultrasonic sensing system is \( O((n_{de} + n_{ce})n_f) \), where \( n_f \) is the number of detectable features in the polygonal environment model.

Figure 7.1 shows the direct echoes, Figure 7.2 the cross echoes with first neighbors, Figure 7.3 the cross echoes with second neighbors, and Figure 7.4 the cross echoes with third neighbors. Figure 7.5 presents the complete simulation model of the ultrasonic sensing system, i.e. the circular and elliptic arcs for all direct and cross echo paths. Echo paths with no detectable echo within the operating range of the sensors are represented by arcs corresponding to the maximum distance detection range \( (d_{max} = 2 \text{ m}) \).
Figure 7.1: Direct echoes

Figure 7.2: Cross echoes with first neighbors
Figure 7.3: Cross echoes with second neighbors

Figure 7.4: Cross echoes with third neighbors
7.3 Comparison of Real and Simulated Sensor Readings

In the previous section, it has been described how the shortest detectable distance is assigned to each direct and cross echo path in order to create the simulation model of the ultrasonic sensing system. Within this section, the first received direct and cross echoes from the ultrasonic sensing system are compared with the results obtained from the simulation model. Figure 7.6 displays the circular and elliptic arcs for the real distance readings.
It can be observed that discrepancies exist between the real and the simulated distance readings. A table edge at \((x, y) = (1.3 \text{ m}, 0.0 \text{ m})\) could not be recognized by the laser range finders due to the lower height of this object. Consequently, the table edge is not contained in the polygonal environment model created from laser range finder data, and is therefore also not considered in the simulation model of the ultrasonic sensing system. A chair back-rest at \((x, y) = (1.0 \text{ m}, -0.7 \text{ m})\) was only detected at its top edge by the laser range finders, which is not sufficient to yield a straight line segment in the polygonal environment model. A second chair back-rest at \((x, y) = (0.9 \text{ m}, 1.1 \text{ m})\), which is higher than the first one, could be completely recognized by the laser range finders and is correctly considered in the simulation model of the ultrasonic sensing system. However, the second chair is further distant from the robot than the first chair, and it could not be properly observed by the ultrasonic sensors, although this should be the case according to the simulation model.

In order to automatically compare the real and the simulated distance readings of \(n = 24\) sensors, two 4-by-24 matrices are created, matrix \(D_r\) containing the real echoes and matrix \(D_s\) containing the simulated echoes. Considering only first received echoes, the ultrasonic sensing system provides one direct echo and six cross echoes for each sensor. The cross echo...
paths exist two-fold, since both sensors involved are able to transmit and receive ultrasound. Regarding the simulation model, the distance results for both directions are identical. For comparison with the simulated distance readings, the original 7-by-24 matrix obtained from the sensing system is reduced to a 4-by-24 matrix. To accomplish this, the average length of the two-fold existing echo paths is calculated if the echoes are consistent, or an error code is returned if the echoes are inconsistent. The averaged echo path lengths are converted into distance readings, considering the displacements of the involved sensors. Distance readings exceeding the detection range are confined to the maximum value.

Subsequently, the 4-by-24 matrices $D_r$ and $D_s$ are subtracted and a “simulation-based difference matrix” $\Delta_s$ is obtained. The element values $x$ of the simulation-based difference matrix $\Delta_s$ are processed by the following parameterized function $f(x)$:

$$f(x) = e^{-\left(\frac{x}{\sigma}\right)^2}$$

(7.1)

with the heuristically determined parameter $|\alpha| = 0.1$ m. The parameterized function $f(x)$, which has the nature of a centered normal distribution function (with the scaling factor being omitted), is shown in Figure 7.7.
7.3 Comparison of Real and Simulated Sensor Readings

Processing the simulation-based difference matrix $\Delta_s$ by the parameterized function $f(x)$ yields a so-called “simulation-based confidence matrix” $C_s$ of the same dimension. The element values of the simulation-based confidence matrix $C_s$ are a measure for the conformity between real and simulated distance readings.

Now, the simulation-based confidence matrix $C_s$ is expanded to the dimension 7-by-24, which is achieved by adding the redundant echo paths to each column. The expanded simulation-based confidence matrix $C_s$ then contains confidence values for one direct echo path and six cross echo paths per sensor. An average confidence value is calculated for all sensors by accumulating over each column of the expanded simulation-based confidence matrix $C_s$ and dividing the sum by the number of added rows ($m_s = 7$), thus obtaining a “simulation-based confidence vector” $c_s$. For the simulated distance readings presented in Figure 7.5 and the real distance readings displayed in Figure 7.6, in the case of a faultless ultrasonic sensing system, the simulation-based confidence vector $c_s$ is the following:

$$c_s = \{ 1.000 \ 0.714 \ 0.286 \ 0.143 \ 0.701 \ 0.552 \ 0.557 \ 0.598 \ \ldots \ 
0.163 \ 0.181 \ 0.585 \ 0.556 \ 0.702 \ 0.848 \ 0.286 \ 0.571 \ \ldots \ 
0.149 \ 0.292 \ 0.286 \ 0.143 \ 0.376 \ 0.429 \ 0.429 \ 1.000 \ \}$$
7.4 Mutual Comparison of Real Sensor Readings

Within this section, the first received direct and cross echoes from the ultrasonic sensing system are compared mutually. Consequently, one direct echo and six cross echoes are considered for each sensor, i.e. \((1+6) \times 24 = 168\) echoes altogether. Since the sensors are placed close to each other, and the detection angles for various direct and cross echo paths multiply overlap, neighbor echoes can be assumed to be similar. This allows comparison between distance readings of overlapping echo paths.

The following echo path overlaps exist for each sensor \((S_i)\):

direct echo \(S_i\) with
- direct echoes \(S_{i3}, S_{i2}, S_{i1}, S_{i+1}, S_{i+2}, S_{i+3}\)
- cross echoes \(S_{i3}/S_{i2}, S_{i2}/S_{i1}, S_{i1}/S_i, S_i/S_{i+1}, S_{i+1}/S_{i+2}, S_{i+2}/S_{i+3}\)
- cross echoes \(S_{i3}/S_i, S_{i2}/S_{i+1}, S_{i1}/S_{i+2}, S_i/S_{i+3}\)

cross echo \(S_i\)/\(S_{i+1}\) with
- cross echoes \(S_{i3}/S_{i+2}, S_{i2}/S_{i+1}, S_i/S_{i+1}, S_{i+1}/S_{i+2}\)
- cross echoes \(S_{i3}/S_{i+1}, S_{i2}/S_i, S_{i1}/S_{i+1}, S_i/S_{i+2}\)
- cross echoes \(S_{i3}/S_i, S_{i2}/S_{i+1}, S_{i1}/S_{i+2}\)

cross echo \(S_i\)/\(S_{i+2}\) with
- cross echoes \(S_{i2}/S_{i+1}, S_{i1}/S_i, S_{i+1}/S_{i+2}, S_{i+2}/S_{i+3}\)
- cross echoes \(S_{i2}/S_i, S_{i1}/S_{i+1}, S_i/S_{i+2}, S_{i+1}/S_{i+3}\)
- cross echoes \(S_{i2}/S_{i+1}, S_{i1}/S_{i+2}, S_i/S_{i+3}\)

cross echo \(S_i\)/\(S_{i+3}\) with
- cross echoes \(S_{i3}/S_{i+1}, S_{i2}/S_{i+1}, S_{i1}/S_{i+1}\)
- cross echoes \(S_{i3}/S_i, S_{i2}/S_{i+1}\)
cross echo $S_i / S_{i+2}$ with
- cross echoes $S_{i+1} / S_{i+1}$, $S_{i+1} / S_{i+3}$
- cross echoes $S_{i+1} / S_{i+2}$, $S_i / S_{i+3}$

Thus, 51 overlaps of direct and cross echo paths exist for each sensor. In order to compare the real distance readings for these echo paths, the following procedure is proposed.

Firstly, all circular and elliptic arcs displayed in Figure 7.6 are represented by a predefined number of equidistant points per arc (e.g. $n_p = 11$). The discretization of all ultrasonic echo arcs by equidistant points is illustrated in Figure 7.8. As described in Subsection 4.3.1, it is assumed that an echo could originate from the surface of an object being located at each discrete point on the echo arc and having a perpendicular reflection line to the real or virtual sensor. The potential reflecting surfaces are represented by straight line elements using Hesse’s normal form of the equation of a line with the perpendicular distance $\rho$ from the origin in the coordinate system and the angle $\theta$ between the normal of the line and the $x$-axis. Figure 7.9 shows the straight line elements representing potential reflecting surfaces.

![Figure 7.8: Discretization of the ultrasonic echo arcs by equidistant points](image-url)
Subsequently, for all combinations of overlapping echo paths, the \( n_p \) straight line elements representing potential reflecting surfaces are compared with each other by calculating a form of Mahalanobis distance (statistical distance) \( d_{1,2} \):

\[
d_{1,2} = \sqrt{\frac{\left(\rho_2 - \rho_1\right)^2}{\sigma_{\rho_1} + \sigma_{\rho_2}} + \frac{\left(\theta_2 - \theta_1\right)^2}{\sigma_{\theta_1} + \sigma_{\theta_2}}} \tag{7.2}
\]

where \((\sigma_{\rho_1}, \sigma_{\theta_1})\) and \((\sigma_{\rho_2}, \sigma_{\theta_2})\) are the uncertainties associated with the respective straight line elements. For calculating the statistical distances \( d_{1,2} \) between straight line elements from two echo arcs, the number of required computations is \( \# = n_p^2 \).

Finally, for each combination of overlapping echo paths, the two straight line elements with the minimum statistical distance \( d_{1,2} \) are determined, and the \( \rho \)-parameters of these elements are subtracted in order to derive a “neighborhood-based difference matrix” \( \Lambda_n \) of dimension 51-by-24. The element values \( x \) of the neighborhood-based difference matrix \( \Lambda_n \)
are processed by the parameterized function $f(x)$ in Eq. 7.1, with the parameter $|\alpha| = 0.1$ m.

Processing the neighborhood-based difference matrix $\Delta_n$ by the parameterized function $f(x)$ yields a so-called “neighborhood-based confidence matrix” $C_n$ of the same dimension. The element values of the neighborhood-based confidence matrix $C_n$ are a measure for the conformity between real distance readings of overlapping echo paths.

In the proposed procedure it is assumed that consistent distance readings arise from the same potential reflecting surface within the environment. As an approximation, in the case of a circular sensor arrangement on the robot, the real distance readings from the ultrasonic sensing system could also be compared directly. However, the approach via potential reflecting surfaces represents a general solution for universal sensor configurations.

The intention is to compare distance readings of echo paths where the sensor to be tested is involved with distance readings of overlapping echo paths where this sensor is not involved in order to only consider reference paths which are independent from the sensor to be tested. For this reason, rows containing dependent echo paths are removed from the neighborhood-based confidence matrix $C_n$, and a reduced neighborhood-based confidence matrix $C_n$ of dimension 35-by-24 is obtained. For each sensor, the reduced neighborhood-based confidence matrix $C_n$ then contains confidence values for all echo path overlaps with reference paths independent from the sensor to be tested. An average confidence value is calculated for all sensors by accumulating over each column of the reduced neighborhood-based confidence matrix $C_n$ and dividing the sum by the number of added rows ($m_n = 35$), thus obtaining a “neighborhood-based confidence vector” $c_n$. For the real distance readings displayed in Figure 7.6, in the case of a faultless ultrasonic sensing system, the neighborhood-based confidence vector $c_n$ is the following:

$$c_n = \{ \begin{array}{ccccccccccc}
0.709 & 0.511 & 0.261 & 0.417 & 0.557 & 0.783 & 0.752 & 0.671 & \ldots \\
0.644 & 0.642 & 0.808 & 0.876 & 0.777 & 0.563 & 0.226 & 0.199 & \ldots \\
0.145 & 0.483 & 0.502 & 0.573 & 0.530 & 0.766 & 0.936 & 0.880 & \end{array} \}$$

The elements of the neighborhood-based confidence vector $c_n$ can adopt confidence values within the interval $[0, 1]$. If an element value is high, the confidence in this sensor will also be high. If an element value is close to zero, the sensor will be suspected to be faulty.

The time complexity of the second step of the fault detection method is $O(m_n n_p^2)$. 
7.5 Overall Sensor Assessment

In Section 7.3, the real distance readings obtained from the ultrasonic sensing system have been compared with the results obtained from the simulation model, and in Section 7.4, the direct and cross echoes obtained from the ultrasonic sensing system have been compared mutually. The simulation-based confidence vector \( \mathbf{c}_s \) and the neighborhood-based confidence vector \( \mathbf{c}_n \) are now combined by a convex combination in order to obtain an “overall confidence vector” \( \mathbf{c}_o \):

\[
\mathbf{c}_o = \lambda_1 \mathbf{c}_s + \lambda_2 \mathbf{c}_n
\]

(7.3)

with

\[
\lambda_1 = \frac{m_s}{m_s + m_n} = \frac{7}{7 + 35}
\]

(7.4)

\[
\lambda_2 = \frac{m_n}{m_s + m_n} = \frac{35}{7 + 35}
\]

(7.5)

where \( \lambda_i \geq 0 \) and \( \sum \lambda_i = 1 \).

For the described distance readings, in the case of a faultless ultrasonic sensing system, the overall confidence vector \( \mathbf{c}_o \) is the following:

\[
\mathbf{c}_o = \{ 0.757 \ 0.544 \ 0.265 \ 0.371 \ 0.581 \ 0.745 \ 0.719 \ 0.659 \ldots \\
0.564 \ 0.565 \ 0.771 \ 0.822 \ 0.764 \ 0.610 \ 0.236 \ 0.261 \ldots \\
0.146 \ 0.451 \ 0.466 \ 0.502 \ 0.505 \ 0.709 \ 0.852 \ 0.900 \ldots \}
\]

The overall confidence vector \( \mathbf{c}_o \) allows to perform reliable overall sensor assessment. As the simulation-based confidence vector \( \mathbf{c}_s \) and the neighborhood-based confidence vector \( \mathbf{c}_n \), the overall confidence vector \( \mathbf{c}_o \) is normalized to the interval \([0, 1]\). If an element within this vector is smaller than a predefined fault threshold \( t_f \), the respective sensor will be declared faulty and it will be removed from the perceptual system. The fault threshold \( t_f \) is determined by the deliberation that perfect agreement of a real distance reading from a sensor under consideration with only one reference value either from the simulation model or from an independent
overlapping echo path shall be sufficient to assume the sensor to be functioning. For this reason, the fault threshold $t_f$ is defined as follows:

$$ t_f = \frac{1}{m_s + m_n} = \frac{1}{7 + 35} = 0.024 $$  \hspace{1cm} (7.6)

In the above example, none of the sensors was faulty and none of the overall confidence vector elements is smaller than the fault threshold $t_f$.

Supposing that the first and second step of the fault detection method are derived separately, the time complexity of the overall sensor assessment is $O(n)$.

In order to simulate a fault and to demonstrate the behavior of the fault detection method, the distance readings with ultrasonic sensor $S_{18}$ being involved are falsified and the following results are obtained:

$$ c_s = \{ \begin{array}{cccccccccccc}
1.000 & 0.714 & 0.286 & 0.143 & 0.701 & 0.552 & 0.557 & 0.598 & \ldots \\
0.163 & 0.181 & 0.585 & 0.556 & 0.702 & 0.848 & 0.143 & 0.429 & \ldots \\
0.143 & 0.000 & 0.286 & 0.143 & 0.376 & 0.429 & 0.429 & 1.000 & \\
\end{array} \}$$

$$ c_n = \{ \begin{array}{cccccccccccc}
0.709 & 0.511 & 0.261 & 0.417 & 0.557 & 0.783 & 0.752 & 0.671 & \ldots \\
0.644 & 0.642 & 0.808 & 0.876 & 0.777 & 0.563 & 0.194 & 0.164 & \ldots \\
0.179 & 0.000 & 0.295 & 0.375 & 0.437 & 0.766 & 0.936 & 0.880 & \\
\end{array} \}$$

$$ c_o = \{ \begin{array}{cccccccccccc}
0.757 & 0.544 & 0.265 & 0.371 & 0.581 & 0.745 & 0.719 & 0.659 & \ldots \\
0.564 & 0.565 & 0.771 & 0.822 & 0.764 & 0.610 & 0.185 & 0.208 & \ldots \\
0.173 & 0.000 & 0.293 & 0.336 & 0.427 & 0.709 & 0.852 & 0.900 & \\
\end{array} \}$$

It can be noted that $c_o(18)$ becomes smaller than the fault threshold, which indicates a fault on ultrasonic sensor $S_{18}$.

To further increase confidence in the assessment results, the overall confidence vector can be averaged over time, and a sensor can be declared faulty when the averaged overall confidence value of the respective sensor becomes smaller than the predefined fault threshold $t_f$.

If all ultrasonic sensors on the front or on the back of the robot appear faulty in the first step and faultless in the second step of the two-step fault detection method, a fault on one of the laser range finders may be inferred. However, this contradiction can also arise when the ultrasonic sensors and the laser range finders observe different objects due to their differing measurement characteristics. Thus, the condition of an suspicious laser range finder should be verified with the aid of the other laser range finder.
If the robot is not equipped with laser range finders, the first step of the fault detection method can be completely omitted, and ultrasonic sensor assessment can be based exclusively on the second step.

Additionally to the explained fault detection model, fault diagnosis has been realized in context with the newly developed ultrasonic sensing system. On hardware level, it can be checked if all sensors are connected to the ultrasonic sensing system, if the supply voltage of the sensors is available, and if all sensors can be properly fired. On software level, the data transmission between the DSP-Board and the robot PC via serial interface is monitored by observing if the received data packages are complete, correct, and accurately timed. Thus, it can be recognized if the data transmission is disturbed or interrupted.

In case of a critical fault, the robot is stopped immediately, and the operator or the service technician is informed by a fault protocol. In case of a less critical fault, a fault protocol is also released, but automatic fault recovery is performed and the robot continues its mission. Automatic fault recovery involves removing faulty sensors from the perceptual system and navigating with a reduced sensor system. The fault of a single ultrasonic sensor can be compensated by the overlapping echo paths of neighbor sensors or by alternative robot control strategies.

### 7.6 Contributions

In this chapter, fault diagnosis for the perceptual system of mobile robots has been covered. Although fault detection for environmental sensors on mobile robots is extremely important in order to ensure safe navigation, this special subject has rarely been considered in research. In one pertinent probabilistic approach [94, 95], fault detection for ultrasonic sensors is based on a grid map of the environment that is constructed using a probabilistic sensor model. With the use of an occupancy grid representation it is possible to compare sensor readings made by different sensors (physical redundancy), at different points of time (temporal redundancy), and from various positions and orientations in space (spatial redundancy). In contrast, the approach presented in this thesis predominantly relies on physical and analytical redundancy, i.e. it is not necessary to make observations by rotating the robot on the spot or moving it within the environment. In order to incorporate the physical properties of the employed ultrasonic sensors, their geometrical configuration, as well as the alignment of direct and cross echo paths, a simulation model of the ultrasonic sensing system has been derived. A developed two-step fault detection method compares in
the first step real with simulated distance readings, and in the second step real distance readings among one another. Based on these two steps, reliable overall sensor assessment can be performed. One ultrasonic scan (measurement cycle) around the robot is sufficient to carry out a complete sensor check for various kinds of faults.
8 Conclusion

Within this dissertation, new approaches in the research fields of ultrasonic sensing, environment mapping and self-localization, as well as fault detection, diagnosis, and recovery for autonomous mobile systems have been presented. A concept of high-resolution ultrasonic sensing by a multi-aural sensor configuration has been proposed, which incorporates cross echoes between neighbor sensors as well as multiple echoes per sensor. In order to benefit from the increased sensor information, algorithms for adequate sensor data processing and sensor data fusion have been developed. In this context, it has been described how local environment models can be created at different robot locations by extracting geometric primitives from laser range finder data and from ultrasonic sensor data. Additionally, the application of an extended Kalman filter to mobile robot self-localization based on previously modeled and newly extracted geometric primitives has been explained. Furthermore, it has been demonstrated how local environment models can be merged for building a global environment map. As a supplement for monitoring the state of environmental sensors, a fault detection model has been developed, which consists of sub-models for data from laser range finders and ultrasonic sensors.

In summary, the results of this research work are as follows:

- Chapter 3: A novel ultrasonic sensing system for autonomous mobile systems
- Chapter 4: Local environment modeling by extracting straight line segments
  - Parameter space clustering (PSC)
  - Tangential regression (TR)
- Chapter 5: Mobile robot self-localization based on straight line segments
- Chapter 6: Global environment mapping by merging local environment models
- Chapter 7: Fault diagnosis for the perceptual system of mobile robots
The contributions of Chapter 3 to Chapter 7 are recapitulated in Section 8.1 to Section 8.5.

### 8.1 Contributions of Chapter 3

In Chapter 3, a novel ultrasonic sensing system for autonomous mobile systems has been presented. The benefits of the newly developed ultrasonic sensing system and the concept of high-resolution ultrasonic sensing are manifold. The application of sensors with a broad beam width supports complete environmental coverage. Additionally, the proposed multi-aural sensor configuration with widely overlapping detection cones yields a high angular resolution due to exploitation of cross echoes between neighbor sensors. Moreover, the acquisition of multiple echoes per sensor further improves object perception. As a result, by a given number of sensors a significantly higher number of echoes can be utilized in comparison with conventional ultrasonic sensing systems for mobile robots. Consequently, it becomes unnecessary to rotate the whole robot or a sensor array in small angular steps around the vertical axis for scanning the environment. Thus, mechanical wear is prevented, battery energy is saved, the measurement rate is significantly increased, and the navigation safety is improved. Up to 672 echoes can be received with 24 sensors per measurement cycle. The sampling rate of currently 4 measurement cycles per second can still be increased. Ultrasonic sensors designed for automotive applications are employed which are robust, reliable, and cheap. Consequently, they are suitable as well for usage on service robots. The proposed ultrasonic sensing system has gained industrial interest to be used on cleaning robots and automated guided vehicles (AGVs) for navigation control, and on electric wheelchairs for collision avoidance (e.g. with furniture or door-posts) during manual operation.

### 8.2 Contributions of Chapter 4

In Chapter 4, local environment modeling by extracting straight line segments has been described. The main contributions of the chapter are two new algorithms, called parameter space clustering and tangential regression, which have been developed for deriving local environment models from ultrasonic sensor data. With the aid of these algorithms, a number of straight line segments can be extracted by a single ultrasonic scan around the robot from a fixed location and without turning the robot around its
own axis for data acquisition. Another important advantage of both algorithms is that the reflection properties of ultrasound are properly exploited by considering the requirement of a perpendicular reflection line from a reflecting surface to a sensor.

Further potential applications of parameter space clustering and tangential regression are in diagnostic medical sonography, seismic investigations, radar imaging, ultrasonic underwater sensing, and wavefront approximation.

### 8.2.1 Contribution of Subsection 4.3.1

The contribution presented in Subsection 4.3.1 is the algorithm of parameter space clustering (PSC). The Hough transform has been extensively used in computer vision for boundary detection in images [101] and has been previously applied in robotics for straight line extraction from sensor data [32, 49, 65]. The distinction of parameter space clustering from related work comprises the discretization of circular and elliptic echo arcs by potential reflecting lines of infinitesimal length, the Hough transform of all potential reflecting lines into an occupancy grid in parameter space, the two-dimensional low-pass filtering of the occupancy grid, the connected components analysis to determine regions of cohesive highly occupied grid cells for cluster formation without requiring the number of expected clusters to be predefined, and the calculation of cluster centers that represent extracted straight line elements in parameter space.

### 8.2.2 Contribution of Subsection 4.3.2

The contribution presented in Subsection 4.3.2 is the algorithm of tangential regression (TR). Standard regression as well as orthogonal regression have been widely employed in robotics for straight line extraction from sensor data [32, 38, 49, 50, 51, 53, 59, 68, 110]. In a closely related approach [60], a common tangent is calculated for two circular arcs by a closed-form solution. Consequently, an over-determined situation of fitting a common tangent to more than two circular arcs is not considered. The idea of tangential regression is to calculate the parameters of a tangential regression line so that it represents the best fit to multiple circular or elliptic echo arcs of distance measurements from different sensor positions. The difference to orthogonal regression is that the distance measurements indicate potential regression points on curves instead of fixed regression points.
8.3 Contributions of Chapter 5

In Chapter 5, mobile robot self-localization based on straight line segments has been explained. Considering the application of an extended Kalman filter to self-localization by multi-target tracking as described by Leonard et al. [56, 57, 58, 112], because of the high angular uncertainty associated with sonar measurements forming regions of constant depth, only distance information (i.e. no orientation information) from the ultrasonic sensor data is used for computing the updated robot location estimate. In contrast, by extracting straight line segments from ultrasonic sensor data, in addition to the distance constraint an angle constraint is obtained. In order to be able to exploit both constraints for calculating the updated robot location estimate, the measurement model has been composed of functions considering distance information and functions considering angle information. Thus, the extended Kalman filter framework has been adapted for self-localization utilizing extracted straight line segments instead of distance measurements. The proposed method is also applicable to self-localization based on straight line segments derived from laser range finder data.

8.4 Contributions of Chapter 6

In Chapter 6, global environment mapping by merging local environment models has been demonstrated. Fusion methods have been developed for spatially integrating straight line segments extracted at different robot locations from laser range finder data and ultrasonic sensor data in order to create detailed maps of complex environments. The underlying idea is to aggregate locally extracted straight line segments in a global environment map, and to preferably replace them by merged straight line segments. As a basis for straight line fusion, an approach of forming regions via Delaunay triangulation has been proposed for subdividing aggregated straight line segments into neighborhood regions in workspace. Straight line fusion has been realized by a clustering approach and by a regression approach. The performance of the developed algorithms has been proved in a typical office environment.

8.5 Contributions of Chapter 7

In Chapter 7, fault diagnosis for the perceptual system of mobile robots has been covered. Although fault detection for environmental sensors on mo-
bile robots is extremely important in order to ensure safe navigation, this special subject has rarely been considered in research. In one pertinent probabilistic approach [94, 95], fault detection for ultrasonic sensors is based on a grid map of the environment that is constructed using a probabilistic sensor model. With the use of an occupancy grid representation it is possible to compare sensor readings made by different sensors (physical redundancy), at different points of time (temporal redundancy), and from various positions and orientations in space (spatial redundancy). In contrast, the approach presented in this thesis predominantly relies on physical and analytical redundancy, i.e. it is not necessary to make observations by rotating the robot on the spot or moving it within the environment. In order to incorporate the physical properties of the employed ultrasonic sensors, their geometrical configuration, as well as the alignment of direct and cross echo paths, a simulation model of the ultrasonic sensing system has been derived. A developed two-step fault detection method compares in the first step real with simulated distance readings, and in the second step real distance readings among one another. Based on these two steps, reliable overall sensor assessment can be performed. One ultrasonic scan (measurement cycle) around the robot is sufficient to carry out a complete sensor check for various kinds of faults.
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Books


Books


Miscellaneous


Publications in the Context of this Research


Zusammenfassung

Es werden neue Methoden zur Ultraschallmessdatenerfassung, zur Umgebungskartierung und Selbstlokalisierung sowie zur Fehlererkennung, -diagnose und -behebung für autonome mobile Systeme vorgestellt.


