Size Illusion on an Asymmetrically Divided Circle

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1. Introduction

In the Poggendorff (1870) illusion a line, inclined by about $45^\circ$ to the horizontal, is occluded by a vertical bar. The thought extensions of the two protruding ends seem to miss each other. The systematic extrapolation error is such as if, unconsciously, one had turned the visible segments of the line slightly towards the horizontal. Taking into account a possible interaction between the edges of the bar and the line, Hotopf (1966) puts it as if “… the illusion manifested a tendency for the angles of intersection of the distorting with the distorted parts to appear nearer to right angles than they are.” The intensity of the illusion is considerably reduced for the crossing line being oriented either horizontally or vertically.

However, what is important in the context of the Poggendorff illusion is not so much the perceived orientation of the protruding line segments but the orientation of their perceived extensions. This was investigated by several authors. Bouma and Andriessen (1968) report that the perceived extension of a horizontal or vertical line segment agrees with its geometric extension. A similar observation is made on a line segment inclined by $\alpha=45^\circ$ with respect to the horizontal, except the error is considerably larger. However, for slightly larger or smaller angles $\alpha$, the perceived direction appears to be tilted towards the nearest horizontal or vertical. Deviations up to $10^\circ$ have been observed. Its dependence on the slant $\alpha$ can be approximated by a function $\sim \sin(4 \alpha)$.

In the presence of an abutting vertical line the perceived extension of a slanted line ($45^\circ$) deviates substantially from its true extension. This was shown by Wenderoth, Beh, and White (1978). Deviations up to $7^\circ$ have been found. Spivey-Knowlton and Bridgeman (1993) showed that context elements exert an influence on the illusion, too. Greene and Fisher (1994) investigated the angular induction process by decomposing the induction segment into an array of scattered points.

In the divided circle illusion (Ehrenstein, 1954; Lingelbach, 1984), a bar partly occludes a circular line such that segments of unequal size are protruding on either side. Similar to the Poggendorff illusion, the thought extensions of the circular arcs of the two segments appear to miss each other. As a consequence, the protruding segments give the impression as if they were not part of the identical circle, i.e., the smaller segment appears too small, and vice versa.

The goal of the first experiment was to measure the perceived size of both segments, with the bar upright and, then, tilted by $45^\circ$. The size of the larger segment was varied, serving as the independent parameter $x$. In a second experiment, where both segments were parts of the identical circle, the apparent size of both segments was measured as a function of the orientation of the occluding bar, its tilt angle being varied in nine steps, from vertical to horizontal orientation.
2. Experiments

2.1 Experiment 1

2.1.1 Subjects. Eight healthy volunteers took part, mainly elderly people. Sight was corrected to normal. All of them were naive.

2.1.2 Stimuli. The stimuli consisted of black circular lines, printed on white glossy paper (DIN A3, 297x420 mm), partly occluded by an empty rectangle, as shown in Fig. 1. The small segment was always of the same size, the diameter of the corresponding complete circle being d=68mm, while the diameter D of the large segment’s circular line was varied between D=63mm and 75mm, in steps of 3 mm. Both circles were concentric. The occluding bar was 23mm wide, its height being 115mm, and its maximum distance from the arc of the small segment was 12mm. Line width of the circles and the bar was 1.8mm. The bar was oriented either vertically or inclined by 45°. Five empty black circles below the target served as a reference, decreasing in size from left to right (outer diameter 79, 74.5, 70, 65.5, 61mm). In total 20 transparencies were shown.

Fig. 1. Examples of stimuli, one with the occluding bar upright and one with the bar tilted at 45°. In separate runs, the subjects chose one of the five circles shown underneath to indicate the perceived size of the right or the left segment, respectively.

2.1.3 Procedure. The stimuli were printed on DIN A3 glossy paper and fixed to a flip chart board. The 10 transparencies were presented in random order. They were shown for eight seconds, followed by a blank of five seconds. In the first run, the size of the small segment had to be estimated, in the second run (different order of the transparencies) the larger one. Mean observation distance was about 3m.

2.1.4 Results. Figs. 2a, b give the perceived size of the segments in relation to their true size, as a function of the true size of the large segment, with the bar oriented vertically. On the average, the perceived size of the smaller segment is 91% of its true size (Fig. 2a), while the
larger segment, on the average, is estimated nearly correctly (Fig. 2b). In addition, with increasing true size of the larger segment, a steady decrease of the apparent size of both segments is observed. This is ascribed to a size constancy effect.

This illusion is less powerful when the bar is rotated counterclockwise by 45° (Fig. 3a). On the average, the small segment is perceived at 95% of its true size, while the size of the large segment is perceived almost correctly again (Fig. 3b).
Fig. 3a. With the bar tilted, the size of the smaller segment is underestimated, but not to the same degree as in case of a vertical bar. In addition, its relative apparent size decreases with increasing true size of the large segment.
Average value: 95.4(24) %.

Fig. 3b. With the bar tilted, the size of the larger segment, on the average, is estimated correctly. Again, there is a slight decrease of the relative apparent size with increasing true size.
Average value: 100.1(1.5) %

Fig. 4 gives the ratio of the illusions, i.e., the apparent size of the small segment divided by the perceived size of the large segment for vertical orientation of the occluding bar. The apparent reduction in size is always more pronounced on the smaller segment than on the larger one. Fig. 5 shows the situation for the bar rotated counterclockwise by 45°. In both diagrams the results are plotted as a function of the true diameter of the larger segment.

Size constancy effect. So far, the intensity of the illusion has been plotted versus the true diameter of the larger segment. The perceived size of both of the segments decreases with
increasing diameter of the larger one. This is ascribed to a size constancy effect. It means that objects of different size are not perceived in strict proportion to their true size, ie, to the size of their retinal images (Lühr, 1898; Cornish, 1937; Schur, 1925; Gilinsky, 1955; Kaufman and Rock, 1962). Concentrating on one particular stimulus, the effect of size constancy occurs when the retinal image varies in size, either because the distance of observation increases or, at constant distance, the size of the object decreases. To give an example, in case the diameter of the retinal image of an object shrinks by a factor of 4, the perceived size of the object will not shrink to one quarter of the original value, but will appear somewhat larger. The degree of size constancy can be quantified by the size constancy parameter, n (Kreiner, 2004). The parameter n can take any value between zero and one. Zero means no size constancy, ie, the apparent size changes in proportion to the size of the retinal image. n=1 refers to the case where the object is always perceived at the same size, independent of the variation of the retinal image. 0 < n <1 means that the apparent size will be larger than one would expect it from the laws of geometrical optics. In general, the effect is triggered by an increase in perceived structural density.
In this particular case, the large segment will be increasingly overestimated as soon it gets smaller. This effect transfers to the perceived size of the small segment, so it will appear larger, too. On the other hand, the two segments have also in common that their perceived size reduces steadily with increasing true size of the larger segment.

In order to determine the size constancy parameter $n$, the function $y(x) = F \cdot x^{-n}$ has been employed for the fitting procedure. $y$ means the apparent size of a segment relative to its true size, as a function of the true size $x$ of the larger segment (Figs. 2 and 3). $F$ is a scaling parameter. Data are collected in Table 1. It may be worth mentioning that the size constancy parameters found in this experiment (between $n=0.28$ and $0.37$) are comparable to values obtained from experiments performed in order to investigate the moon illusion: From data reported by Schur (1925) one can derive values of $n=0.32$ and $n=0.58$, for vertical and horizontal direction of observation, respectively. From experiments performed by Gilinsky (1955) $n$-values between 0.36 and 0.42 are obtained.

Table 1. Results of Experiment 1. Function fitted: $y(x) = F \cdot x^{-n}$

<table>
<thead>
<tr>
<th>Left segment</th>
<th>Average perceived size / %</th>
<th>Size constancy parameter $n$</th>
<th>Scaling parameter $F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bar vertical</td>
<td>100.3(22)</td>
<td>0.284(49)</td>
<td>334(69)</td>
</tr>
<tr>
<td>Bar tilted by 45°</td>
<td>100.1(15)</td>
<td>0.285(94)</td>
<td>297(106)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Right segment</th>
<th>Average perceived size / %</th>
<th>Size constancy parameter $n$</th>
<th>Scaling parameter $F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bar vertical</td>
<td>90.6(24)</td>
<td>0.322(73)</td>
<td>354(109)</td>
</tr>
<tr>
<td>Bar tilted by 45°</td>
<td>95.4(24)</td>
<td>0.37(11)</td>
<td>464(215)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ratio of apparent size (right vs. left) / %</th>
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<tr>
<td>Bar vertical</td>
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<tr>
<td>Bar tilted by 45°</td>
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2.2 Experiment 2

2.2.1 Subjects. In total 11 healthy volunteers took part (among them the author), mainly elderly people. Nine were present in the first run, when the size of the small segment was estimated. Sight was corrected to normal. 10 participants were naive.

2.2.2 Stimuli. 12 Transparencies were employed. Both segments were part of the identical circle. When printed on a DIN A4 paper, the diameter of the circle was 48mm. The orientations of the occluding bar were $0^\circ$ (vertical), $8^\circ$, $23^\circ$, $37^\circ$, $45^\circ$, $53^\circ$, $67^\circ$, $82^\circ$, and $90^\circ$ (horizontal). In addition, three more transparencies were shown ($0^\circ$, $45^\circ$, and $90^\circ$), where the circle’s diameter was 45.7 mm (smaller by 5%). Dots were placed symmetrically right on the edges of the bar in order to enhance the effect. Examples are given in Figs. 6a and b. For comparison, five reference circles were presented (52, 49, 46, 43, and 40 mm in diameter). On the transparencies with the smaller circle the same standards were employed.

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Fig. 6a. Occluding bar inclined by $82^\circ$ with respect to the vertical. Dots are positioned symmetrically on the edge of the bar. In the classic Poggendorff illusion dots enhance the perceived offset:

![Fig. 6a](image1.png)

Fig. 6b. Stimulus with dots, bar inclined by $45^\circ$. For comparison, in the Poggendorff illusion the effect is reduced in case the crossing line is horizontal.

![Fig. 6b](image2.png)
2.2.3. Procedure. The transparencies were presented with a beamer in random order. First, the stimulus was shown for four seconds, then, the five reference circles were added for another six seconds. Viewing distance was 3m and 4.5m, respectively. Depending on the viewing distance, the target circle (48 mm) subtended an angle of $66 \cdot 10^{-3}\text{rad}$ and $44 \cdot 10^{-3}\text{rad}$, respectively. For the 45.7 mm circle the angles are smaller by 5%. In a first run, the small segment was estimated while the perceived size of the large segment was determined in a separate run, presenting the transparencies in different order.

2.2.4 Results

are given in Fig. 7. (The results obtained from the smaller circle have been adjusted proportionally.) The large segment was estimated nearly correctly for all orientations of the occluder except for the extreme positions, where a slight enhancement of the apparent size has been found. The small segment appears to be reduced compared to its true size, except for angles around $45^\circ$. Maximum illusion is observed for tilt angles of the bar slightly off the horizontal and vertical orientation ($82^\circ$ and $8^\circ$, respectively). A sine function has been fitted.

![Fig. 7: Left: Apparent size of the large segment at nine different angles of inclination of the occluder. At $0^\circ$ (vertical) and $90^\circ$ (horizontal) the segment appears to be slightly enlarged. Fourth order polynomial fit was performed. For the small segment, the apparent and the true size coincide only for a tilt angle around $45^\circ$. For the other angles between zero and ninety degrees the segment appears too small. A sine function was fitted. The dotted line gives the true diameter of the circle.](image)

3. Comparison with the Poggendorff illusion

The size illusion is explained in analogy to the Poggendorff phenomenon (Fig. 6a). There, the visual system extends each of the protruding ends of the crossing line beyond the occluding bar, just with a tendency to take a slightly shorter distance. For a vertical bar, one rotates these thought extensions unconsciously towards the horizontal, so they miss each other.
In the split circle illusion, the extensions of the circular arcs seem to miss each other, too. As a consequence, a size illusion is induced on both of the segments. The size of the large segment is recognized almost correctly, most probably because the major part of the circle can be seen. However, the apparent size of the small segment seems to be more susceptible to the influence of the context. There is experimental evidence that the intensity of the illusion depends considerably on the orientation of the bar. In order to judge the diameter of the corresponding circle one has to extrapolate the circular arc. For simplicity, in Fig. 8 these thought extensions are approximated by the tangents (dotted lines) to the arc at the merging points.

In Fig. 8, left, the bar is in vertical position (0 degrees). In analogy to the Poggendorff illusion, the tangents to the small segment will appear to be rotated towards the horizontal. As a result, one concludes the segment to be part of a smaller circle, so the circular lines don’t seem to match. However, in case the occluding bar is tilted by 45°, the tangents to the segments are oriented already nearly horizontally or vertically, respectively. Hardly any rotation will occur (Fig. 8, right), again in analogy to classic Poggendorff illusion. For tilt angles of 8° and 82°, there seems to be a strong tendency to rotate the thought continuation of the tangent off its true orientation. For simplicity, the dots positioned symmetrically at the edges of the bar (enhancing the illusion) have been omitted in the last figure.

4. Conclusion

The divided circle illusion has been investigated. The perceived size of both protruding segments have been determined experimentally. While, on the average, the size of the large segment is estimated almost correctly, the size of the small segment is mostly underestimated. In a second experiment, the occluding bar has been rotated in steps between vertical and horizontal orientation. The illusion on the small segment nearly
disappears when the bar is oriented at 45 degrees. In addition, the perceived size of both segments decreases with increasing diameter of the large segment. This is ascribed to a size constancy effect.

Citations


Ehrenstein, W (1954). *Probleme der ganzheitspsychologischen Wahrnehmungslehre. Fig. 100, p. 154*. Leipzig: Johann Ambrosius Barth /Verlag.


Lingelbach, B. Die Poggendorff-Täuschung.


