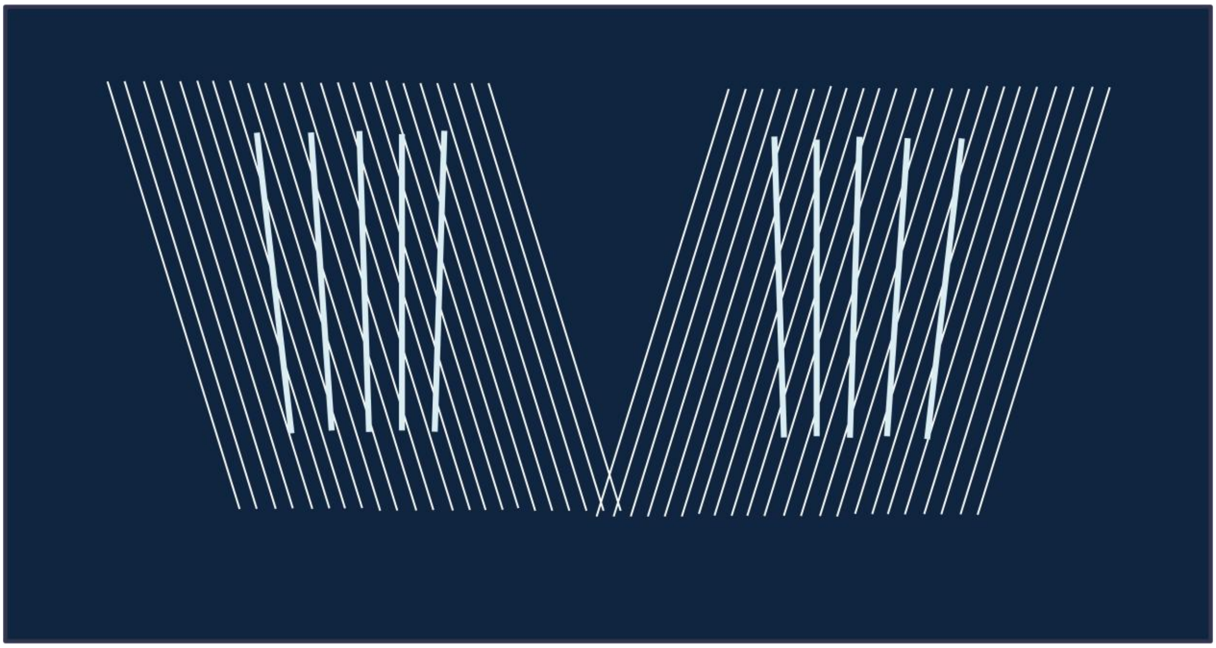
Algebraic functions describing the Zöllner illusion



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1. Introduction

There are several visual illusions where geometric figures are distorted when seen against a lined background. Due to Hotopf (1966), "... these illusions all manifest a tendency for the angles of intersection of the distorting with the distorted parts to appear nearer to right angles than they are." The Zöllner illusion (1860) is one of the best known examples. There, the apparent direction of a line deviates from its true direction if superimposed on a field of parallel lines at an acute intersect angle. Wallace and Crampin (1968) performed several experiments to investigate the dependence of the apparent rotation of a horizontal line on two parameters, i.e., the intersect angle and the spacing of the background lines (pattern density). In their experimental setup, two parallel horizontal lines appeared to diverge due to a pattern of lines in the background. The subjects had to adjust one of them to make them appear parallel. The authors found that the illusion follows a curve which shows a maximum at an intersect angle between 15^0 and 20^0. There was a clear increase of the intensity of the illusion with increased pattern density, but no shift of the maximum. From the existence of a maximum they concluded that the observed illusion may be due to two interacting and opposing effects. The main process is assumed to increase continually as the intersect angle decreases while the second effect offers kind of increasing resistance.

As a variant, this illusion can be also observed as the so called after effect (Gibson and Radner, 1936; Day, Pollack, and Seagrim, 1959; Fig. 1): A single line, tilted counterclockwise (ccl) from the vertical direction by a small angle, say 5^0, and watched for about one minute, has the effect that a true vertical test line presented afterwards appears to be tilted into the opposite direction (negative shift). Maximum illusion is observed when the inducing line is rotated by an angle between 15^0 and 20^0 from the vertical. It turns out that the same inducing line causes a comparable effect on a *horizontal test line* as well (indirect

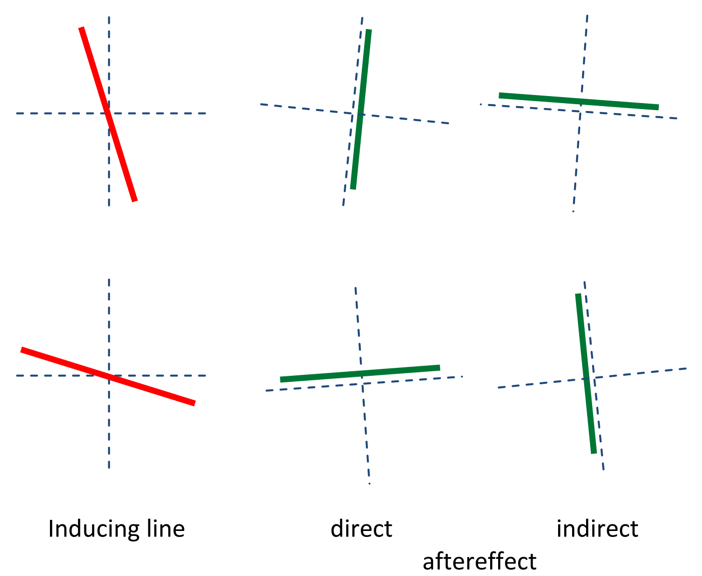


Fig. 1. Upper row: A line which is tilted to the left and watched for about one minute (red) has the effect that a true vertical line shown afterwards appears to be rotated to the right (green; negative aftereffect). A similar, just smaller effect is observed on a horizontal target line as well (indirect effect; top, right). It appears as if the spatial framework of the visual system had suffered from a small clockwise rotation. A tilted horizontal inducing line has a similar effect (bottom row).

Hardly any after effect is observed when the inducing line is rotated by exactly 45^0.

aftereffect). It seems as if perceptually the vertical and horizontal directions belonged to a single spatial framework. At 45^0 both effects are almost zero. If the inducing line is inclined beyond 45^0, it appears to deviate from the horizontal rather than from the vertical. Gibson and Radner (1936) investigated the tilt effect up to an intersect angle of 90^0. Now the aftereffect on the horizontal and vertical occurs in the opposite direction (Fig. 1, bottom).

2. Experiments

The experiments are variants of the classic Zöllner illusion. In the first experiment five lines were superimposed simultaneously on a pattern of parallel lines which was rotated in steps from 4^0 to 45^0 with respect to one of the principal directions. Only one of the five target lines was oriented vertically/horizontally, the others were tilted to different degrees. The participants were asked to indicate the line which appeared as being oriented vertically or horizontally, respectively. In a second experiment the distortion of a square and a diamond shaped figure was observed under the influence of the same background field of parallel lines. The goal was to compare the intensity of the illusion for horizontal and vertical orientation of the target, respectively and to discuss algebraic functions which would reproduce the results obtained in different kinds of experiments.

Experiment 1.

Stimuli. Tilt illusion on a vertical or horizontal line: Examples of the stimuli are given in Fig. 2. There were two series, 8 transparencies each, for measurement of the apparent deviation from the vertical and the horizontal, respectively. They show a field of 24 parallel blue lines and, superimposed, five red target lines, one of them oriented vertically or horizontally, one tilted by 2^0 such that the intersect angle was increased and the remaining three tilted into the opposite direction by 2^0, 4^0, and 6^0. In both cases, the context field was rotated by 4^0, 7^0, 12^0, 17^0, 22^0, 27^0, 32^0, and 45^0, clockwise in order to investigate the effect on a horizontal line, and counterclockwise for the vertical target line. The five lines were numbered for identification.

Participants. 11 elderly people above 60, healthy volunteers, took part (among them the author). Ten of them were naive. Sight was corrected to normal.

Procedure. The transparencies were presented with a beamer in random order. Each of them was shown for seven seconds, followed by a blank of 2 seconds. Watching distance was 3m (5 subjects) and 4.5m (6 subjects). The length of the red target lines subtended an angle of 71 and $47 \cdot 10^{-3}$rad, respectively. The separation of the blue lines of the inducing field was 3.97 and $2.65 \cdot 10^{-3}$rad, depending on the viewing distance. The participants indicated the line they perceived to be oriented vertically or horizontally by putting down its number.

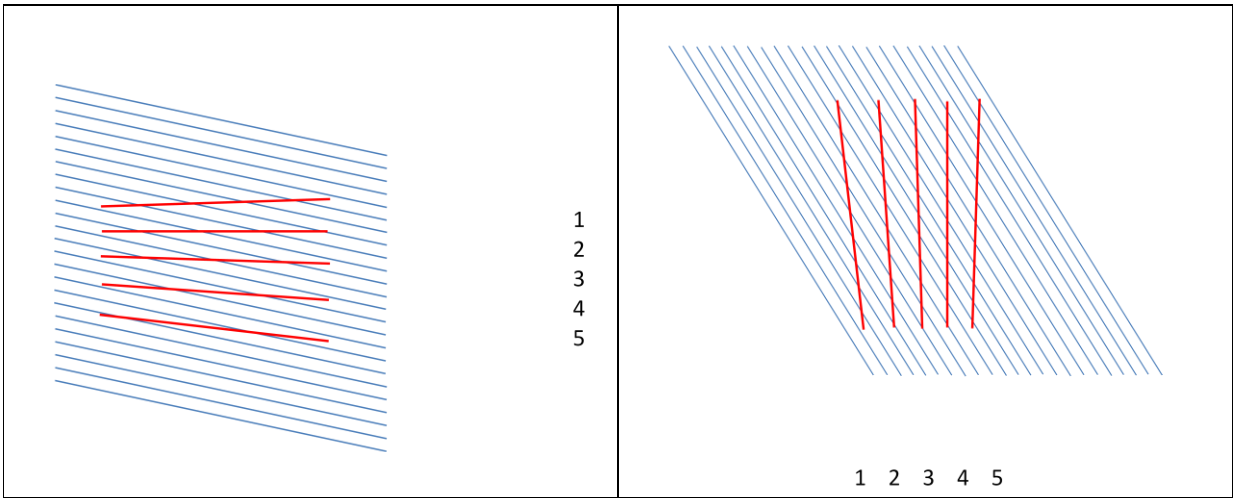


Fig. 2. Stimuli. Left: Five lines are presented in front of a field of 24 blue parallel lines. The deviation from the horizontal is 2^0 (ccw), 0^0, and 2^0, 4^0, and 6^0 (clockwise). Right: The deviation of the target lines from the vertical is mirror-like, three lines are tilted into ccw direction from the vertical, on clockwise. The inclination of the field relative to the horizontal/vertical was varied in steps, from 4^0 to 45^0. Each of the participants indicated the line which appeared to be oriented horizontally/vertically.

Results. A true horizontal or a vertical red line appears to be tilted depending on the intersect angle between the field and the horizontal or vertical, respectively. The sense of the illusion is such as if the visual system would perceive the inducing field being rotated by a small amount towards the nearest principal direction (horizontal or vertical, respectively), while the target line seems to take part into this rotation, as if both were attached to a common framework.

The algebraic functions (chapter 4), together with the fitted parameters, are given in Table 1. The maximum perceived deviation from the true horizontal is 3.3^0, while the vertical deviates by 2.6^0, utmost (Fig. 3). The maximum deviation of the horizontal target line from the true horizontal orientation occurs at an intersect angle of 0.33rad (19^0), the corresponding intersect angle for the vertical target is 0.28rad (16^0).

Table 1. Functions describing the apparent tilt of lines. Parameters fitted.

	y=A·x·exp(-B·x)		y=A1·exp(-B1·(log(x/C1))^2)		
	A	B	A1	B1	C1
horizontal	27.6(25)	3.12(23)	3.33(20)	0.52(12)	0.284(23)
vertical	25.3(22)	3.55(23)	2.54(13)	0.363(78)	0.235(18)

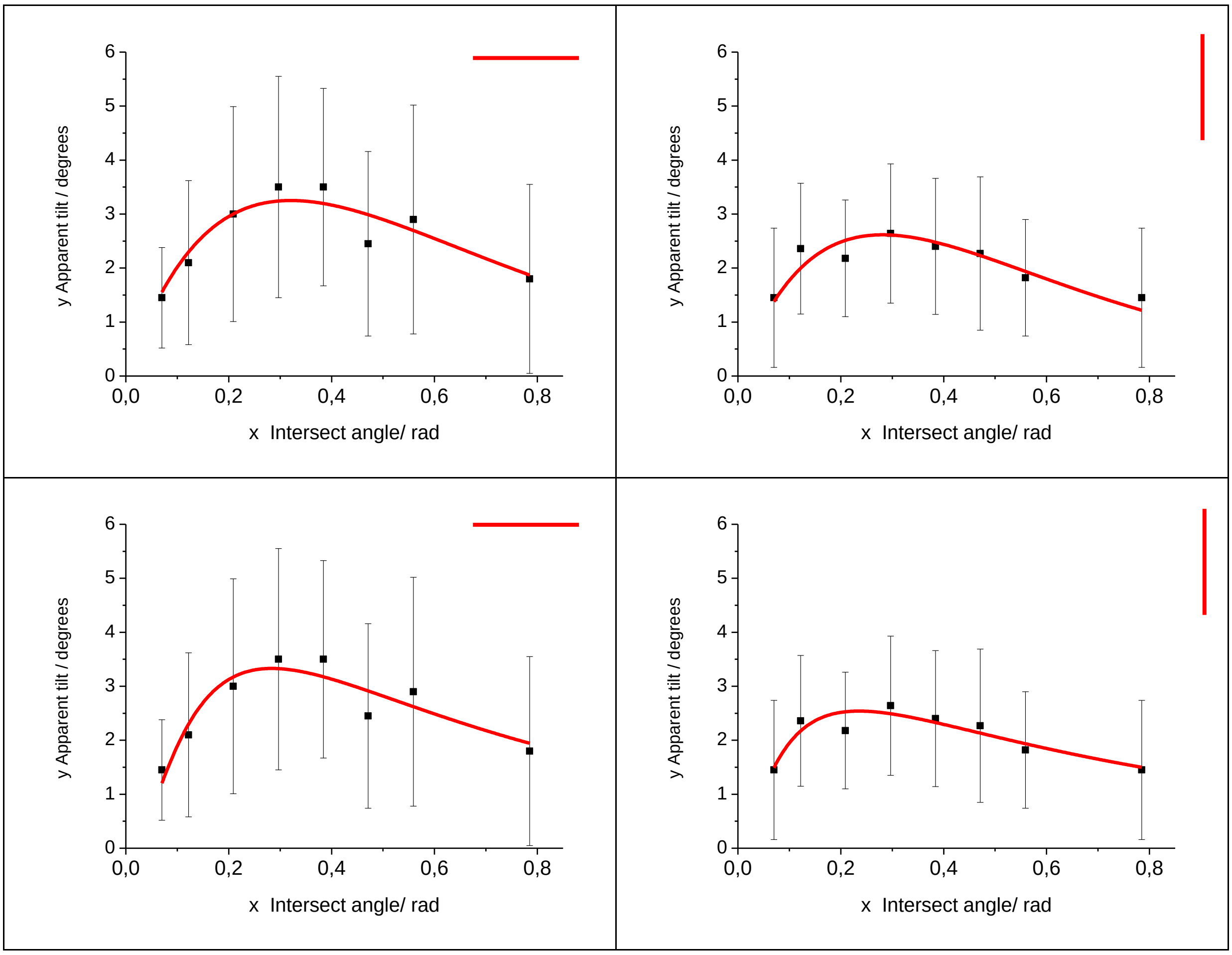


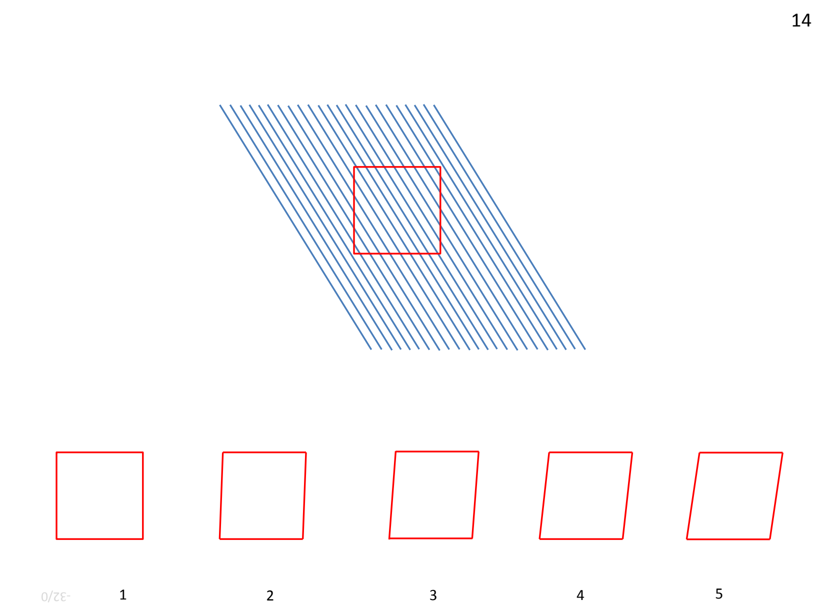
Fig.3. Left: Apparent inclination of a horizontal target line observed against a field of parallel lines. Right: Perceived tilt of a vertical line. In the upper row, the function y=A·x·exp(-B·x) was fitted, in the second row a lognormal function, y=A1·exp(-B1·(log(x/C1))^2), was employed. The illusion is such that the line appears to be rotated together with the background field.

Experiment 2.

Stimuli. Square and diamond. Figs. 4 and 5 give examples of the transparencies. There were two series of 8 transparencies each, one showing squares and the other one diamonds. The targets were to be seen against a field of parallel lines which was rotated by 4^0, 7^0, 12^0, 17^0, 22^0, 27^0, 32^0, and 45^0, counterclockwise with respect to the vertical. Below the stimulus, five standards were given. Starting from the left, there was a square and four diamond shaped figures, their smaller angle ranging from 88^0 to 82^0, in steps of 2^0.

Participants. 11 Elderly healthy volunteers above 60 took part (among them the author). Sight was corrected to normal.

Procedure. The transparencies were presented with a beamer, in random order. Watching distance was 3m (5 subjects) and 4.5m (6 subjects). The length of one side of the square/diamond subtended an angle of 36 and $24{\cdot}10^{-3}$rad, respectively. The separation of the blue lines of the inducing field was 3.97 and $2.65 \cdot 10^{-3}$ rad. The participants indicated the perceived shape of the target by putting down the number of one of the standards.



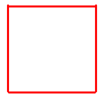
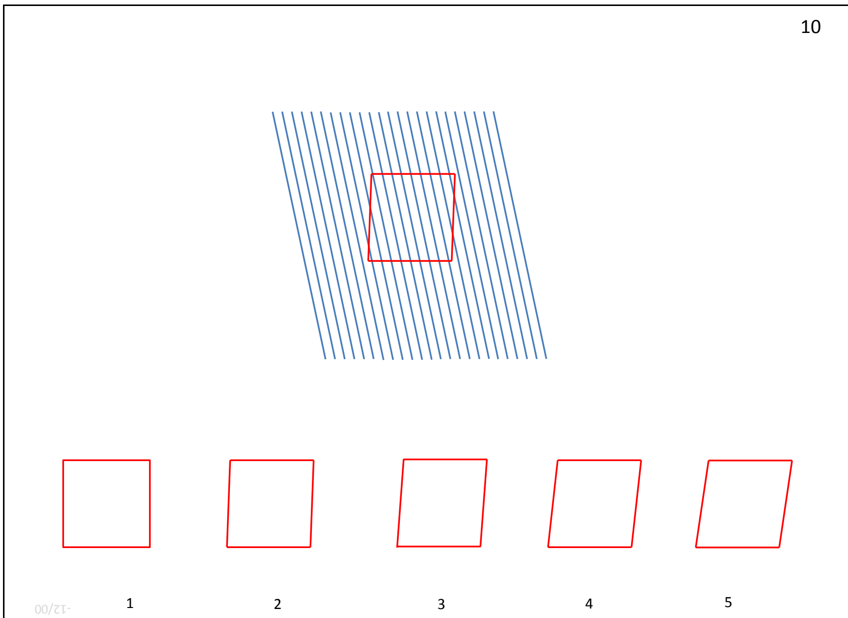


Fig. 4. A square appears slightly distorted under the influence of the field of lines in the sense that the vertical sides appear to be tilted clockwise.



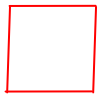


Fig. 5. A diamond appears slightly distorted in the sense that the smaller angle gets even smaller.

Table 2. Square and rhombus. Fitting functions and parameters

	y=A·x·exp(-B·x)		y=A1·exp(-B1·(log(x/C1))^2)		
	A	B	A1	B1	C1
Square	35.7(60)	3.77(46)	3.54(46)	0.53(23)	0.237(37)
Rhombus	25.11(72)	3.545(76)	2.63(11)	0.488(76)	0.246(14)

Results. Table 2 and Fig. 6. The square appears distorted as it was expected from the vertical line experiment. The maximum deviation is 3.4^0, occurring when the inducing field is inclined by 15^0. In case of the diamond, the small angle appeared to be even smaller. The additional apparent tilt is 2.6^0 at an intersect angle of 16^0.

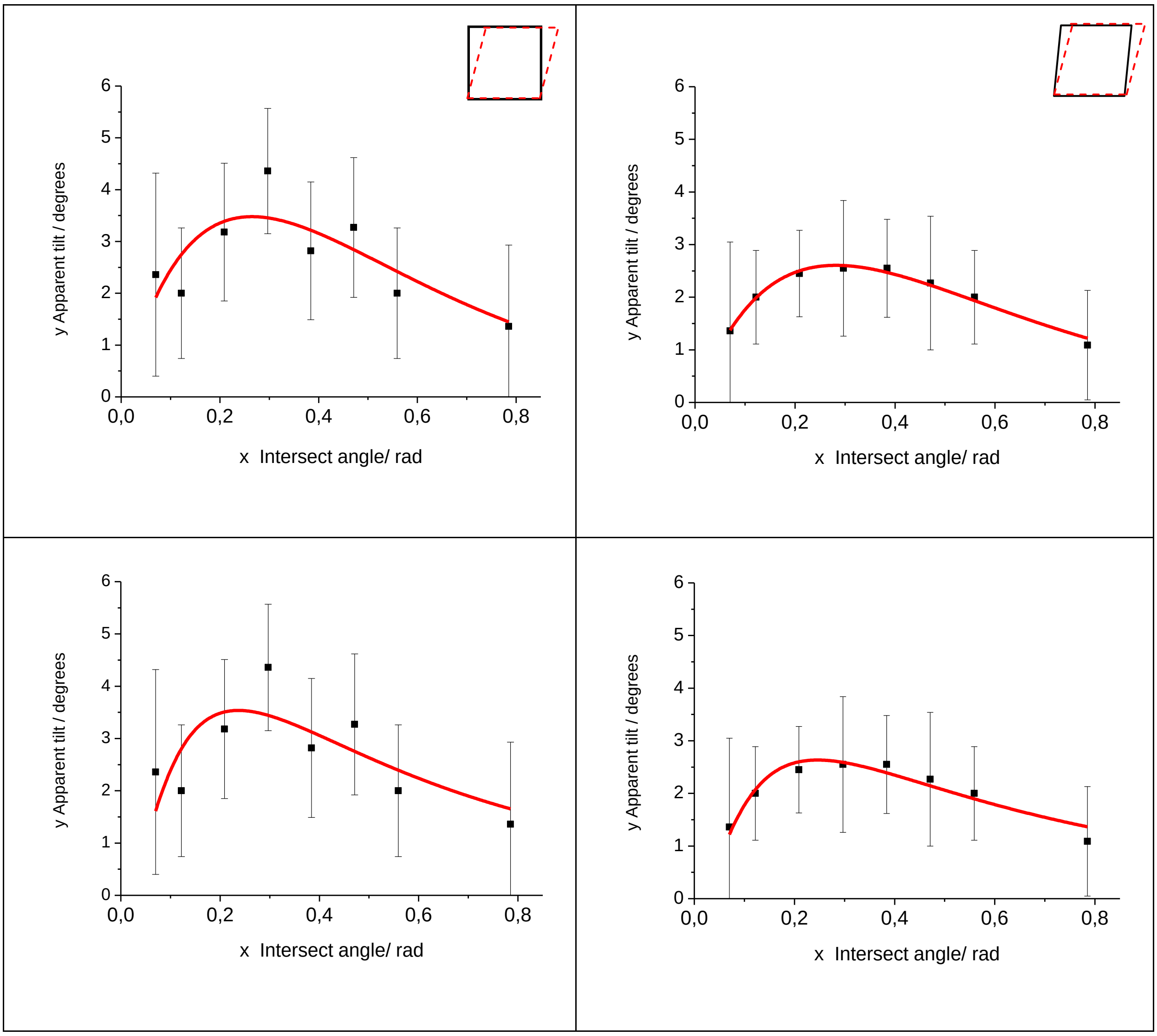


Fig. 6. Left: Apparent tilt of the vertical sides of a square relative to the true vertical as a function of the intersect angle. Right: Perceived additional inclination of the opposite sides of a rhombus as a function of the intersect angle. In the upper row, the function y=A·x·exp(-B·x) was fitted, in the second row a lognormal function, $y=A1\cdot\exp(-B1\cdot(\log(x/C1))^2)$, was employed. The maxima occur at an intersect angle of 0.28rad (15^0) for the square and at 0.27rad (16^0) for the rhombus.

Strictly speaking, the upper and the lower sides of the geometric figures are slightly affected, too. Only the perceived tilt of the vertical sides is plotted in the diagrams.

3. Experiments by other authors

Two examples of experimental data are chosen in order to check whether the functions discussed in chapter 4 can be fitted within the error limits. Although the experimental procedures were different, the illusions are assumed to rely on the same basic effect. The first experiment, by Gibson and Radner (1936), refers to an aftereffect on a vertical line, where the tilt angle of the inducing line had been varied between 0^0 and 90^0. Results obtained from two subjects are shown. For the fitting, the average of the two sets of values is taken. The function y=D+A·sin(4x) ·exp(-B·sin(2x)) reproduces the data within their scattering.

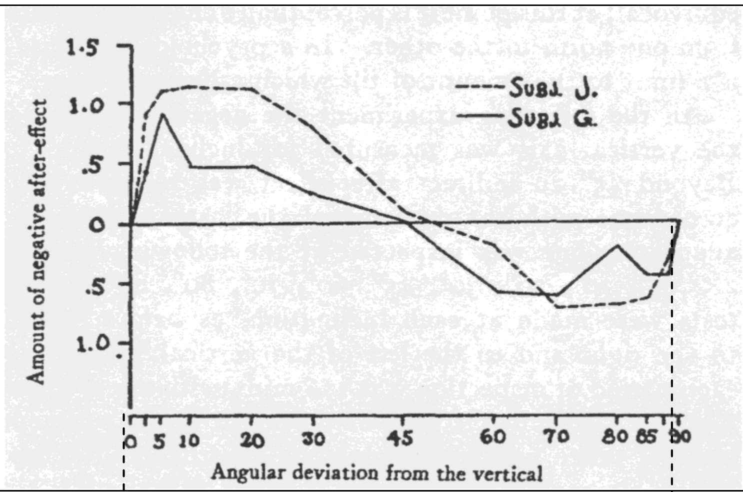


Fig. 7a

Gibson Radner (1936)
Two sets of data

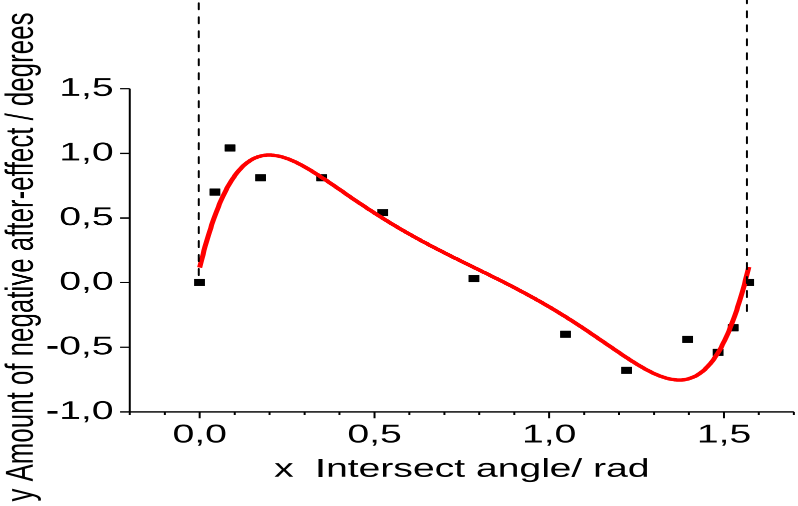


Fig. 7b

For fitting, the two sets of data were averaged.

y=D+A·sin(4x) · exp(-B·sin(2x))

D=0.117(48)
A=2.78(49)
B=2.13(39)

Another example refers to one of the experiments by Wallace and Crampin (1969) on the Zöllner illusion. The amount of the illusion was determined for different intersect angles with a pattern of constant line density, ranging from 2^0 to 45^0. Two target lines were seen against a field of diverging lines. The subjects adjusted one of the lines to make both appear parallel. Fig. 8a gives the original drawing of the curve, 8b the fit of the function $y=D+A\cdot x\cdot \exp(-B\cdot x^2)$. In Fig. 8c, a lognormal function has been fitted.

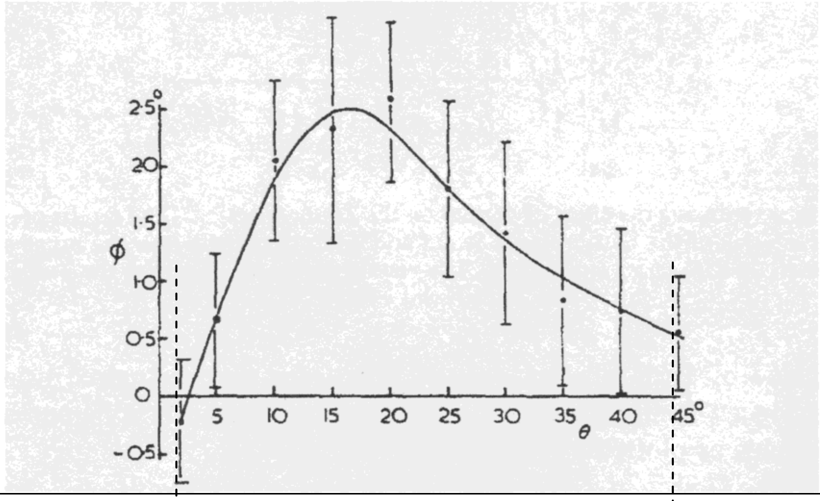


Fig. 8a
Measurement of the Zöllner illusion by Wallace and Crampin (1969).

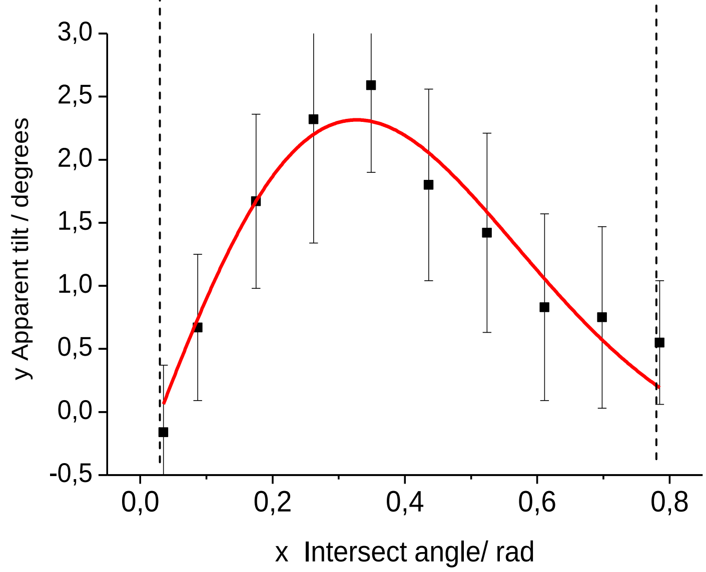


Fig. 8b

Data of Wallace and Crampin (1969).

Fit of the function

$y=D+A\cdot x\cdot \exp(-B\cdot x^2)$

D= -0.42(21)
A= 13.8(1.4)
B= 4.67(39)

4. Algebraic functions

The conceptual model assumes that, in case of a small intersect angle of a vertical target line with an inducing field of parallel lines, the visual system will adopt a new vertical principal orientation. Relative to this new vertical, a true vertical target line will appear to be tilted. It seems as if the field and the target line had undergone a common rotation (Fig. 1). To first order of approximation, the apparent rotation y(x) of a vertical line is assumed to be in proportion to the intersect angle, x:

$$y(x) = A\cdot x; \qquad \text{(Eq. 1)}$$

This holds for small angles x. For larger angles, however, the visual system may be less prepared to accept a new vertical which is deviating to such a large amount from the

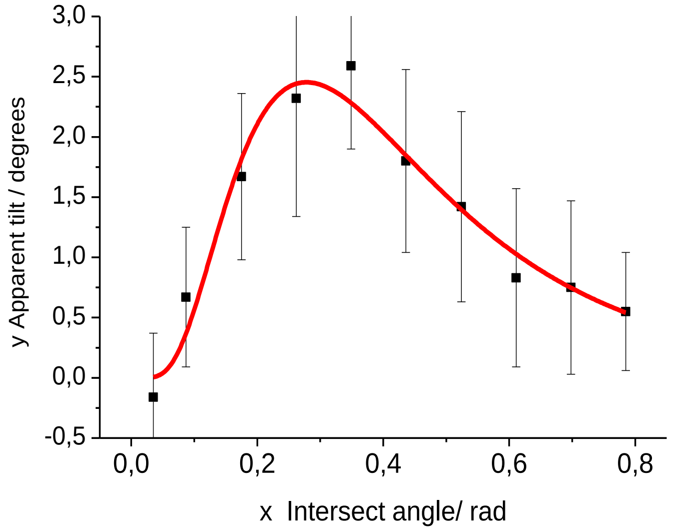


Fig. 8c

Zöllner illusion;
Data of Wallace and Crampin (1969).

Fit of the function

$y = A1 \cdot \exp(-B1 \cdot (\log(x/C1))^2)$

A1= 2.45(14)
B1= 1.40(21)
C1= 0.278(12)

true one, and joint rotation will lag behind the intersect angle more and more. This is taken into account by a nonlinear term, i.e., an exponential decay term, with the negative intersect angle in the exponent:

$$y(x) = A \cdot x \cdot \exp(-B \cdot x); \qquad \text{(Eq. 2)}$$

This function seems reasonable in order to describe the observed illusions within their error limits below an intersect angle of 45^0. At this angle, the orientation of the inducing field deviates equally from the vertical as well as from the horizontal. Due to the results obtained by Gibson et al. (1936) on the aftereffect, the positive and negative effects will cancel. The authors extended their experiment up to 90^0. Effects due to variable angles can be quantified by trigonometric functions. The goal is to find an expression describing the intensity of the illusion as a function of intersect angles beyond 45^0. The observations are:

1) At small angles of rotation, the tilt illusion seems to increase approximately with the angle of rotation. 2) There is no effect at 45^0. 3) Beyond 45^0, there is a tilt effect into the opposite direction, and there is no illusion right at 90^0again.

This can be taken into account by replacing x in equation (1) by a sine function being zero at 45^0 ad 90^0 degrees, which is sin(4x). The exponential function, describing the decrease of the effect when approaching 45^0, must exhibit a minimum there. Therefore, the function sin(2x) is chosen for the exponent:

$$y(x) = D + A \cdot \sin(4x) \cdot \exp(-B \cdot \sin(2x)) \qquad \text{(eq. 3)}$$

A curve, starting from $y \approx 0$ at x=0, and then, after a maximum, coming down again asymptotically towards the x-axis, can often be approached by a lognormal function, too:

$$y(x) = A1 \cdot \exp(-B1 \cdot (\log(x/C1))^2). \qquad \text{(eq. 4)}$$

5. Summary

The apparent tilt of vertical and horizontal lines has been measured as a function of the inclination of an inducing background field of parallel lines. The results are similar to the ones obtained from other Zöllner type experiments. Distortion illusions on a square and a diamond shaped target are reported. There are algebraic functions suitable to describe the results of Zöllner type illusion experiments within their error limits.

Citations

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