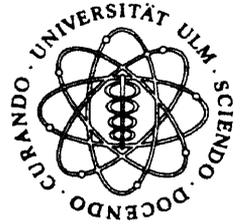


Universität Ulm
Fakultät für Informatik



On the Largest Common Subgraph Problem

Oleg Verbitsky

Nr. 95-01

Ulmer Informatik-Berichte

Januar 1995

0
QAA 5
A 4.95,1
EX.2

On the Largest Common Subgraph Problem

Oleg Verbitsky*

Department of Mathematics, Lviv State University
Universytetska 1, 290602 Lviv, Ukraine

Abstract

Given two graphs $G_1 = \langle V_1, E_1 \rangle$ and $G_2 = \langle V_2, E_2 \rangle$, $|V_1| = |V_2| = n$, to determine whether they have a size- k common subgraph is one of the earliest examples of an NP-complete problem (by a trivial reduction from the Maximum Clique problem). We show that this problem for equally sized G_1 and G_2 , i.e. when $|E_1| = |E_2| = m$, remains NP-complete. Moreover, the restriction to the case $k = m - \sqrt[n]{c}$, $c > 1$, is also NP-complete. In this result k and m can hardly be made tighter because the largest Common Subgraph problem for equally sized graphs is reducible to the Graph Isomorphism problem in time $n^{O(m-k)}$.

Further, we consider the optimization problem of computing the maximum common subgraph size. It is only known that this problem is not harder than computing the maximum clique size (V.Kann, STACS'92), and that it is approximable within factor $O(\frac{n}{\log n})$ (M.Hall-dórsson, 1994). For some $\epsilon \in (0, 1)$, we prove that the largest common subgraph size is not approximable within addend n^ϵ unless $\text{NP} = \text{P}$.

The techniques used are reductions from the problem of distinguishing between graphs with large and small clique size.

*Supported in part by grant No. MGT 000 from the International Science Foundation. Part of this work was done while visiting the Ulm university.

1 Introduction

We address the complexity properties of the Largest Common Subgraph problem (further on LCS), one of the earliest examples of an NP-complete problem. Given two graphs $G_1 = \langle V_1, E_1 \rangle$ and $G_2 = \langle V_2, E_2 \rangle$, we denote the size of the largest isomorphic subgraphs of G_1 and G_2 by $\sigma(G_1, G_2)$. The problem is, given G_1 and G_2 , $|V_1| = |V_2| = n$, and a natural k , to determine whether $\sigma(G_1, G_2) \geq k$. This problem is NP-complete by a trivial reduction from the Maximum Clique problem.

In this paper we consider a natural version of LCS for *equally sized graphs*, i.e. graphs with $|E_1| = |E_2| = m$. The reduction from Maximum Clique does not work now. To show that LCS OF EQUALLY SIZED GRAPHS remains NP-complete, we suggest a reduction from a generalization of Maximum Clique, namely, from the problem of distinguishing between graphs with large and small maximum clique size. The last problem was shown to be NP-hard owing to the progress in the area of the probabilistically checkable proofs [8, 2, 1].

Furthermore, we consider the restriction of LCS to the case $k = m - l(n)$ where l is a prespecified function of n . This problem denoted by $\text{LCS}[l(n)]$ is of interest due to its connections to the Graph Isomorphism problem. We show that whereas for superpolynomially small l the problem $\text{LCS}[l(n)]$ is closely related to Graph Isomorphism (see Proposition 3.1), for polynomially small l it is NP-complete.

Finally, we address the problem of approximating $\sigma(G_1, G_2)$. M.Halldórsson showed that the maximum common subgraph size is approximable within factor $O\left(\frac{n}{\log n}\right)$ [6, GT36]. It is known nothing about the hardness of approximating this problem within a constant factor. The method of the present paper allows us to prove that for some $\epsilon \in (0, 1)$, the value $\sigma(G_1, G_2)$ is not approximable within addend n^ϵ unless $\text{NP} = \text{P}$. In this connection it should be remarked that some NP-hard optimization problems, e.g. Maximum Packing in Two Containers, are approximable within a constant addend ([11], see also [10]).

Related work. NP-completeness of some problems related to Graph Isomorphism was established by A.Lubiw [15].

Provided $\text{NP} \neq \text{P}$, non-approximability within a constant addend was shown for some problems admitting FPTAS, e.g. for Knapsack [10].

The developments in the probabilistically checkable proofs allowed one

to prove that many of the well-known optimization problems are hard to approximate (see survey [3]). First this technique was applied to the optimization problem Max Clique. V.Kann [13] showed that Maximum Common Subgraph is reducible to Maximum Clique with respect to an approximation preserving reduction. Some approximation problems reducible to Maximum Clique was shown to be hard in [4] and [17].

The approximation complexity of some variations of Maximum Common Subgraph was investigated in [13].

The paper is organized as follows. Section 2 contains the needed definitions and some auxiliary propositions along with well-known results cited in the paper. We give an account of our results consequently in Theorems 3.1, 3.2, 3.3 of Section 3. A proof of every theorem extends a proof of the previous one. The paper is concluded by the discussions of related open questions in Section 4.

2 Preliminaries

By $G = \langle V, E \rangle$ we denote a graph G whose set of vertices is V and set of edges is E . The number of vertices $|V|$ will be denoted by n , ν or N . $G' = \langle V', E' \rangle$ is a *subgraph* of $G = \langle V, E \rangle$ if $E' \subseteq E$ and V' is the vertex set of all the edges from E' . Sometimes we will identify a graph with its set of edges. Under this convention, a subgraph is just an arbitrary subset of edges.

Two graphs $G_1 = \langle V_1, E_1 \rangle$ and $G_2 = \langle V_2, E_2 \rangle$ are *isomorphic* iff there exists a bijection $\phi : V_1 \rightarrow V_2$ preserving the adjacent relation. Given two graphs $G_1 = \langle V_1, E_1 \rangle$ and $G_2 = \langle V_2, E_2 \rangle$, by $\sigma(G_1, G_2)$ we denote the maximum size of a *common subgraph* of G_1 and G_2 , that is, the maximum $|E'_i|$ over isomorphic $E'_1 \subseteq E_1$ and $E'_2 \subseteq E_2$.

Given two graphs $G_1 = \langle V_1, E_1 \rangle$ and $G_2 = \langle V_2, E_2 \rangle$ with $V_1 \cap V_2 = \emptyset$, we define their (*disjoint*) *union* by $G_1 \cup G_2 = \langle V_1 \cup V_2, E_1 \cup E_2 \rangle$.

Lemma 2.1 *For any graphs $G_1, G_2, G = \langle V, E \rangle$ it is true*

$$\sigma(G_1 \cup G, G_2 \cup G) = \sigma(G_1, G_2) + |E|.$$

The proof of Lemma 2.1 will appear in the full version of the paper.

Definition 2.1 Definitions of the problems that occur in the paper.

LCS—LARGEST COMMON SUBGRAPH

INPUT: graphs $G_1 = \langle V_1, E_1 \rangle$ and $G_2 = \langle V_2, E_2 \rangle$, $|V_1| = |V_2|$, a natural number k

QUESTION: $\sigma(G_1, G_2) \geq k$?

LCS OF EQUALLY SIZED GRAPHS

INPUT: graphs $G_1 = \langle V_1, E_1 \rangle$ and $G_2 = \langle V_2, E_2 \rangle$, $|V_1| = |V_2|$, $|E_1| = |E_2|$, a natural number k

QUESTION: $\sigma(G_1, G_2) \geq k$?

LCS[$l(n)$] (the problem is specified by a function $l : \mathbf{N} \rightarrow \mathbf{R}$ such that $\lceil l(n) \rceil$ is computable in polynomial time)

INPUT: graphs $G_1 = \langle V_1, E_1 \rangle$ and $G_2 = \langle V_2, E_2 \rangle$, $|V_1| = |V_2|$, $|E_1| = |E_2|$, a natural number k

QUESTION: $\sigma(G_1, G_2) \geq |E_i| - l(|V_i|)$?

GI—GRAPH ISOMORPHISM is the problem LCS[0]

K_n is a complete graph on n vertices. E_n stands for a graph on n vertices with the empty set of edges. $\omega(G)$ denotes the size of the largest clique in a graph G , that is, the maximal s such that G has a subgraph K_s .

Given functions a and b , we define the problem of distinguishing between graphs with large and small clique size CLIQUE[$a(n), b(n)$] as follows. Given a graph G on n vertices, it is required to output

- 1 if $\omega(G) \geq a(n)$,
- 0 if $\omega(G) < b(n)$;

otherwise the output does not matter.

We use \leq_m^P to denote the many-one polynomial time reducibility. We say that a language L is many-one reducible to CLIQUE[$a(n), b(n)$] in polynomial time iff there is a polynomial time computable transformation r mapping every w to a graph G on n vertices with $\omega(G) \geq a(n)$ if $w \in L$, and with $\omega(G) < b(n)$ if $w \notin L$. We say the problem CLIQUE[$a(n), b(n)$] is NP-hard if any NP-language is \leq_m^P -reducible to it.

We say that the problem CLIQUE[$a(n), b(n)$] is many-one reducible to a language L in polynomial time iff there is a polynomial time computable transformation r such that for any graph G on n vertices if $\omega(G) \geq a(n)$ then $r(G) \in L$, and if $\omega(G) < b(n)$ then $r(G) \notin L$.

Obviously, if $L_1 \leq_m^P \text{CLIQUE}[a(n), b(n)]$ and $\text{CLIQUE}[a(n), b(n)] \leq_m^P L_2$ then $L_1 \leq_m^P L_2$.

Proposition 2.1 ([8, 2, 1]) i) For some $Q \geq 8$,
the problem $\text{CLIQUE}[n/Q, n/4Q]$ is NP-hard.

ii) There are $q \geq 3$ and $C > 1$ such that for

$$\alpha = \frac{C+1}{(q+1)C+1}, \quad \beta = \frac{C}{(q+1)C+1}, \quad (1)$$

the problem $\text{CLIQUE}[n^\alpha, n^\beta]$ is NP-hard.

Remark 2.1 i) is a corollary from the equality $\text{NP} = \text{PCP}(\log n, 1)$ [2, 1]. $Q = 2^q$ where q is a number of oracle queries of an PCP machine recognizing an NP-complete language. More efficient using random sources by the PCP machine [12] allows one to improve i) to ii). The constant C depends on the expanders used.

As a matter of fact, the restrictions of $\text{CLIQUE}[n/Q, n/4Q]$ and $\text{CLIQUE}[n^\alpha, n^\beta]$ to graphs of order n such that $n/Q, n/4Q$ and n^α, n^β are integer remain NP-complete. Due to this, we can simplify the exposition assuming $n/Q, n/4Q$ and n^α, n^β integer throughout the paper.

Proposition 2.2 (Turán's theorem, see e.g. [7, 5]) Let $f(a, b)$ denote the maximal possible number of edges in a graph on a vertices without an b -clique. Then

$$f(a, b) = a(a-1)/2 - (b-1)d(d-1)/2 - dr$$

where

$$a = (b-1)d + r, \quad 0 \leq r < b-1.$$

The extremal graph $G = \langle V, E \rangle$ with $|V| = a$, $|E| = f(a, b)$, $\omega(G) < b$ can be effectively constructed in time (a^2) .

Corollary 2.1 A graph obtained from K_a by deleting arbitrary a edges contains a subgraph $K_{\lfloor a/3 \rfloor}$.

By F_m we denote a graph consisting of m independent edges. $\sigma(F_{\lfloor n/2 \rfloor}, G)$ is the size of the maximal matching in a graph G of order n . It is well known that $\sigma(F_{\lfloor n/2 \rfloor}, G)$ can be computed in time (n^3) (see e.g. [9]).

3 The Complexity of the Largest Common Subgraph problem

Theorem 3.1 LCS OF EQUALLY SIZED GRAPHS is NP-complete.

Theorem 3.1 is a direct corollary from Proposition 2.1 and the following lemma.

Lemma 3.1 Let a and b be polynomial time computable functions, and $a(n) \geq 3b(n)$ for sufficiently large n . Then

$$\text{CLIQUE}[a(n), b(n)] \leq_m^P \text{LCS OF EQUALLY SIZED GRAPHS}.$$

Proof: We describe a reduction transforming an graph $G = \langle V, E \rangle$, $|V| = n$, $|E| = m$, into a pair of graphs G_1, G_2 and a natural k so that

$$\omega(G) \geq a(n) \quad \text{implies} \quad \sigma(G_1, G_2) \geq k, \quad (2)$$

$$\omega(G) < b(n) \quad \text{implies} \quad \sigma(G_1, G_2) < k. \quad (3)$$

For the brevity we will sometimes denote $a(n)$ just by a , and $b(n)$ by b . Of course, we can consider only G without isolated vertices. We assume $m \geq a(a-1)/2 + n/2$, otherwise it is clear that $\omega(G) < a$. Given such G , we take $G_1 = K_a \cup F_{m-a(a-1)/2}$ and $G_2 = G \cup E_{2m-a(a-2)-n}$. Clearly, G_1 and G_2 are equally sized.

Let s be the size of the maximal matching in G . We take $k = a(a-1)/2 + (s-a)$.

As easily seen, if $\omega(G) \geq a$ then G_1 and G_2 have a common subgraph $K_a \cup F_{s-a}$ of size k . Thus, $\sigma(G_1, G_2) \geq k$ and (2) is true.

Suppose now that $\sigma(G_1, G_2) \geq k$. This means there is a subgraph H of G_1 that can be embedded into G . It is clear that $|H \cap K_a| \geq k - s = a(a-1)/2 - a$. By Corollary 2.1, $H \cap K_a$ contains a subgraph K_b . Thus, $\omega(G) \geq b$ and (3) follows. ■

The following proposition shows the relations between the problems LCS[$l(n)$] and GI.

Proposition 3.1 i) $\text{GI} \leq_m^P \text{LCS}[l(n)]$ provided $l(n) \leq \sqrt{n}/3$.

ii) $\text{LCS}[l(n)]$ is many-one reducible to GI in time $n^O(l(n))$.

Proof: The proof of i) will appear in the full version of the paper. As to ii), the straightforward Turing reduction can be turned into a many-one reduction by techniques from [14] involving OR-function for GI. ■

Theorem 3.2 *For any $c > 1$, the problem $\text{LCS}[\sqrt[c]{n}]$ is NP-complete.*

Theorem 3.2 follows from the first part of Proposition 2.1 and the following lemma by trivial padding arguments.

Lemma 3.2 *For any rational Q , $\text{CLIQUE}[n/Q, n/4Q] \leq_m^P \text{LCS}[\nu/4]$.*

Proof: We have to describe a reduction transforming a graph $G = \langle V, E \rangle$, $|V| = n$, $|E| = m$, into a pair of graphs $G_1 = \langle V_1, E_1 \rangle$ and $G_2 = \langle V_2, E_2 \rangle$, $|V_1| = |V_2| = \nu$, $|E_1| = |E_2| = \mu$, so that

$$\omega(G) \geq n/Q \quad \text{implies} \quad \sigma(G_1, G_2) \geq \mu - \nu/4, \quad (4)$$

$$\omega(G) < n/4Q \quad \text{implies} \quad \sigma(G_1, G_2) < \mu - \nu/4. \quad (5)$$

Denote $a = n/Q$, $b = n/4Q$. We can assume $m \leq f(n, b)$; on all the other instances $\text{CLIQUE}[a, b]$ can be easily solved because then $\omega(G) \geq b$.

First we take an extremal graph on n vertices without K_b -subgraph and delete m edges from it. Denote the obtained graph by \hat{G} . Take the disjoint union of G and \hat{G} and add n more edges to it so that every vertex of one graph is matched to a vertex of the other graph. Denote the obtained graph by G' . Obviously, G' has $2n$ vertices, $f(n, b) + n$ edges and a size- n matching. Clearly, $\omega(G) \geq a$ implies $\omega(G') \geq a$, and $\omega(G) < b$ implies $\omega(G') < b$.

Now we apply to G' the reduction from the proof of Lemma 3.1, obtaining two graphs $G'_1 = \langle V'_1, E'_1 \rangle$ and $G'_2 = \langle V'_2, E'_2 \rangle$, $|V'_1| = |V'_2| = N$, $|E'_1| = |E'_2| = M$. We have

$$M = f(n, b) + n, \quad N = 2M - a(a - 2). \quad (6)$$

For $K = a(a - 1)/2 + (n - a)$, (2) and (3) give

$$\omega(G) \geq a \quad \text{implies} \quad \sigma(G'_1, G'_2) \geq K, \quad (7)$$

$$\omega(G) < b \quad \text{implies} \quad \sigma(G'_1, G'_2) < K. \quad (8)$$

By Turán's theorem, $f(n, b) = \frac{1}{2}n^2 - 2Qn - 2Q$. Therefore,

$$\begin{aligned} M &= \frac{1}{2}n^2 - (2Q - 1)n - 2Q, \\ N &= (1 - Q^{-2})n^2 - 2(2Q - 1 - Q^{-1})n - 4Q, \\ K &= Q^{-2}n^2 + (1 - \frac{3}{2}Q^{-1})n. \end{aligned}$$

Finally, take a graph H on $4(M - K) - N$ vertices with $M - 2K$ edges and construct the desired graphs $G_1 = G'_1 \cup H$ and $G_2 = G'_2 \cup H$. As easily seen,

$$\nu = 4(M - K), \quad \mu = 2(M - K). \quad (9)$$

By Lemma 2.1,

$$\sigma(G_1, G_2) = \sigma(G'_1, G'_2) + (M - 2K). \quad (10)$$

Thus,

$$\begin{aligned} \sigma(G'_1, G'_2) \geq K &\quad \text{implies} \quad \sigma(G_1, G_2) \geq M - K, \\ \sigma(G'_1, G'_2) < K &\quad \text{implies} \quad \sigma(G_1, G_2) < M - K. \end{aligned}$$

Summing up this with (7), (8), and (9), we get (4) and (5). ■

In the rest of this section we consider an optimization problem of computing the value $\sigma(G_1, G_2)$, given a pair of graphs $G_1 = \langle V_1, E_1 \rangle$ and $G_2 = \langle V_2, E_2 \rangle$, $|V_1| = |V_2| = n$.

Definition 3.1 σ is approximable within addend $g(n)$ iff there is a polynomial time algorithm A such that

$$\sigma(G_1, G_2) - g(n) \leq A(G_1, G_2) \leq \sigma(G_1, G_2)$$

Theorem 3.3 For some $\epsilon \in (0, 1)$, $\sigma(G_1, G_2)$ is not approximable within addend n^ϵ unless $\text{NP} = \text{P}$.

Theorem 3.3 immediately follows from the second part of Proposition 2.1 and the following lemma.

Lemma 3.3 For any $\epsilon < \alpha - \beta/2$ where α and β are defined by (1), $q, C > 1$, the problem $\text{CLIQUE}[n^\alpha, n^\beta]$ is reducible to approximation of $\sigma(G_1, G_2)$ within addend N^ϵ , that is, a polynomial time algorithm approximating the value $\sigma(G_1, G_2)$ within addend N^ϵ can be transformed into a polynomial time algorithm solving $\text{CLIQUE}[n^\alpha, n^\beta]$.

Proof: Denote $a = n^\alpha$ and $b = n^\beta$. Consider the reduction from the proof of Lemma 3.2 transforming a graph $G = \langle V, E \rangle$, $|V| = n$, $|E| = m$, into a pair of graphs $G'_1 = \langle V'_1, E'_1 \rangle$ and $G'_2 = \langle V'_2, E'_2 \rangle$, $|V'_1| = |V'_2| = N$, $|E'_1| = |E'_2| = M$. By (7),

$$\omega(G) \geq a \text{ implies } \sigma(G'_1, G'_2) \geq n + \frac{1}{2}n^{2\alpha} - \frac{3}{2}n^\alpha. \quad (11)$$

By Turán's theorem,

$$f(a, b) = \frac{1}{2}n^{1\alpha} - \frac{1}{2}n^{2\alpha-\beta} - \frac{1}{2}n^{2\alpha-2\beta} - \frac{1}{2}n^{\alpha-\beta}.$$

Assume $\sigma(G'_1, G'_2) \geq n + \frac{1}{2}n^{2\alpha} - \frac{1}{2}n^{2\alpha-\beta}$. Then for G' (see the description of the reduction) we have $\sigma(K_a, G') \geq \frac{1}{2}n^{2\alpha} - \frac{1}{2}n^{2\alpha-\beta} > f(a, b)$. Therefore, $\omega(G') \geq b$ and $\omega(G) \geq b$. It is proved that

$$\omega(G) < b \text{ implies } \sigma(G'_1, G'_2) < n + \frac{1}{2}n^{2\alpha} - \frac{1}{2}n^{2\alpha-\beta}. \quad (12)$$

(11) and (12) show that an algorithm approximating $\sigma(G_1, G_2)$ within addend n^γ , for any $\gamma < 2\alpha - \beta$, allows one to distinguish between the cases $\omega(G) \geq a$ and $\omega(G) < b$. To conclude the proof it suffices to notice that $N = n^2(1 - o(1))$. This follows from (6) and the simple estimates

$$f(n, b) = n(n-1)/2 - (n^\beta - 1)d(d-1)/2 - dr$$

where

$$d = \sum_{i=1}^{q+1} n^{1-i\beta} = \frac{n^{1-\beta}(1 - n^{-(q+1)\beta})}{1 - n^{-\beta}} < n^{1-\beta}(1 + 2n^{-\beta}), \quad r = n^{\alpha-\beta},$$

obtained by Turán's theorem. ■

Remark 3.1 The slightly updated proof of Theorem 3.3 gives also the following proposition. For any $\lambda \in (0, 1/2)$ there exist $\epsilon, \delta \in (0, 1)$ such that the difference $n^\lambda - \sigma(G_1, G_2)$, if positive, is not approximable within factor n^ϵ even for G_1 and G_2 with $\frac{1}{2}n(1 - n^{-\delta}) < m < \frac{1}{2}n$ unless $\text{NP} = \text{P}$.

4 Discussions and Open Problems

4.1 Approximation properties of Maximum Common Subgraph

In this paper we obtained the first hardness approximation result for the Maximum Common Subgraph problem saying that approximating it within *addend* n^ϵ is NP-hard. The challenging problem is to show that $\sigma(G_1, G_2)$ is hard to approximate within *factor* c , at least for some constant $c > 1$. In particular, it is unclear whether Maximum Common Subgraph is Max SNP-hard.

4.2 The case of half-sized graphs

We say that $G = \langle V, E \rangle$ is a *half-sized graph* if $|V| = n$ and $|E| = n(n-1)/4$. The Largest Common Subgraph problem restricted to half-sized graphs $G_1 = \langle V_1, E_1 \rangle$, $G_2 = \langle V_2, E_2 \rangle$, $|V_1| = |V_2|$, is of interest due to the following fact.

Proposition 4.1 *For any two graphs $G_1 = \langle V_1, E_1 \rangle$ and $G_2 = \langle V_2, E_2 \rangle$ with $|V_1| = |V_2| = n$, $|E| = n(n-1)/4$,*

$$\sigma(G_1, G_2) \geq \frac{n(n-1)}{8}.$$

It is not hard to show that LARGEST COMMON SUBGRAPH OF EQUALLY SIZED GRAPHS is equivalent to its restriction on half-sized graphs. Denote the restriction of $\text{LCS}[l(n)]$ to half-sized graphs by $\text{LCS}[l(n)]$ OF HALF-SIZED GRAPHS. Theorem 3.2 can be somewhat strengthened.

Proposition 4.2 *For any $c > 1$, the problem $\text{LCS}[\sqrt[n]{n}]$ OF HALF-SIZED GRAPHS is NP-complete.*

In view of Propositions 4.1 and 4.2 the following question looks interesting. Is it possible to establish the NP-completeness of $\text{LCS}[l(n)]$ OF HALF-SIZED GRAPHS for $l(n) = n(n-1)/8 - o(1)$? I cannot solve this question even for $l(n) = cn(n-1)$, $c \in (0, 1/8)$.

Proposition 4.1 shows that $\sigma(G_1, G_2)$ is trivially approximable within factor 2 for half-sized graphs. What is complexity of finding a size- $\frac{n(n-1)}{8}$ common subgraph in this case? Can the performance ratio be improved?

4.3 A generalized version of the Graph Automorphism problem

The Graph Isomorphism and Graph Automorphism problems are of particular interest in structural complexity theory (see [14]). It is natural to regard the problem $\text{LCS}[l(n)]$ as an edge deletion generalization of GI. Whereas Proposition 3.1 asserts that for superpolynomially small l the problem $\text{LCS}[l(n)]$ is closely related to GI, Theorem 3.2 shows that for polynomially small l $\text{LCS}[l(n)]$ is NP-complete.

In this subsection we introduce an NP-problem related to Graph Automorphism. The problem deals with the asymmetry measure of a graph that has been studied in combinatorics.

Given a graph $G = \langle V, E \rangle$, define $A(G) = \{|D| : E \Delta D \text{ has nontrivial automorphism} \}$ where D is a set of edges on V . In words, $A(G)$ is the minimal number of edges to be changed in G in order to obtain a symmetrical graph. A natural problem is defined as follows.

Given a graph G and a natural k , to determine whether $A(G) \leq k$.

Is this problem NP-complete?

Concluding, we remark some relevant combinatorial results. It is known that $A(G) < n/2$ for any $G = \langle V, E \rangle$, $|V| = n$, and $A(G) = n(1/2 - o(1))$ for some sequence of graphs [7]. The proof of these facts in [7] is non-constructive. Some constructivization was achieved in [16] where it was shown that a sequence of graphs G_n with $A(G_n) \geq (1/12 - \epsilon)n$ can be encoded by boolean function computable by constant depth $\text{poly}(\log n)$ -size circuits over basis $\{\&, \oplus\}$.

Acknowledgement

I am grateful to Johannes Köbler and Pavel Pudlák for conversations on this subject, and to Alexander Razborov for valuable suggestions. I am indebted to Uwe Schöning for his hospitality during my research stay at the Ulm university.

References

- [1] S. Arora, C. Lund, R. Motwani, M. Sudan, and M. Szegedy. Proof verification and hardness of approximation problems. In *Proc. of the 33rd IEEE Ann. Symp. on Foundations of Computer Science (FOCS)*, pages 14–23, 1992.
- [2] S. Arora and S. Safra. Probabilistic checking of proofs; a new characterization of NP . In *Proc. of the 33rd IEEE Ann. Symp. on Foundations of Computer Science (FOCS)*, pages 2–13, 1992.
- [3] L. Babai. Transparent proofs and limits to approximation. In *Proc. of the 1st Europ. Congr. Math.* Birkhäuser, 1994. *To appear.*
- [4] P. Berman and G. Schnitger. On the complexity of approximating the Independent Set problem. *Information and Computation*, 96:77–94, 1992.
- [5] B. Bollobas. *Extremal graph theory*. Academic Press, London, New York, and San Francisco, 1978.
- [6] P. Crescenzi and V. Kann. A compendium of NP optimization problems. 1994.
- [7] P. Erdős and J. Spenser. *Probabilistic methods in combinatorics*. Akademiai Kiado, Budapest, 1974.
- [8] U. Feige, S. Goldwasser, L. Lovász, S. Safra, and M. Szegedy. Approximating clique is almost NP -complete. In *Proc. of the 32nd IEEE Ann. Symp. on Foundations of Computer Science (FOCS)*, pages 2–12, 1991.
- [9] H. N. Gabow. An efficient implementation of edmond’s algorithm for maximum matching on graphs. *J. Assoc. Comput. Mach.*, 23:221–234, 1976.
- [10] M. R. Garey and D. S. Johnson. *Computers and Intractability. A guide to the theory of NP -completeness*. W. H. Freeman, 1979.
- [11] E. Horowitz and S. Sahni. *Algorithms: design and analysis*. Computer Science Press, Potomak, MD.

- [12] R. Impagliazzo and D. Zuckerman. How to recycle random bits. In *Proc. of the 30th IEEE Ann. Symp. on Foundations of Computer Science (FOCS)*, pages 248–253, 1989.
- [13] V. Kann. On the approximability of the maximum common subgraph problem. In *Proc. of the 9th Ann. Symp. on Theoretical Aspects of Comput. Sci. (STACS)*, Lecture Notes in Computer Science 247, pages 377–388, New York/Berlin, 1992. Springer-Verlag.
- [14] J. Köbler, U. Schöning, and J. Torán. *The Graph Isomorphism problem: its structural complexity*. Birkhäuser, 1993.
- [15] A. Lubiw. Some *NP*-complete problems similar to graph isomorphism. *SIAM Journal on Computing*, 10:11–21, 1981.
- [16] A. A. Razborov. Formulas of bounded depth in basis $\{\&, \oplus\}$ and some combinatorial problems. In *Voprosy Kibernetiki*, S. I. Adian editor, pages 149–166, Moscow, 1988. In Russian.
- [17] O. Verbitsky. On the hardness of approximating some optimization problems which are supposedly easier than Max Clique. Steklov Mathematical Institute of the Russian Academy of Sciences. Department of Mathematical Logic. Prepublication series: Mathematical Logic and Theoretical Computer Science, No. 5., November 1992.

Liste der bisher erschienenen Ulmer Informatik-Berichte

Einige davon sind per FTP von `ftp.informatik.uni-ulm.de` erhältlich

Die mit * markierten Berichte sind vergriffen

List of technical reports published by the University of Ulm

Some of them are available by FTP from `ftp.informatik.uni-ulm.de`

Reports marked with * are out of print

- 91-01 *Ker-I Ko, P. Orponen, U. Schöning, O. Watanabe*
Instance Complexity
- 91-02* *K. Gladitz, H. Fassbender, H. Vogler*
Compiler-Based Implementation of Syntax-Directed Functional Programming
- 91-03 *Alfons Geser*
Relative Termination
- 91-04* *J. Köbler, U. Schöning, J. Toran*
Graph Isomorphism is low for PP
- 91-05 *Johannes Köbler, Thomas Thierauf*
Complexity Restricted Advice Functions
- 91-06 *Uwe Schöning*
Recent Highlights in Structural Complexity Theory
- 91-07 *F. Green, J. Köbler, J. Toran*
The Power of Middle Bit
- 91-08* *V. Arvind, Y. Han, L. Hamachandra, J. Köbler, A. Lozano,
M. Mundhenk, A. Ogiwara, U. Schöning, R. Silvestri, T. Thierauf*
Reductions for Sets of Low Information Content
- 92-01* *Vikraman Arvind, Johannes Köbler, Martin Mundhenk*
On Bounded Truth-Table and Conjunctive Reductions to Sparse and Tally Sets
- 92-02* *Thomas Noll, Heiko Vogler*
Top-down Parsing with Simultaneous Evaluation of Noncircular Attribute Grammars
- 92-03 *Fakultät für Informatik*
17. Workshop über Komplexitätstheorie, effiziente Algorithmen und Datenstrukturen
- 92-04 *V. Arvind, J. Köbler, M. Mundhenk*
Lowness and the Complexity of Sparse and Tally Descriptions
- 92-05 *Johannes Köbler*
Locating P/poly Optimally in the Extended Low Hierarchy
- 92-06 *Armin Kühnemann, Heiko Vogler*
Synthesized and inherited functions - a new computational model for syntax-directed semantics
- 92-07 *Heinz Fassbender, Heiko Vogler*
A Universal Unification Algorithm Based on Unification-Driven Leftmost Outermost Narrowing

Liste der bisher erschienenen Ulmer Informatik-Berichte

Einige davon sind per FTP von `ftp.informatik.uni-ulm.de` erhältlich

Die mit * markierten Berichte sind vergriffen

List of technical reports published by the University of Ulm

Some of them are available by FTP from `ftp.informatik.uni-ulm.de`

Reports marked with * are out of print

- 91-01 *Ker-I Ko, P. Orponen, U. Schöning, O. Watanabe*
Instance Complexity
- 91-02* *K. Gladitz, H. Fassbender, H. Vogler*
Compiler-Based Implementation of Syntax-Directed Functional Programming
- 91-03 *Alfons Geser*
Relative Termination
- 91-04* *J. Köbler, U. Schöning, J. Toran*
Graph Isomorphism is low for PP
- 91-05 *Johannes Köbler, Thomas Thierauf*
Complexity Restricted Advice Functions
- 91-06 *Uwe Schöning*
Recent Highlights in Structural Complexity Theory
- 91-07 *F. Green, J. Köbler, J. Toran*
The Power of Middle Bit
- 91-08* *V. Arvind, Y. Han, L. Hamachandra, J. Köbler, A. Lozano,
M. Mundhenk, A. Ogiwara, U. Schöning, R. Silvestri, T. Thierauf*
Reductions for Sets of Low Information Content
- 92-01* *Vikraman Arvind, Johannes Köbler, Martin Mundhenk*
On Bounded Truth-Table and Conjunctive Reductions to Sparse and Tally Sets
- 92-02* *Thomas Noll, Heiko Vogler*
Top-down Parsing with Simultaneous Evaluation of Noncircular Attribute Grammars
- 92-03 *Fakultät für Informatik*
17. Workshop über Komplexitätstheorie, effiziente Algorithmen und Datenstrukturen
- 92-04 *V. Arvind, J. Köbler, M. Mundhenk*
Lowness and the Complexity of Sparse and Tally Descriptions
- 92-05 *Johannes Köbler*
Locating P/poly Optimally in the Extended Low Hierarchy
- 92-06 *Armin Kühnemann, Heiko Vogler*
Synthesized and inherited functions - a new computational model for syntax-directed semantics
- 92-07 *Heinz Fassbender, Heiko Vogler*
A Universal Unification Algorithm Based on Unification-Driven Leftmost Outermost Narrowing

- 92-08 *Uwe Schöning*
On Random Reductions from Sparse Sets to Tally Sets
- 92-09 *Hermann von Hasseln, Laura Martignon*
Consistency in Stochastic Network
- 92-10 *Michael Schmitt*
A Slightly Improved Upper Bound on the Size of Weights Sufficient to Represent Any Linearly Separable Boolean Function
- 92-11 *Johannes Köbler, Seinosuke Toda*
On the Power of Generalized MOD-Classes
- 92-12 *V. Arvind, J. Köbler, M. Mundhenk*
Reliable Reductions, High Sets and Low Sets
- 92-13 *Alfons Geser*
On a monotonic semantic path ordering
- 92-14* *Joost Engelfriet, Heiko Vogler*
The Translation Power of Top-Down Tree-To-Graph Transducers
- 93-01 *Alfred Lupper, Konrad Froitzheim*
AppleTalk Link Access Protocol basierend auf dem Abstract Personal Communications Manager
- 93-02 *M.H. Scholl, C. Laasch, C. Rich, H.-J. Schek, M. Tresch*
The COCOON Object Model
- 93-03 *Thomas Thierauf, Seinosuke Toda, Osamu Watanabe*
On Sets Bounded Truth-Table Reducible to P-selective Sets
- 93-04 *Jin-Yi Cai, Frederic Green, Thomas Thierauf*
On the Correlation of Symmetric Functions
- 93-05 *K.Kuhn, M.Reichert, M. Nathe, T. Beuter, C. Heinlein, P. Dadam*
A Conceptual Approach to an Open Hospital Information System
- 93-06 *Klaus Gäßner*
Rechnerunterstützung für die konzeptuelle Modellierung
- 93-07 *Ullrich Keßler, Peter Dadam*
Towards Customizable, Flexible Storage Structures for Complex Objects
- 94-01 *Michael Schmitt*
On the Complexity of Consistency Problems for Neurons with Binary Weights
- 94-02 *Armin Kühnemann, Heiko Vogler*
A Pumping Lemma for Output Languages of Attributed Tree Transducers
- 94-03 *Harry Buhrman, Jim Kadin, Thomas Thierauf*
On Functions Computable with Nonadaptive Queries to NP
- 94-04 *Heinz Faßbender, Heiko Vogler, Andrea Wedel*
Implementation of a Deterministic Partial E-Unification Algorithm for Macro Tree Transducers

- 94-05 *V. Arvind, J. Köbler, R. Schuler*
On Helping and Interactive Proof Systems
- 94-06 *Christian Kalus, Peter Dadam*
Incorporating record subtyping into a relational data model
- 94-07 *Markus Tresch, Marc H. Scholl*
A Classification of Multi-Database Languages
- 94-08 *Friedrich von Henke, Harald Rueß*
Arbeitstreffen Typtheorie: Zusammenfassung der Beiträge
- 94-09 *F.W. von Henke, A. Dold, H. Rueß, D. Schwier, M. Strecker*
Construction and Deduction Methods for the Formal Development of Software
- 94-10 *Axel Dold*
Formalisierung schematischer Algorithmen
- 94-11 *Johannes Köbler, Osamu Watanabe*
New Collapse Consequences of NP Having Small Circuits
- 94-12 *Rainer Schuler*
On Average Polynomial Time
- 94-13 *Rainer Schuler, Osamu Watanabe*
Towards Average-Case Complexity Analysis of NP Optimization Problems
- 94-14 *Wolfram Schulte, Ton Vullings*
Linking Reactive Software to the X-Window System
- 94-15 *Alfred Lupper*
Namensverwaltung und Adressierung in Distributed Shared Memory-Systemen
- 94-16 *Robert Regn*
Verteilte Unix-Betriebssysteme
- 94-17 *Helmuth Partsch*
Again on Recognition and Parsin of Context-Free Grammars: Two Exercises in Transformational Programming
- 94-18 *Helmuth Partsch*
Transformational Development of Data-Parallel Algorithms: an Example
- 95-01 *Oleg Verbitsky*
On the Largest Common Problem