Transmission Methods for Wireless Multi Carrier Systems in Time-Varying Environments

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1 Introduction 1

2 Fundamentals 5
  2.1 Basic Channel Models .......................... 6
    2.1.1 Time-Variant Channels ....................... 6
    2.1.2 Discrete-Time Model ......................... 10
    2.1.3 Example Channels ........................... 11
  2.2 OFDM Transmission Model ....................... 14
  2.3 Time Synchronization and Frequency Offset Estimation ........... 19
  2.4 Channel Coding ................................ 21
  2.5 Capacity of Time-Variant Multipath Channels .................. 23

3 Robust Multi Carrier Transmission Methods 29
  3.1 Modulation Schemes for Noncoherent Reception ................. 30
    3.1.1 OFDM-MFSK .................................. 31
    3.1.2 The PAPR Problem ............................. 33
    3.1.3 Hybrid Modulation Schemes .................... 34
  3.2 Iterative Receivers ................................ 37
  3.3 Model for Coded Transmissions .......................... 43

4 Noncoherent Signal Detection 45
  4.1 Receive Metrics for Noncoherent Detection .................... 45
    4.1.1 AWGN Channel ................................ 47
    4.1.2 Rayleigh Channel ............................. 48
    4.1.3 WSSUS Channel ............................... 48
  4.2 Single Symbol Detection ................................ 49
    4.2.1 Performance of OFDM-MFSK ..................... 50
    4.2.2 Iterative Detection ........................... 54
    4.2.3 Serial Detection of Hybrid Modulation Schemes ............ 59
  4.3 Multiple Symbol Detection ................................ 62
    4.3.1 Multiple Symbol Detection for OFDM-MFSK ............. 62
    4.3.2 Joint Multiple Symbol Detection of Hybrid Modulation Schemes 68
  4.4 Transinformation for OFDM-MFSK-based modulation schemes ........ 74
  4.5 Extended Mapping .................................. 76
    4.5.1 Choice of the Mapping ........................ 76
    4.5.2 Choice of the Channel Code .................... 79
4.5.3 Numerical Results ............................................. 80
4.6 Precoding ......................................................... 81
4.6.1 Three-Dimensional EXIT Chart Analysis .................. 83
4.6.2 Numerical Results ............................................. 86

5 Noncoherent Communication Based on Subspaces 87
5.1 Subspace Representation of Noncoherent MIMO Transmission 87
5.2 Stiefel and Grassmann Manifolds ................................ 89
5.3 Subspace Representation of OFDM-MFSK-Based Modulation Schemes 91
5.3.1 OFDM-MFSK .................................................... 91
5.3.2 Hybrid Modulation Schemes .................................. 94
5.3.3 Multitone FSK .................................................. 96
5.4 General Remarks .................................................. 97

6 Summary and Conclusions ........................................... 99

A Pairwise Error Probability for OFDM-MFSK with MSD 103
A.1 Approximation of the PEP for OFDM-MFSK with MSD for High SNR ... 104

B Metrics for noncoherent MIMO channels 107
B.1 ML Metric ......................................................... 107
B.2 GLRT Metric ...................................................... 108

C List of Frequently Used Operators, Symbols, and Acronyms 111

Bibliography .......................................................... 117
Nowadays, it is almost taken for granted that mobile wireless internet access with high data rates is available at any time and at any place. However, there are many scenarios where multipath propagation and the time variance of the physical radio channel imply significant challenges for the design of transmission systems. For example, the data link to a high speed train is such a scenario. Here a transmission scheme is necessary that can cope with such challenging conditions in order to provide a reliable data transmission. Already today, maglev and conventional wheeled trains can reach speeds of almost 600 km/h [56, 57]. This leads to a radio channel that changes very rapidly. In addition to the time variance of the channel, that is caused by the movement of the train, reflections at buildings, bridges, metal pylons, or tunnel entrances lead to a frequency selective channel [44, 49]. Many standard mobile communication systems like the global system for mobile communications (GSM), GSM-Rail, or the universal mobile telecommunications system (UMTS) are either not suited for very high speeds or provide only low throughput for fast time-variant channels [27, 45]. Newer technologies such as worldwide interoperability for microwave access (WiMAX) and long term evolution (LTE) support higher mobility up to 500 km/h [74], but their performance also degrades under such conditions. Besides the high speed train scenario, the radio link to a low flying airplane in a hilly environment might be mentioned as another example for such a difficult channel.

Most of the transmission schemes that are currently used in wireless networks are based on modulation schemes that need coherent detection. This means that the receiver requires precise knowledge of the radio channel, which is a very challenging task in scenarios where the receiver and/or the transmitter is moving with high speed. Usually, channel knowledge is obtained by transmitting pilot symbols or sequences that are known to the receiver in order to estimate the channel coefficients. However, due to the time variance of mobile channels, the channel knowledge might be outdated very quickly, which either leads to a degradation of the performance or requires a large pilot overhead, which in turn reduces the useful data rate. The second challenge is the frequency selectivity of the channel that is caused by multipath propagation. Multi car-
rier techniques such as orthogonal frequency division multiplexing (OFDM) are widely used in this case as an efficient method to avoid inter symbol interference (ISI) and therefore a complex equalizer.

In many application scenarios there are several data streams from different sources that require different levels of reliability. Let us take the high speed train scenario again where we assume that the passengers can use a base station inside the train to access the internet via a standard wireless local area network (WLAN) [1]. This base station has to be connected to a fixed base station along the railway track in order to provide a connection to the network outside the train. At the same time, there is important control data and security relevant information from the train itself that also has to be transmitted to the base station along the track and vice versa. While a larger delay or loss of data is only a minor problem for the internet users, the transmission of security relevant data has much higher requirements regarding the reliability. So it is necessary to use a transmission scheme that has a high robustness and reliability also in difficult environments at least for a part of the transmitted data.

In this work, we focus on channels that are both frequency selective and fast time-variant. There are several approaches to tackle this problem. One possibility is to describe the channel using the Doppler-variant impulse response [29]. This has the advantage that only a limited number of constant parameters have to be estimated within a data block. Other approaches are based on OFDM to allow low-complexity equalization. For example, in [39, 43, 44] directive antennas or multiple antennas and special signal processing are used to compensate for the effects of time-variant channels. Furthermore, blind channel estimation methods can be employed where the redundancy in the transmit symbols is used to implicitly estimate the channel within a transmitted block without any additional pilot symbols. For an example, see [59] and references therein.

Our approach is to use transmission schemes that allow noncoherent detection. Therefore, explicit channel knowledge is not needed at the receiver and we do not make any attempt to gain it. In order to avoid ISI, we employ OFDM-based transmission and a cyclic prefix. The OFDM parameters have to be chosen such that a good trade off is found between the degradation due to time variance and the energy loss due to the cyclic prefix.

The outline of this thesis is as follows: Chapter 2 introduces the basic system model that is used throughout this thesis. The mathematical model for the description of time-variant channels is explained and some example channels that are used for simulations are defined. After this, the vector transmission model describing the OFDM system is presented including the case of time-variant channels. Finally the capacity of time-variant multipath channels under certain assumptions is addressed.

In Chapter 3, we present some OFDM-based transmission methods that allow noncoherent detection. The basis is formed by a combination of OFDM and frequency shift keying (FSK), which leads to a very simple and robust scheme that can do without equalization and channel estimation even for time-variant multipath channels. In order to increase the bandwidth efficiency, we propose the use of hybrid modulation...
schemes where the phases of the occupied OFDM subcarriers are used to transmit additional data. Furthermore, the principle of iterative receivers is presented. At the end of the chapter, a detailed model for the total transmission system including hybrid modulation and iterative detection is given.

In Chapter 4, we derive the receive metrics for noncoherent signal detection that are needed for soft decision decoding. We propose iterative multiple symbol detection for our OFDM-MFSK-based modulation schemes and give simulation results for different channel models. Another topic is the interaction between the mapping and channel coding in systems with iterative receivers. In order to obtain a good receiver performance, it is necessary that the mapping and the channel code are matched to each other. For this, we examine methods that are based on optimized bit mappings and employ an additional intermediate code to improve the convergence of the iterative receiver.

Chapter 5 introduces a new representation for the OFDM-MFSK-based schemes discussed in the previous chapters. We show that, assuming noncoherent detection, these schemes can be regarded as information transmission using vector subspaces. After introducing the general subspace-based transmission model for multiple input multiple output (MIMO) channels including the corresponding receive metrics, we can establish the link between OFDM-MFSK-based schemes and the subspace-based interpretation of noncoherent space time modulation.

The main contributions of this thesis are contained in Chapter 3, Chapter 4, and Chapter 5. Parts of this work have been published in [94–98].
This chapter describes the fundamentals that are needed to model a digital wireless communication as it is used in this work. Fig. 2.1 shows a very coarse model of such a communication system in the equivalent lowpass domain. After we have introduced all the necessary fundamentals and specified the transmitter and receiver, we give a more detailed model of the whole system in Section 3.3.

In Fig. 2.1, a data sequence \( q(k_q) \) from a digital source \((Q)\) is transmitted to a sink \((S)\). The data symbols are encoded and interleaved in the block COD. After this, the digital modulation (MOD) takes place, where the coded data symbols are mapped to transmit symbols and the samples of the discrete-time transmit signal \( s(k) \) are generated. In our case, this block also contains the OFDM modulation that will be explained in Section 2.2. After Dirac-delta sampling and interpolation by a lowpass filter (IP-LP), the continuous-time transmit signal \( s(t) \) is obtained, which is transmitted over the channel. At the receive side, the signal is band limited by an anti-aliasing lowpass filter (AA-LP) and sampled again. The receive samples are processed in the detector (DET) to obtain the received data symbols \( \hat{q}(k_q) \) at the sink. In the case of an OFDM
2 Fundamentals

![Continuous-time channel model](image)

Fig. 2.2: Continuous-time channel model.

transmission, the detector in Fig. 2.1 also contains the digital OFDM demodulation of the discretized receive signal.

This chapter is organized as follows: After introducing the basic channel model in the first section, we focus on the description of linear time-variant channels. Stochastic system functions are used to describe the behavior of ergodic channels and some example channels are given. A vector model for OFDM will be given, that is used throughout this work. Finally the capacity of dispersive channels is addressed.

2.1 Basic Channel Models

Fig. 2.2 shows the basic continuous-time channel model for transmission of a signal over a linear time-variant channel with additive noise. Throughout this thesis we model the system in the equivalent lowpass domain, which is possible under the assumptions that the corresponding physical signals are bandpass signals and the widening of the spectra due to Doppler spreading is small compared to the bandwidth $B$ of the receive bandpass [53]. For further details about representing systems in the equivalent lowpass domain, the reader is referred to standard text books such as [53] and [66].

Let us assume a transmit signal $s(t)$, which is transmitted over a multipath channel with time-variant channel impulse response $h(\tau, t)$. In this representation of the channel impulse response (CIR), $t$ denotes the absolute time and $\tau$ the delay time. The receive signal is then given as

$$g(t) = \int_{-\infty}^{\infty} h(\tau, t)s(t - \tau)d\tau + n(t), \quad (2.1)$$

where $n(t)$ stands for an additive white Gaussian noise (AWGN) term. Because we model the channel in the equivalent lowpass domain, the noise term is a complex-valued zero-mean process where the real and imaginary part have a spectral noise power density of $N_0/2$.

2.1.1 Time-Variant Channels

In (2.1) we have used the time-variant channel impulse response $h(\tau, t)$ to describe the influence of the channel on a transmitted signal. To emphasize the fact that both
variables are in time domain we also denote it by \( hh(\tau, t) \). By using the Fourier transform, it is possible to derive three additional functions to describe the system \([53, 61]\). One is the time-variant transfer function. It is obtained by taking the Fourier transform of \( hh(\tau, t) \) with respect to the delay time \( \tau \):

\[
HH(f, t) = \int_{-\infty}^{\infty} hh(\tau, t)e^{-j2\pi ft}d\tau.
\]

(2.2)

The receive signal can therefore also be described as

\[
g(t) = \int_{-\infty}^{\infty} HH(f, t)S(f)e^{j2\pi ft}df + n(t).
\]

(2.3)

The time variance of the transfer function, i.e., the dependence on \( t \), leads to an amplitude and phase modulation of the transmit signal. Under the assumption that the bandwidth \( B \) of the signals is small compared to the center frequency \( f_c \), this causes a shift and/or widening of the transmit signal spectrum.

To get a better insight into the effects of time variance, it is useful to take the Fourier transform of \( hh(\tau, t) \) with respect to the absolute time \( t \) to get the Doppler-variant impulse response:

\[
hH(\tau, \nu) = \int_{-\infty}^{\infty} hh(\tau, t)e^{-j2\pi \nu t}dt.
\]

(2.4)

The Doppler-variant impulse response shows the spectral widening/shift of the multipath components. Using the inverse Fourier transform of (2.4) we can express the receive signal as a weighted infinite sum of delayed and Doppler shifted versions of the input signal \( s(t) \):

\[
g(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} hH(\tau, \nu)s(t - \tau)e^{j2\pi \nu t}d\nu d\tau + n(t).
\]

(2.5)

The last system function is obtained by taking the Fourier transform of the time-variant transfer function with respect to the absolute time \( t \) and is denoted as Doppler-variant transfer function:

\[
HH(f, \nu) = \int_{-\infty}^{\infty} HH(f, t)e^{-j2\pi \nu t}dt.
\]

(2.6)

All four functions are equivalent and completely describe the influence of a time-variant multipath channel. However, as described above, each of the representations gives insight into different properties of the channel. Fig. 2.3 shows the four system functions for time-variant channels and their relationship by the Fourier transform.

Because we frequently use the time-variant impulse response \( hh(\tau, t) \) and the time-variant transfer function \( HH(f, t) \) in our work, we use the simplified notation \( h(\tau, t) \) and \( H(f, t) \), respectively. The time domain description of the absolute time-variance should be clear from the variables.
### Statistical Description of Time-Variant Channels

So far, we have described the influence of a deterministic channel on a transmitted signal. To characterize the time variance of realistic fading channels, it is often useful to use a statistical description. For our system model we assume that the channel statistics are not changing over time. A second assumption is that scattering components at different delay times are uncorrelated. This leads to the widely used \textit{wide sense stationary uncorrelated scattering (WSSUS)} model \cite{8}.

The statistical behavior of the channel can be described by using the autocorrelation function of the CIR $hh(\tau, t)$:

$$
\varphi \varphi(\tau_1, \tau_2, t_1, t_2) = E \{hh^*(\tau_1, t_1)hh(\tau_2, t_2)\}.
$$

Because of the assumption of wide sense stationarity, the above expectation value does not depend on the absolute times $t_1$ and $t_2$ but only on the time difference $\Delta t = t_2 - t_1$. The uncorrelated scattering assumption implies that (2.7) is vanishing for $\tau_1 \neq \tau_2$, so that we can use the function $\varphi \varphi(\tau, \Delta t)$ for our purposes, which is called the \textit{delay time correlation function} or \textit{delay cross-power spectral density}. For $\Delta t = 0$ we obtain the average power at the output of the channel for a certain delay $\tau$ which is called the \textit{power delay profile (PDP)} $\varphi \varphi(\tau)$. The \textit{multipath spread} $T_m$ of the channel is defined as the range of values of $\tau$ where the PDP is substantially larger than zero. Similarly, the \textit{coherence time} $\Delta t_c$ is given as the range of $\Delta t$ where $\varphi \varphi(\tau, \Delta t)$ has significant nonzero values.

The \textit{time frequency correlation function} provides a measure of the frequency correlation of the channel for time difference $\Delta t$ and is defined as

$$
\Phi \varphi(\Delta f, \Delta t) = E \{Hh^*(f, t)Hh(f + \Delta f, t + \Delta t)\}.
$$
It can be obtained also by taking the Fourier transform of the delay time correlation function with respect to the delay time, which can be written as

$$\Phi \varphi (\Delta f, \Delta t) = \int_{-\infty}^{\infty} \varphi \varphi (\tau, \Delta t) e^{-j2\pi \Delta f \tau} d\tau.$$  \hspace{1cm} (2.9)

The coherence bandwidth $\Delta f_c$ of the channel is defined as the range of $\Delta f$ where $\Phi \varphi (\Delta f, \Delta t)$ is essentially nonzero. Its relation to the multipath spread is given as

$$\Delta f_c \approx \frac{1}{T_m}.$$  \hspace{1cm} (2.10)

Corresponding to the Doppler-variant impulse response (2.4) of deterministic channels, we can use the scattering function to describe the spectral widening of all multipath components in a statistical way. It is defined as

$$\varphi \Phi (\tau, \nu) = E \{h^H (\tau, \nu) hH (\tau, \nu)\}$$  \hspace{1cm} (2.11)

and can be obtained by taking the Fourier transform of the delay time correlation function (2.7) with respect to $\Delta t$. Here we can define the Doppler spread $B_d$ as the range of $\nu$ where the scattering function is nonzero. As the scattering function and the delay time correlation function are connected by the Fourier transform, it is not surprising that the relation of the Doppler spread to the coherence time is given as

$$\Delta t_c \approx \frac{1}{B_d}.$$  \hspace{1cm} (2.12)

To make the set complete, we give the Doppler cross-power spectral density, which can be obtained as the Fourier transform of the time frequency correlation function with respect to $\Delta t$:

$$\Phi \Phi (\Delta f, \nu) = \int_{-\infty}^{\infty} \Phi \varphi (\Delta f, \Delta t) e^{-j2\pi \nu \Delta t} d\Delta t.$$  \hspace{1cm} (2.13)

For $\Delta f = 0$, (2.13) yields the Doppler power spectrum $\Phi \Phi (\nu)$.

The spread factor of a channel is defined as the product $T_m B_d$ [66]. A channel is called underspread if the spread factor is smaller than one. Basically, all practical channels are underspread [21]. However, the larger the spread factor, the more difficult it is to get an accurate estimate of the CIR before it is outdated again.

Fig. 2.4 gives an overview of the system functions that can be used to statistically describe a time-variant channel. Like in the case of a deterministic channel (cf. Fig. 2.3), the four stochastic system functions are equivalent and linked by the Fourier transform.

The explanations given in this section on the description of time-variant channels should give a brief overview over the topic. More detailed descriptions can be found in, e. g., [66] and [61].
2.1.2 Discrete-Time Model

To simplify the mathematical representation and to allow digital signal processing, it is useful to use a discrete-time channel model. Let us assume ideal transmit and receive lowpass filters with cutoff frequency $f_g$ corresponding to a bandwidth $B = 2f_g$ in the bandpass domain. The sampling rate is $1/T \geq B$ so that the sampling theorem is fulfilled. Because the transmit and receive signals are band limited, we can also regard the channel as being band limited so that we can use a discrete-time representation. A further assumption is that the spectral widening or shift of the signals due to the time variance of the channel is small compared to the signal bandwidth $B$, which justifies the use of equal sampling rates and filter bandwidths at the transmitter and the receiver. This leads to the following discrete-time channel model

$$g(k) = \sum_{l=-\infty}^{\infty} h(l, k)s(k-l) + n(k), \quad (2.14)$$

where the discrete-time CIR is given as

$$h(l, k) = hh(l, k) = \int_{-\infty}^{\infty} h(\tau, kT) \frac{\sin(\pi B (\tau - lT))}{\pi B (\tau - lT)} d\tau. \quad (2.15)$$

We assume a causal CIR with finite duration, i.e., $h(\tau, t) = 0$ for $\tau < 0$ and $\tau \geq LT = T_m$. For finite bandwidth, the discrete-time CIR is not completely zero for $l < 0$ and $l \geq L$ in general. In this thesis, we assume $B = T$ and use channel models where the path delays are integer multiples of the sampling period $T$ so that (2.14) can be
written as

\[ g(k) = \sum_{l=0}^{L} h(l, k) s(k - l) + n(k). \]  

(2.16)

The noise samples \( n(k) \) are modeled as a complex white Gaussian noise process with variance \( \sigma_n^2 = N_0 B \). The discrete-time channel is normalized so that the average channel power gain is

\[ E \left\{ \sum_{l=0}^{L-1} |h(l, k)|^2 \right\} = 1. \]  

(2.17)

### 2.1.3 Example Channels

We use several channel models to analyze the robustness and performance of methods and algorithms that are presented in this work. The three channel models we mainly use are described in the following.

**AWGN Channel**

The simplest channel model is the *AWGN-channel*. It assumes a time-invariant channel without multipath propagation. The channel impulse response is given as

\[ h(\tau, t) = \delta(\tau), \]  

(2.18)

which leads to the discrete-time model

\[ g(k) = \sum_{l=\infty}^{\infty} \delta(l) s(k - l) + n(k) = s(k) + n(k). \]  

(2.19)

The receive signal is simply the transmitted signal plus additive noise as defined at the beginning of this section.

**WSSUS Channel Model**

As a second model, we use a very general time-variant channel model. Let us begin with the definition of the Doppler frequency \( f_D \). It is given as the frequency shift of a sinusoidal signal that can be observed at the receiver due to the movement of the receiver. This shift can be calculated as [61]

\[ f_D = f_{D_{\text{max}}} \cos \alpha, \]  

(2.20)

where \( \alpha \) is the angle of the incident wave relative to the movement direction of the mobile receiver. The maximum absolute value of the Doppler frequency is obtained for \( \alpha = 0 \) or \( \alpha = \pi \), i.e., if the direction of the incident wave is equal or opposite to the
direction of movement. The maximum Doppler frequency \( f_{D\text{max}} \) itself is depending on the receiver velocity \( v \), the propagation velocity \( c \) and the carrier frequency \( f_c \):

\[
f_{D\text{max}} = f_c \frac{v}{c}.
\] (2.21)

We assume the carrier frequency \( f_c \) to be large compared to the signal bandwidth \( B \), so that the Doppler frequency can be approximated as a constant within the bandwidth \( B \).

The channel model we use is based on the WSSUS channel model already mentioned before. As we are interested in mobile wireless channels, we assume a scenario, where several uncorrelated multipath components arrive at a moving receiver. For simplicity, we assume that the delay difference between the resolvable multipath components is the sampling time \( T \). Furthermore, each component is assumed to consist of many unresolved paths that arrive from all directions at the receiver, which leads to a Rayleigh distribution of the amplitudes of each path. The mobility of the receiver causes a time-variant CIR. In the stochastic channel model, this is modeled by Jakes spectra for all paths with maximum Doppler frequency \( f_{D\text{max}} \). In this case, the scattering function can be written as the product of the PDP and the Doppler power spectrum [61]:

\[
\varphi \Phi(l, \nu) = \varphi \varphi(\tau)|_{\tau = lT} \cdot \Phi \Phi(\nu),
\] (2.22)

where the Doppler power spectrum is given as [14]

\[
\Phi \Phi(\nu) \propto \frac{1}{\pi \sqrt{f_{D\text{max}}^2 - \nu^2}}, |\nu| < f_{D\text{max}}.
\] (2.23)

The corresponding discrete-time correlation function corresponding to the Doppler power spectrum above is given as [14]

\[
\Phi \varphi(\Delta t)|_{\Delta t = kT} \propto J_0(2\pi f_{D\text{max}}Tk),
\] (2.24)

where \( J_0(\cdot) \) is the zero-order Bessel function of the first kind.

**Two-Path Channel Model**

The third channel model is a deterministic two-path channel where a transceiver is moving with high velocity and communicating with a fixed base station. The first path is a direct LOS path which is Doppler shifted due to the movement of the transceiver. A second path is reflected at an object in the opposite direction of the base station. This causes a Doppler shift with opposite sign compared to the LOS path. Fig. 2.5 shows such a scenario, where a high speed train is leaving a tunnel and the signal is reflected at the tunnel entrance. Assuming equal path amplitudes \( A_1 \) and \( A_2 \), this scenario serves as a worst case scenario, as the channel experiences deep fades and
2.1 Basic Channel Models

Fig. 2.5: Scenario for the two-path channel. A receiver that is moving with high velocity receives the transmitted signal via a direct LOS path and a second path which is caused by a reflection behind the receiver.

\[ h(l, k) = h h(l, k) = A_1 \delta(l) e^{j2\pi f_{D_{\text{max}}} k T} + A_2 \delta(l - \Delta \tau / T) e^{-j2\pi f_{D_{\text{max}}} k T}. \]  

(2.25)

For an OFDM transmission with \( N \) subcarriers, we can assume periodic signals with periodicity \( N \). Therefore, we can apply the discrete Fourier transformation of size \( N \) with respect to \( l \) in order to obtain the time-variant transfer function

\[ H(m, k) = H h(m, k) = A_1 e^{j2\pi f_{D_{\text{max}}} k T} + A_2 e^{-j2\pi (f_{D_{\text{max}}} k T + \frac{m \Delta \tau}{N})}. \]  

(2.26)

Fig. 2.6 shows the PDF of the fading amplitudes \(|H(m, k)|\) for the two-path channel for equal path amplitudes. In contrast to the WSSUS channel model where the fading amplitudes follow a Rayleigh distribution, we obtain a PDF that has a finite nonzero value for \(|H(m, k)| \to 0\). This means that the probability of deep fades is quite high. The Doppler spread in this case is given as \( B_d = 2f_c \frac{v}{c} \). In (2.25) the common delay

Fig. 2.6: PDF of the fading amplitudes for the two-path channel with equal path amplitudes.
time was neglected as it has no influence on the basic behavior of the channel. Furthermore, it was assumed that the absolute delay times are large compared to change of the delay times caused by the transceiver movement, so that $\Delta \tau$ is not changing significantly within the observation time.

## 2.2 OFDM Transmission Model

The demand for high data rates in wireless communication systems is continuously increasing. If single carrier systems are used, this means that the symbol rate has to be increased and thus the symbol time gets smaller. The short symbol period leads to the problem that in multipath environments, the maximum delay time can be much greater than one symbol period, leading to severe ISI. The necessary equalizers to resolve the ISI can therefore be quite complex. One elegant solution to this problem is the use of multi carrier systems, where the data stream that is to be transmitted is divided into several substreams that are transmitted in parallel using different subcarriers at a lower rate. Therefore, a larger symbol period can be used. A very efficient implementation is OFDM, which is used in many wireless communication standards such as DAB, DVB-T, WLAN, WiMAX, and LTE. As the name says, the subchannels are chosen such that they are orthogonal and the frequency spacing is minimum. A cyclic prefix is used as a guard interval to avoid ISI and to maintain the orthogonality of the subchannels in time-dispersive environments. The subchannels experience flat fading and the equalization task is reduced to a multiplication of each subchannel with a complex factor. This behavior is due to the fact that sinusoidal functions are eigenfunctions of any linear time-invariant system [53]. To generate the transmit signal and to demodulate the receive signal, Weinstein et al. proposed to use the discrete Fourier transform (DFT) [93]. This makes the implementation of OFDM computationally efficient, because fast Fourier transform (FFT) algorithms can be employed. A more detailed description of the basics of OFDM can be found in, e.g., [60] and [34].

In this work, we will use a vector transmission model [22, 52] to describe the OFDM transmission, which is depicted in Fig. 2.7 and explained in the following. First, the data symbols are mapped to transmit symbols (MAP). We assume OFDM with $N$ subcarriers and a subcarrier spacing of $f_\Delta = 1/(TN)$. Therefore, the serial to parallel converter forms blocks of $N$ transmit symbols that are represented by the OFDM symbol vectors $x(k')$. The discrete Fourier transform can be represented as a matrix multiplication with the DFT matrix $F$, where the elements are defined as

$$[F]_{m,n} = \frac{1}{\sqrt{N}} e^{-j \frac{2\pi}{N} (m-1)(n-1)}, \quad m, n = 1, \ldots, N. \quad (2.27)$$

The modulation of the $N$ elements of the OFDM symbol vectors onto the $N$ subcarriers is achieved by an inverse discrete Fourier transform (IDFT), which can be mathematically expressed as a multiplication of $x(k')$ with the inverse of $F$:

$$s(k') = F^{-1} x(k'). \quad (2.28)$$
Each time domain OFDM symbol \( s(k') \) is extended by a cyclic prefix of \( N_G \) time domain samples to avoid interference between subsequent OFDM symbols. This operation can also be represented by a matrix multiplication:

\[
\mathbf{s}_{\text{cp}}(k') = \mathbf{G}_{\text{ap}} \mathbf{s}(k'),
\]

(2.29)

where

\[
\mathbf{G}_{\text{ap}} = \begin{pmatrix}
\mathbf{0}_{N_G \times (N-N_G)} & \mathbf{I}_{N_G}
\end{pmatrix}.
\]

(2.30)

In (2.30) \( \mathbf{0} \) denotes an all zero matrix and \( \mathbf{I} \) the identity matrix. After conversion to a serial stream of time domain samples, the signal is transferred to an analog signal by Dirac sampling and interpolation by a lowpass filter (IP-LP) with cutoff frequency \( f_g = \frac{1}{2T} \). The continuous time domain signal \( s(t) \) is then transmitted over the channel as described in Section 2.1.

After band limitation at the receiver side with \( f_g = \frac{1}{2T} \) in the receive lowpass filter (AA-LP), the signal is sampled and converted back to a parallel OFDM symbol stream. We assume that the noise after the receive filter can still be approximated as being white in the considered bandwidth. The inverse operations of the transmit side, i.e., removal of the cyclic prefix and demodulation by the DFT at the receive side, can again be represented by matrix multiplications:

\[
\tilde{x}(k') = \mathbf{F} \mathbf{G}_{\text{rp}} \mathbf{g}_{\text{cp}}(k'),
\]

(2.31)

where the matrix for removing the prefix is defined as

\[
\mathbf{G}_{\text{rp}} = \begin{pmatrix}
\mathbf{0}_{N \times N_G} & \mathbf{I}_N
\end{pmatrix}.
\]

(2.32)
The received OFDM symbols are then transferred to the detector (DET) for further processing in order to recover the transmitted data.

Let us have a closer look at the description of the channel. In the vector model, the influence of the channel can be described as a matrix vector convolution in the following form:

\[
g_{\text{cp}}(k') = \sum_{\kappa=-\infty}^{\infty} H(\kappa, k') s_{\text{cp}}(k' - \kappa) + n(k'),
\]

where \( n(k') \) denotes the noise after the receive filter and is assumed to be white in the considered bandwidth \( B \). \( H(\kappa, k') \) contains the time-variant CIR and has size \((N + N_G) \times (N + N_G)\). Throughout this work, we assume that \( L - 1 \leq N_G \), i.e., the cyclic prefix is long enough to avoid ISI. In this case, \( H(\kappa, k') \) is only nonzero for \( \kappa = 0 \) and \( \kappa = 1 \):

\[
H(0, k') = \begin{pmatrix}
    h(0, k') & 0 & \ldots & 0 \\
    (N + N_G) & h(0, k') & \ldots & 0 \\
    \vdots & \vdots & \ddots & \vdots \\
    h(L-1, k') & 0 & \ldots & 0 \\
    (N + N_G) + L - 1 & h(L-1, k') & \ldots & 0 \\
    \vdots & \vdots & \ddots & \vdots \\
    0 & \ldots & 0 & h(L-1, (k' + 1)) & h(0, (k' + 1)) \\
    (N + N_G) + L & \ldots & 0 & h(L-1, (k' + 1)) & \ldots & (N + N_G) + L - 1 \\
\end{pmatrix},
\]

\[
H(1, k') = \begin{pmatrix}
    0 & \ldots & 0 & h(L-1, k'(N+N_G)) & \ldots & h(1, k'(N+N_G)) \\
    \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
    \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
    \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
    0 & \ldots & 0 & 0 & \ldots & 0 \\
\end{pmatrix}.
\]

Please note that, in order to simplify the notation, the CIR \( h(l, k) \) in 2.34 also contains the influence of non-ideal transmit and receive filters. Combining (2.31), (2.33), (2.29), and (2.28) leads to

\[
\tilde{x}(k') = \sum_{\kappa=-\infty}^{\infty} F G_{\text{tp}} H(\kappa, k') G_{\text{ap}} F^{-1} x(k' - \kappa) + n(k').
\]
2.2 OFDM Transmission Model

The assumption of a cyclic prefix with \( L - 1 \leq N_G \) from above has also the effect that \( G_{rp} \mathbf{H}(1, k')G_{ap} \) equals a zero matrix, such that (2.35) can be simplified to a matrix product:

\[
\tilde{x}(k') = \mathbf{F} G_{rp} \mathbf{H}(0, k')G_{ap} \mathbf{F}^{-1}x(k') + n(k').
\]  

(2.36)

Defining the OFDM channel matrix \( \mathbf{H}_{OFDM}(k') = \mathbf{F} G_{rp} \mathbf{H}(0, k')G_{ap} \mathbf{F}^{-1} \), the whole OFDM transmission can therefore be modeled as a matrix-vector multiplication plus additive noise:

\[
\tilde{x}(k') = \mathbf{H}_{OFDM}(k')x(k') + n(k').
\]  

(2.37)

In the case of time-invariant channels, \( G_{rp} \mathbf{H}(0, k')G_{ap} = G_{rp} \mathbf{H}(0)G_{ap} \) in (2.36) is a circulant matrix which is diagonalized by the DFT [31]:

\[
\mathbf{F} G_{rp} \mathbf{H}(0)G_{ap} \mathbf{F}^{-1} = \text{diag}(\sqrt{N} \mathbf{F} (h(0), \ldots, h(L-1), 0_{1 \times N})^T) = \mathbf{H}_{OFDM}.
\]  

(2.38)

This means, the equivalent channel matrix for the OFDM transmission \( \mathbf{H}_{OFDM} \) is a constant diagonal matrix with the discrete channel transfer function

\[
[H_{OFDM}]_{m,m} = H(m) = \sum_{l=0}^{L-1} h(l) e^{-j \frac{2\pi}{N} lm}, m = 0 \ldots N - 1
\]  

(2.39)

on its diagonal. The diagonal structure of \( \mathbf{H}_{OFDM} \) shows that the orthogonality of the subchannels is maintained and no inter subchannel interference (ISCI) occurs.

**OFDM Model for Time-Variant Channels**

If the time variance of the channel is slow, so that the CIR is nearly constant within one OFDM symbol, i.e., \( h(l, k'(N + N_G)) \approx h(l, (k' + 1)(N + N_G) - 1) \), the OFDM channel matrix \( \mathbf{H}_{OFDM}(k') \) is approximately a diagonal matrix. The ISCI for such channels is still negligible but \( \mathbf{H}_{OFDM}(k') \) is not constant any more. Similar to (2.39) the diagonal elements contain the discrete channel transfer function

\[
[H_{OFDM}(k')]_{m,m} = H(m, k') = \sum_{l=0}^{L-1} h(l, k'(N + N_G)) e^{-j \frac{2\pi}{N} lm}, m = 0 \ldots N - 1,
\]  

(2.40)

that is depending on the OFDM symbol index \( k' \) now.

For fast time-variant channels, the CIR changes during an OFDM symbol period. It can be seen from (2.34) that the matrix \( G_{rp} \mathbf{H}(0, k')G_{ap} \) is not circulant, because \( h(l, k) \) changes with the absolute time sample index \( k \). Therefore, \( \mathbf{H}_{OFDM}(k') \) is not a
diagonal matrix any more. The time variance causes ISCI because the orthogonality of the subchannels is lost. The elements of $H_{\text{OFDM}}(k')$ can be written as

$$
H_{\text{OFDM}}(k')_{m,n} = \frac{1}{N} \sum_{p=0}^{N-1} \sum_{l=0}^{L-1} h(l, p + N_G + k'(N + N_G)) e^{-j \frac{2\pi}{N} (n-1)l} e^{j \frac{2\pi}{N} p(n-m)}.
$$

(2.41)

Using (2.41), the elements of the receive vector $\tilde{x}(k')$ in (2.37) can be split up in a wanted term, a second term which contains the ISCI, and the additive noise term:

$$
\tilde{x}(k')_m = [H_{\text{OFDM}}(k')]_{m,m} [x(k')]_m + \sum_{n=1}^{N} [H_{\text{OFDM}}(k')]_{m,n} [x(k')]_n + [n(k')]_m.
$$

(2.42)

It can be seen that the wanted part contains the transmitted symbol of the $m$-th subchannel, weighted with the corresponding diagonal element of the channel matrix. The ISCI part contains the sum of all symbols from the other subchannels, weighted with the off-diagonal elements of $H_{\text{OFDM}}(k')$. Fig. 2.8 shows a visualization of $H_{\text{OFDM}}(k')$ for a time-variant multipath channel. Like in (2.42), the diagonal elements represent the frequency selective channel transfer function, where we can observe that some subcarriers are strongly attenuated by fading. The off-diagonal elements represent ISCI.

If the OFDM channel matrix is known at the receiver, an equalizer can be used to remove the ISCI [38]. However, obtaining exact knowledge of the complete channel
2.3 Time Synchronization and Frequency Offset Estimation

Every transmission system needs a form of synchronization between the transmitter and the receiver. The receiver needs to recover the symbol clock in order to detect the transmitted data. For an OFDM transmission, the receiver needs to know the position of the OFDM symbols on the time axis in order to cut out the useful part of the OFDM symbol and remove the guard interval. This is usually referred to as timing recovery. Another task is to estimate the carrier frequency offset of the received signal compared to the receiver oscillator. Frequency offsets can result from mismatched oscillators or time-variant channels and have to be removed by the receiver. There is a vast amount of literature on synchronization of digital communication systems. For OFDM, time synchronicity has to be established before any processing is done in the frequency domain because it is necessary to choose the correct position of the receiver FFT window. Initial timing recovery and carrier frequency offset estimation can be

matrix may have prohibitive complexity [101], because for fast time-variant channels, the CIR is changing significantly even within a single OFDM symbol. Therefore, in practice, usually only the diagonal elements are estimated and used to equalize the receive symbols, while the ISCI is regarded as additional noise. Due to the central limit theorem, it is appropriate to assume this additive noise to be Gaussian for large \( N \) [69]. Another approach is to make certain assumptions about the channel in order to simplify the estimation of \( H_{\text{OFDM}}(k') \) and the equalization task. For example in [37], it is assumed that the channel variation is slow enough, so that the variations can be modeled by a linear change and the interference is limited to neighboring subcarriers.

In this work, we will focus on noncoherent detection and receivers that do not have exact knowledge of the channel matrix. Therefore we follow the approach of considering the ISCI as additional noise. For the WSSUS channel mentioned in Section 2.1.3, Russel and Stüber have derived the variance of the ISCI noise [69]. It is given as

\[
\sigma_{\text{ISCI}}^2 = E_s - \frac{E_s}{N^2} \left( N + 2 \sum_{n=1}^{N-1} (N - n) J_0 (2\pi f_{D_{\text{max}}} T n) \right),
\]

(2.43)

where \( E_s \) is the average symbol energy transmitted on each subchannel, assuming a normalized channel. Results for a channel with uniform Doppler power spectrum and for a two-path channel have been derived in [67]. In [51], Li and Cimini presented a general upper bound of the interference power for OFDM signals. It only depends on the maximum Doppler frequency and the subcarrier spacing and is given as

\[
\sigma_{\text{ISCI}}^2 \leq \frac{E_s}{12} \left( 2\pi f_{D_{\text{max}}}/f_{\Delta} \right)^2.
\]

(2.44)

For the two-path channel with equal path amplitudes presented in Section 2.1.3, this bound is tight.
done jointly by exploiting periodic segments of the time domain signal. Such periodic segments can be obtained by, e.g., transmitting pilot sequences that consist of repeated sub-sequences [65, 71]. Due to the cyclic prefix, OFDM signals already have a certain periodic structure. In [89] and [76], this property is used for synchronization so that no additional pilot overhead is needed in this phase.

Fig. 2.9 shows the structure of an OFDM signal. As described in the previous section, a part of each OFDM symbol is copied and added to the transmit signal as a cyclic prefix to avoid interference between subsequent OFDM symbols. However, the cyclic prefix usually contains interference from the previous OFDM symbol caused by multipath propagation (cf. Fig. 2.9). Therefore, only the part of the cyclic prefix that is not influenced by ISI is periodic and can be exploited for synchronization [42, 76]. A metric for the timing offset is given in [71] as

$$\Delta \hat{k} = \arg \max_{\Delta k} \frac{\left| \sum_{k=0}^{N_P-1} g^*(k + \Delta k)g(k + \Delta k + P) \right|^2}{\left( \sum_{k=0}^{N_P-1} |g(k + \Delta k + P)|^2 \right)^2} \quad (2.45)$$

where $N_P$ denotes the number of samples that are jointly checked for periodicity. If using the cyclic prefix of OFDM, $N_P$ must be less than $N_G - L + 1$ to avoid ISI, which would lead to an increase of the synchronization error [42]. $P$ is the distance between the periodic segments of the signal which means that $P = N$ if using the cyclic prefix of OFDM. To improve the synchronization accuracy, the metric can be extended to include several OFDM symbols (see, e.g., [42]).

The frequency offset can be estimated by evaluating the phase between the two periodic segments [71, 89]

$$\varphi_{offs} = \angle \left( \sum_{k=0}^{N_P-1} g^*(k + \Delta k)g(k + \Delta k + P) \right), \quad (2.46)$$

where $\angle(\cdot)$ denotes the angle of its argument. The estimated frequency offset is then
given as

\[ \hat{f}_{\text{offs}} = \frac{\hat{\phi}_{\text{offs}}}{2\pi PT} + \frac{n}{PT}, \quad (2.47) \]

where \( n \) is an arbitrary integer introduced due to the ambiguity of the phase. If we assume a limited frequency offset \( |f_{\text{offs}}| < 1/(2PT) \), then \( n = 0 \). This can be assured by an initial coarse frequency estimation or by resolving the ambiguity subsequently by using a special OFDM training symbol [71].

As long as the timing offset \( \Delta k \) is within the area A in Fig. 2.9, there will only be a phase offset depending on the OFDM subcarrier number \( n \) given by

\[ \theta_{\text{offs},n} = 2\pi n \frac{\Delta k}{N}. \quad (2.48) \]

If the timing offset is outside the area A, additional ISI will occur. This means that a timing offset that leads to a late FFT window as shown in Fig. 2.9 is worse than an early FFT window and should be avoided.

In order to avoid a mixing of the effects of time-variant channels and different synchronization methods, we assume perfect synchronization throughout this work. A possibly remaining frequency offset after synchronization can also be seen as part of the time-variant channel. Note that as long as the remaining timing offset only leads to a phase shift according to (2.48), OFDM-MFSK (cf. Section 3.1.1) with noncoherent detection is not affected at all. Only multiple symbol detection and hybrid modulation schemes that use the phase relation of neighboring subcarriers are affected.

### 2.4 Channel Coding

When we use OFDM to transmit over frequency selective and/or time-variant channels, the received signal power on the subcarriers can have large variations. This is referred to as fading. Some subcarriers can even be faded out completely at certain times. There are two main methods to combat the negative effects of fading. The first one is spreading. In this case, the transmit symbol energy is distributed over several subchannels in frequency and/or in time direction by a linear mapping. If several transmit antennas are used, the symbols can also be spread over several antennas (see, e.g., [100] and references therein). Distributing the transmit symbol energy over multiple independently fading subchannels introduces diversity, which can be exploited to reduce the error probability. However, spreading usually requires channel knowledge and an equalizer because the subchannel orthogonality is lost. The second method, which is the most common method used in practice today, is channel coding. While spreading is done on transmit symbol basis using a continuous range of values in \( \mathbb{R}/\mathbb{C} \), channel coding is performed on code symbol basis. In the most common case of binary codes, this means coding on bit level so that we obtain a discrete range in \( GF(2) \). A channel code adds redundancy to the transmitted information that allows the receiver to detect and correct transmission errors.
Convolutional Codes

In this work, we will use binary convolutional codes for channel coding. As the name says, the coding can be described by a convolution operation of $k_{\text{cod}}$ input sequences with the corresponding impulse response $g^{(i)}(k_q)$ of a linear time invariant (LTI) system to obtain $n_{\text{cod}}$ output sequences [11]:

$$c_j(k_q) = \sum_{i=1}^{k_{\text{cod}}} q_i(k_q) \ast g_i^{(j)}(k_q), \quad j = 1, \ldots, n_{\text{cod}}. \quad (2.49)$$

This leads to a code rate of $r = k_{\text{cod}}/n_{\text{cod}}$. The convolution operation is performed with a modulo 2 calculation for binary codes. Fig. 2.10 shows a convolutional encoder, where the convolution operation is implemented by a shift register. We have one input sequence ($k_{\text{cod}} = 1$) and two output sequences that are multiplexed to obtain the total output sequence. It can be seen, that one output value depends on the current input value and $m_C$ past input values, i.e., the code has memory $m_C$. The impulse responses are also called generator sequences and usually given in octal notation. In the example of Fig. 2.10 the coefficients of the impulse responses are $g^{(1)} = (1, 0, 1, 1, 0, 1, 1)$ and $g^{(2)} = (1, 1, 1, 1, 0, 0, 1)$, leading to $g^{(1)} = 133_8$ and $g^{(2)} = 171_8$ in octal notation. This example is a very common code, used for example in the IEEE standard 802.11a [1]. We will also use it in this work as our standard channel code. To allow code sequences with finite length, we terminate our code blocks by appending $m_C$ zeros to each information block.

One of the advantages of convolutional codes is the possibility to use reliability information in the decoding process. This is of particular importance as we intend to use iterative receivers that are based on processing reliability information. An efficient decoding algorithm is the BCJR algorithm [5] that can process and deliver reliability information. The BCJR algorithm is a symbol based maximum a posteriori probability algorithm, which leads to minimum symbol error probability.
2.5 Capacity of Time-Variant Multipath Channels

Interleaving
Convolutional codes are sensitive to burst errors. Especially in fading channels, the code symbol errors are in general not uncorrelated but occur in bursts when the channel is bad. To avoid such error bursts, convolutional codes are usually used in conjunction with interleavers. An interleaver distributes the code symbols over a large block, so that the errors appear uncorrelated in the receiver after deinterleaving. The interleaving pattern must be known to the receiver in order to recover the original ordering. In our work, we use a pseudo random interleaver. This means the code bits are distributed over a transmit block using a quasi random pattern.

Puncturing
To increase the flexibility of a certain convolutional code, it is possible to reduce the amount of redundancy that is transmitted by deleting some of the code symbols. This is called puncturing. By puncturing, the rate of the original mother code is increased, which makes it possible to transmit at a higher bandwidth efficiency if the channel conditions allow it. The advantage is, that both the encoder and the decoder remain unchanged. In particular, the decoding complexity remains also unchanged for the punctured code. At the receive side, the puncturing pattern has to be known and the punctured code symbols are simply inserted as dummy symbols. Because there is no information about these code symbols at the receiver before decoding, their reliability is set to zero. The puncturing patterns have to be chosen carefully and are usually found by a computer search [11]. As we are using the rate 1/2 code from the IEEE standard 802.11a, we also use the corresponding puncturing patterns given in [1].

2.5 Capacity of Time-Variant Multipath Channels

The capacity of time-variant multipath fading channels without channel knowledge at the transmitter and the receiver is in general unknown. There is no unique definition of the capacity of fading channels as for the AWGN channel that is applicable in all scenarios [85]. However, the topic can be approached under different constraints, which will be discussed in this section.

Let us start with a flat fading channel, where the channel coefficient $h$ is known to the receiver. Like in the rest of this work, we assume a normalized channel where $E\{|h|^2\} = 1$. The ergodic capacity is obtained by averaging the capacity over all channel realizations and can be expressed as a function of the average signal to noise ratio (SNR) at the receiver [85]:

$$C = E\left\{ \log_2(1 + |h|^2\text{SNR}) \right\} \text{bits/s/Hz.} \tag{2.50}$$

However, in a real scenario this definition of capacity is only meaningful, if it is possible to average over many independent realizations of the channel. This may be the case
for a fast fading channel or a frequency selective channel, that is divided into many parallel flat fading channels like in OFDM.

If the channel is time-invariant or very slowly fading, it is not useful to give a capacity that is averaged over many channel realizations. A small channel coefficient will lead to a low capacity over a long period of time. In this case, it is more useful to use the "outage capacity". It is defined as the largest spectral efficiency $\eta$ (in bits/s/Hz) where the instantaneous capacity of the channel is below that value with a certain probability $P_o(\eta) = \epsilon$, which is given as [85]

$$P_o(\eta) := \text{Prob} \left\{ \log_2(1 + |h|^2\text{SNR}) \leq \eta \right\}$$

$$= \text{Prob} \left\{ |h|^2 \leq \frac{2^\eta - 1}{\text{SNR}} \right\}.$$  \hfill (2.51)

For Rayleigh fading, $|h|^2$ has an exponential distribution and the outage probability is therefore given as

$$P_o(\eta) = 1 - e^{-\frac{2^\eta - 1}{\text{SNR}}}.$$  \hfill (2.52)

Solving (2.52) for $\eta$ leads to

$$C_\epsilon = \eta = \log_2 \left( 1 - \ln(1 - \epsilon) \text{SNR} \right).$$  \hfill (2.53)

For small $\epsilon$ this can be approximated by

$$C_\epsilon \approx \log_2 \left( 1 + \epsilon \text{SNR} \right).$$  \hfill (2.54)

In the high SNR region this can be further approximated by

$$C_\epsilon \approx \log_2 (\epsilon \text{SNR}).$$  \hfill (2.55)

If we compare this result to the capacity of an AWGN channel for high SNR, we can observe a constant difference of the capacities, depending on $\epsilon$:

$$C_\epsilon = C_{\text{AWGN}} - \log_2 \left( \frac{1}{\epsilon} \right).$$  \hfill (2.56)

Figure 2.11 compares the capacity of the AWGN channel with the ergodic capacity and the outage capacity of a Rayleigh fading channel. The channel coefficients are assumed to be known at the receiver. The outage capacity is plotted for an outage probability of $\epsilon = 0.1$ and $\epsilon = 0.01$, respectively. The constant difference in capacity to AWGN for high SNR is clearly visible.

So far we assumed perfect channel knowledge at the receiver. However, the channel coefficients are in general unknown. Especially for fast fading multipath channels it is the question, whether the channel can be properly estimated in order to transmit data using coherent transmission schemes. Another approach is therefore to calculate the capacity without a priori knowledge of the channel. The ergodic capacity (in bits/s)
of such a noncoherent time-variant multipath channel with coherence time $\Delta t_c$ and multipath spread $T_m$ was shown to be at least [80]

$$\left(1 - 2 \frac{T_m}{\Delta t_c}\right) \frac{P}{N_0} \log_2 e$$  \hspace{1cm} (2.57)

in the wideband limit ($B \to \infty$) and assuming no peak power constraint. This is called the power-limited case where $P$ denotes the average receive power and $N_0$ the spectral noise power density. If the coherence time is much larger than the multipath spread ($\Delta t_c \gg T_m$), the capacity is close to that of an infinite bandwidth AWGN channel [26, 80], which is given as

$$C_\infty = B \log_2 (1 + \text{SNR})_{B \to \infty} \approx B \left(\frac{P}{BN_0}\right) \log_2 e = \frac{P}{N_0} \log_2 e.$$  \hspace{1cm} (2.58)

To achieve this capacity for time-variant multipath fading channels, the signals must be "peaky" in time and frequency [80]. This condition is fulfilled by FSK with low duty cycle. However, due to the infinite bandwidth assumption and the lack of a peak power constraint, this is not a practical system and more of theoretical interest.

A closed form solution for the capacity of fading channels with finite bandwidth and peak power constraint has not been found yet. However, bounds on the capacity of such channels can be given [21, 70]. Let us assume an OFDM system with $N$ subcarriers and a cyclic prefix of $N_G$ samples. Using the same assumption of slowly time-variant channels as for (2.40), the CIR is assumed to be constant within an OFDM symbol so that ISCI can be neglected. The channel is assumed to be unknown a priori. Information about the channel coefficients must be extracted from the receive signal.
An upper bound on the achievable rate depending on the finite bandwidth $B$ is given as [21]:

$$C(B) \leq \frac{N}{(N+N_G)} B \log_2 \left( 1 + \frac{N+N_G}{N} \frac{P}{BH_0} \right)$$

$$- \frac{B}{\beta} \int_{\nu} \int_{\tau} \log_2 \left( 1 + \frac{\beta P}{BN_0} \varphi \Phi (\tau, \nu) \right) d\tau d\nu, \quad (2.59)$$

where $\beta$ is a parameter, limiting the ratio of the peak power of the transmit symbols $x_n(k')$ to the average transmit power. The first term in (2.59) is the capacity of an AWGN channel suffering from a loss due to the cyclic prefix of $N_G$ samples. The second term is a penalty considering the loss in capacity due to the channel uncertainty. The larger the spread of the scattering function $\varphi \Phi (\tau, \nu)$, i.e., the larger the multipath spread $T_m$ and Doppler spread $B_d$, the higher the channel uncertainty and therefore the loss in capacity.

For channels with larger time variance, the CIR cannot be considered constant during an OFDM symbol. The resulting ISCI can be approximated as additional noise with variance given in (2.43) for a Jakes Doppler spectrum. Using the same normalization as in (2.43) the average SNR at the receiver can be expressed as $\text{SNR} = \frac{P}{BN_0} = \frac{E}{N_0} = \frac{1}{\sigma_n^2}$. Taking into account the additional ISCI noise term, (2.59) can be rewritten as

$$C(B) \leq \frac{N}{(N+N_G)} B \log_2 \left( 1 + \frac{N+N_G}{N} \frac{1}{\text{SNR} + \sigma_{ISCI}^2} \right)$$

$$- \frac{B}{\beta} \int_{\nu} \int_{\tau} \log_2 \left( 1 + \frac{\beta}{\text{SNR} + \sigma_{ISCI}^2} \varphi \Phi (\tau, \nu) \right) d\tau d\nu. \quad (2.60)$$

By dividing (2.59) and (2.60) by the bandwidth $B$, the capacity bounds in bits/s/Hz can also be written as functions of the SNR. For (2.59) we obtain:

$$C \leq \frac{N}{(N+N_G)} \log_2 \left( 1 + \frac{N+N_G}{N} \text{SNR} \right)$$

$$- \frac{1}{\beta} \int_{\nu} \int_{\tau} \log_2 (1 + \beta \text{SNR} \varphi \Phi (\tau, \nu)) d\tau d\nu. \quad (2.61)$$

Let us also have a look at the case where we assume no knowledge about the fading coefficients at all. This can be modeled as a memoryless Rayleigh fading process, where the channel coefficients $h$ are independent. In this case, it is impossible to estimate the fading coefficient and transmit information at the same time. Therefore the receive symbols do not carry any phase information. Only the magnitude can be used to transmit data. The ergodic noncoherent capacity supremum of such a memoryless channel is given as [78]

$$C_{\text{nonc, sup}} = (\mu - \gamma - 1) \log_2 e + \log_2 \Gamma(\mu) - \mu \Psi(\mu) \log_2 e, \quad (2.62)$$
where $\mu$ can be calculated from

$$\text{SNR} = \mu e^{-\gamma - \Psi(\mu) - 1}.$$  \hfill (2.63)

In (2.62) $\Gamma(\cdot)$ is the gamma function, $\Psi(\cdot)$ denotes the digamma function, and $\gamma = -\Psi(1) \approx 0.5772$ is the Euler constant. For $\text{SNR} \to \infty$ we obtain a double logarithmic increase in capacity \cite{78}:

$$C_{\text{nonc,sup}} \simeq \log_2 (\log_2 (1 + \text{SNR})).$$  \hfill (2.64)

The capacity achieving input amplitude distribution of such channels is reported to be discrete with a finite number of mass points, where one of the mass points is zero \cite{2}. For low SNR this results in on-off keying, while for higher SNR more than two amplitude levels are optimal.

Fig. 2.12 shows the capacity bounds for WSSUS channels, assuming an OFDM system with sufficiently long guard interval. For comparison, the AWGN capacity is included both without and with the loss due to a cyclic prefix, if an OFDM system is used (dashed lines). A fixed cyclic prefix of 25% of the useful symbol time, i.e., $N_G/N = 1/4$ is assumed. The dashed lines with markers represent the capacity of a WSSUS channel under the assumption that the channel is constant during an OFDM symbol according to (2.61) and for the time-variant case where the ISCI is modeled as additional noise, respectively. It can be seen that the loss for the upper bound due to
channel uncertainties is quite small for the given parameters of an underspread channel with $f_{D_{\text{max}}}T_m = 0.01$. The capacity bound that includes the loss due to ISCI assumes a Jakes Doppler spectrum, where the maximum Doppler frequency normalized to the subcarrier distance is $f_{D_{\text{max}}}/f_\Delta = 0.064$. The solid line represents the noncoherent capacity supremum according to (2.62). We can see that in this case, where no knowledge about the current channel state is available, the capacity is significantly lower compared to the other cases for high SNR. Only for low SNR the capacity is similar. This motivates the use of FSK modulation for such channels, because FSK is known to be bandwidth inefficient but capacity achieving for high bandwidth or low SNR, respectively (cf. Section 3.1.1).
Robust Multi Carrier Transmission Methods

The performance of coherent detection methods highly depends on the quality of the channel estimate at the receiver. Especially in fast time-variant environments it is difficult to obtain a reliable channel estimate. The channel information is outdated very quickly and therefore the update rate for the channel estimate has to be adapted to the fading rate. This means the number of inserted pilot symbols has to be increased, which leads to a reduction of the useful data rate. In addition, the complexity of the channel estimation is growing because it has to be updated more frequently. One possibility to circumvent this problem is to use transmission methods that allow signal detection at the receiver without explicit channel knowledge. This is usually called noncoherent detection. Due to the fact that noncoherent detection does not rely on an exact channel estimate, such transmission methods are in general more robust against time variance of the channel.

The transmission schemes that we consider in this work are based on OFDM. As described in Section 2.2, a cyclic prefix can avoid ISI if it is longer than the maximum delay of the multipath channel. The energy contained in the cyclic prefix leads to a loss in energy efficiency so that the fraction of the cyclic prefix compared to the total OFDM symbol duration should be as small as possible. At the same time, the symbol duration should be as short as possible in order to minimize the changes of the channel during one OFDM symbol. This leads to a trade off for the number of subcarriers for a given system bandwidth. Therefore, during the design of the system the expected multipath spread $T_m$ and Doppler spread $B_d$ have to be considered and the OFDM parameters have to be chosen accordingly.

In the first section of this chapter, we present some modulation schemes that allow noncoherent detection and can be combined with multi carrier transmission. After this, an overview of iterative receivers that can be used for detection is given so that we are finally able to give a detailed model of the whole transmission system.
3.1 Modulation Schemes for Noncoherent Reception

Already at the beginning of the research about OFDM, it was proposed to use differential modulation to avoid the need for equalization. As originally proposed by Weinstein et al. [93], differential phase shift keying (DPSK) can be used in frequency direction. In this case, the data symbols are contained in the phase difference of neighboring subcarriers. The channel transfer function should be sufficiently flat, i.e., the channel coefficients of neighboring subcarriers should not differ significantly, to avoid a high error rate. Using the vector model (2.37) for OFDM, the components of the transmit vector $x(k')$ can be obtained as

$$[x]_i (k') = Ae^{\theta_i (k')},$$  \hspace{1cm} (3.1)

where

$$\theta_i (k') = \theta_i - 1 (k') + \Delta \theta_i (k').$$  \hspace{1cm} (3.2)

The phase differences $\Delta \theta_i (k')$ are chosen according to a phase shift keying (PSK) alphabet.

The second possibility is to apply DPSK in time direction (see, e.g., [55, 68] and references therein). The data symbols are now contained in the phase difference of subsequent transmit symbols on each subcarrier. In this case, the subcarrier phase is given as

$$\theta_i (k') = \theta_i (k' - 1) + \Delta \theta_i (k').$$  \hspace{1cm} (3.3)

The condition for the applicability of DPSK in time direction is that the variation of the channel transfer function between subsequent OFDM symbols should be small, i.e., the channel should be sufficiently slowly time-variant.

An extension to the OFDM-DPSK scheme is to use differential amplitude and phase shift keying (DAPSK) in combination with OFDM [23]. For large constellation size $M$, $M$-DAPSK outperforms $M$-DPSK and therefore OFDM-DAPSK is suited to increase the bandwidth efficiency of OFDM-DPSK by additionally using the amplitude relation of subsequent symbols.

For channels where it is unknown in advance whether the channel is more frequency selective or more time-variant, on-off keying (OOK) is a good choice. Information is transmitted by either sending an arbitrary symbol on a subcarrier or leaving it empty. The symbol alphabet can be written as $a_i \in \{0, e^{j\theta}\}$, where $\theta$ is an arbitrary symbol phase. The receiver makes an amplitude threshold decision on each subcarrier. Although the receiver does not need explicit channel knowledge for noncoherent detection, the threshold depends on the receive symbol energy and the noise variance, which have to be estimated.

Another possibility is to combine OFDM with $M$-ary frequency shift keying (MFSK). In this case, no knowledge about the channel is necessary at all. We presented modulation schemes based on OFDM-MFSK in [97, 98] and will explain the details in the following subsection.
3.1 Modulation Schemes for Noncoherent Reception

3.1.1 OFDM-MFSK

MFSK is a well known modulation scheme for digital communications [53, 66]. Information is transmitted by choosing from a set of $M$ orthogonal signal waveforms that differ in frequency. In the equivalent lowpass domain, the waveforms are defined as [66]

$$s_m(t) = Ae^{j2\pi mf\Delta t}, \quad m = 0, \ldots, M - 1, \quad 0 \leq t \leq T_s. \quad (3.4)$$

$A$ denotes a common amplitude factor and orthogonality is obtained for a frequency separation of $f\Delta = 1/T_s$, i.e.,

$$\int_{-\infty}^{\infty} s_m(t)s_n(t)dt = 0, \quad \text{for } m \neq n. \quad (3.5)$$

Let us look at the following discrete-time representation of (3.4) with sampling at $t = kT$ and $T = 1/(Mf\Delta)$ to fulfill the sampling theorem:

$$s_m(k) = Ae^{j\frac{2\pi}{M}mk}, \quad m, k = 0, \ldots, M - 1. \quad (3.6)$$

Comparing (3.6) with the model of the OFDM transmitter (2.28) in Section 2.2, we can see that the OFDM model can also be used for the signal generation of MFSK waveforms.

The number of subcarriers $N$ in an OFDM system with high data rate is usually large, so that the rate loss due to the cyclic prefix is only a small fraction of the useful symbol time, while the prefix is still long enough to cover all multipath components. The idea of OFDM-MFSK [97, 98] is therefore to combine OFDM and MFSK by splitting the $N$ OFDM subcarriers into groups of $M$ and to transmit information by assigning energy to one of the $M$ subcarriers in each group while leaving the other $M - 1$ subcarriers empty. Like this, OFDM-MFSK allows to transmit several MFSK symbols in parallel.

A related idea of using an OFDM transmitter to generate the transmit signal for FSK has been reported in [58] for application in an ultra wideband communication system. However, while in our case the FSK modulation is done by assigning energy to one subcarrier per group and the OFDM symbol duration is equal to the FSK symbol duration, the modulation in [58] is done based on assigning energy to subcarrier groups and a reduced FSK symbol duration.

Fig. 3.1 shows the principle of OFDM-MFSK, with $M = 4$ as an example. The OFDM subcarriers are segmented into groups of $M$ subcarriers. By choosing one out of $M$ subcarriers in each group, $m = \log_2 M$ bits of the sequence $c(kc)$ can be transmitted per group. In Fig. 3.1, the bit labels are plotted below each subcarrier. According to the transmit bits, energy is assigned to the corresponding subcarriers (solid lines), whereas the other subcarriers remain empty (dashed lines). In this example, bit labels of neighboring subcarriers within each group differ in only one bit. Although all subcarriers are orthogonal, i.e., all subcarriers have equal distance to each other in the signal space, we call such a mapping "Gray mapping". The reason for this is that the
transmission over channels with Doppler spread leads to symbol errors where wrong decisions are made in favor of neighboring subcarriers with higher probability. Gray mapping can minimize the number of bit errors under such conditions.

The transmission with OFDM-MFSK can be described using the vector model from Section 2.2:

\[
\tilde{x}(k') = H_{\text{OFDM}}(k')x(k') + n(k'). \tag{3.7}
\]

As described above, the elements of the OFDM transmit symbol vectors \( x(k') \) are segmented into groups of \( M \) elements, where one element in each group is nonzero and the others are zero. The following vector is an example for OFDM-4FSK again, where all \( N \) subcarriers are used, i.e., they are segmented into \( \frac{N}{4} \) groups:

\[
x(k') = \begin{pmatrix}
0, Ae^{j\theta_1(k')}, 0, 0, 0, 0, Ae^{j\theta_2(k')}, \ldots, 0, 0, 0, Ae^{j\frac{\theta_N}{4}(k')} \\
\text{group 1} & \text{group 2} & \ldots & \text{group} & \frac{N}{4}
\end{pmatrix}^T. \tag{3.8}
\]

Each group represents an MFSK symbol and \( A \) is an amplitude factor, which is assumed to be equal for all FSK symbols. For noncoherent reception, the receiver calculates the absolute value of all elements in \( \tilde{x}(k') \) and decisions are made corresponding to the highest value in each group. Hence, the phases \( \theta_n(k') \) in the transmit vector are arbitrary. The exact derivation of the receive metric will be given in Section 4.1.

The basic idea of OFDM-MFSK is to create a simple and robust transmission method, that combines the advantages of OFDM and MFSK. By using OFDM it is possible to implement a transmission with high data rate without ISI and hence without the need for a complicated equalizer. MFSK on the other hand can be received using noncoherent signal detection, making channel estimation obsolete. The application of OFDM-DPSK requires at least a little bit of knowledge about the channel, as the performance of OFDM-DPSK in frequency direction degrades for frequency selective channels and OFDM-DPSK in time direction is sensitive to fast time variance. OFDM-MFSK does
3.1 Modulation Schemes for Noncoherent Reception

not need any reference symbols and is therefore not degraded by channel variations between OFDM symbols or in frequency direction. Like all other schemes, OFDM-MFSK is influenced by the loss of subcarrier orthogonality due to a time variance within one OFDM symbol. Because for OFDM-MFSK the OFDM subcarriers can not really be regarded as separate subchannels any more, we use the term inter carrier interference (ICI) instead of ISCI for the interference caused by this loss of orthogonality. It is well known, that the bit error ratio (BER) of orthogonal MFSK improves for increasing \( M \). For \( M \to \infty \) it approaches the Shannon bound for the infinite bandwidth channel \([53, 66]\). This holds also true when MFSK is combined with OFDM.

The big drawback of OFDM-MFSK is its low bandwidth efficiency. In an uncoded system it is given as the average number of bits transmitted per second and Hertz:

\[
\eta = \log_2 M \frac{N}{M + N_G}. \tag{3.9}
\]

While the number of bits per MFSK symbol only grows logarithmically with \( M \), the occupied bandwidth grows linearly. So if \( M \) is increased, the bandwidth efficiency decreases. A trade off has to be found for \( M \). On the one hand the BER improves for increasing \( M \) while on the other hand the bandwidth efficiency tends to zero. The second factor in (3.9) accounts for the guard interval. During this time, no additional information is transmitted and the bandwidth efficiency is reduced accordingly, independent of the used modulation scheme. From (3.9) it can be seen that the bandwidth efficiency of OFDM-MFSK is at most \( \eta = 0.5 \) bits/s/Hz (for \( M = 2 \) and \( M = 4 \)). So these schemes are only suited to come close to capacity for low SNR and difficult channels (cf. Fig. 2.12).

One way to increase the bandwidth efficiency is the use of multitone FSK \([54]\) where \( N_{MT} \) subcarriers in a group of \( M \) are occupied. With this \( \lfloor \log_2 (\frac{M}{N_{MT}}) \rfloor \) bits per group can be transmitted. However, the power efficiency as well as the robustness against frequency selectivity is reduced.

3.1.2 The PAPR Problem

A fundamental drawback of OFDM is the high peak to average power ratio (PAPR) of the transmit signal. To avoid nonlinear distortion, which causes out-of-band radiation and in-band distortion, the transmit power amplifier must be operated in its linear region. For a transmit signal with high PAPR it is therefore necessary to use a large input backoff, i.e., the average input power of the power amplifier must be much lower than its saturation level. This in turn reduces the efficiency of the power amplifier and the transmission range. Another possibility is to reduce the PAPR before amplification. Clipping and filtering is such a method, where after limiting the amplitudes of the digital baseband signal, the arising out-of-band radiation is filtered out. This reduces large signal peaks but introduces in-band distortion which degrades the BER. In the literature, many methods for OFDM have been presented and examined that are used
3 Robust Multi Carrier Transmission Methods

to reduce the large PAPR of OFDM signals. In [33] an overview of such methods is given.

The reason for the high PAPR of multi carrier signals is the superposition of many, in general independently modulated, subcarriers. For unfavorable combinations of subcarrier phases, large peaks can occur in the transmit signal. For OFDM-MFSK with noncoherent symbol detection, the PAPR problem can be approached by choosing the subcarrier phases in the transmitter such, that the PAPR is minimum. The phases of the subcarriers of OFDM-MFSK do not carry any information, so adapting them in order to reduce the PAPR will neither influence the performance of OFDM-MFSK nor is it necessary to transmit any side information about the phase modification to the receiver. As PAPR reduction methods are not within the scope of this work, we would like to refer to [98, 99], where we examined several algorithms to reduce the PAPR of OFDM-MFSK.

3.1.3 Hybrid Modulation Schemes

Section 3.1.1 mentioned the low bandwidth efficiency of OFDM-MFSK as a major drawback. Apart from using the subcarrier phases for PAPR reduction as mentioned in the previous subsection, there is the possibility to use this degree of freedom for the transmission of additional information in order to increase the bandwidth efficiency. Also in this case, there is the advantage that the subcarrier phases can be used without influencing the noncoherent detection of the OFDM-MFSK symbols.

One possibility would be to use the subcarriers that are occupied by OFDM-MFSK and modulate the phase and/or amplitude according to a PSK or quadrature amplitude modulation (QAM) modulation scheme. Because a second modulation scheme is used on top of OFDM-MFSK, we call these “hybrid modulation schemes”. PSK and QAM modulation require coherent signal detection, so it is necessary to estimate the phase and amplitude reference of each subcarrier, i.e., channel estimation is needed.

A second possibility is the use of differential modulation schemes on top OFDM-MFSK. Like this, the signals can still be received noncoherently as in the case of plain OFDM-MFSK. In [97, 98] we presented such a hybrid modulation scheme, where the phases of the subcarriers that are occupied according to the OFDM-MFSK modulation are differentially modulated in frequency direction.

Our work was inspired by [40], where hybrid modulation schemes were proposed that combine \(M\)-ary Walsh symbols with additional DPSK or PSK modulation for the IS-95 system. In the past, also combinations of frequency and phase modulation were reported for single carrier transmission. In [28], Ghareeb analyzes the performance of noncoherent joint frequency-phase modulation (JFPM) where the phase of a carrier is differentially modulated in addition to the frequency. Korn and Wei discuss a combination of binary FSK and DPSK modulation with noncoherent detection for transmission over the satellite mobile channel [47, 92]. In [48] the author presents a hybrid modulation scheme called hybrid FSK-QAM modulation (HQFM) that combines FSK and coherent QAM modulation which is claimed to be applicable to OFDM. While in our
Fig. 3.2: Principle of OFDM-4FSK-4DPSK modulation (DPSK in frequency direction). The arrows indicate the phase of the occupied subcarriers. DPSK information is contained in the phase difference between the occupied subcarriers in neighboring groups.

system the tone separation of the MFSK frequencies corresponds to the OFDM subcarrier spacing $f_\Delta = 1/(NT)$, the FSK tone separation in [48] corresponds to the total OFDM bandwidth $1/T$.

Let us go into more detail about the hybrid modulation schemes used here in this work. Fig. 3.2 shows the principle for an example where OFDM-4FSK is combined with 4DPSK. The occupied subcarriers (gray squares) are determined according to the OFDM-MFSK scheme explained in Section 3.1.1. Like this, $\log_2 M$ bits of information can be transmitted over each group of subcarriers. The additional differential modulation of the subcarrier phases can now be applied in time or frequency direction. The only difference to conventional DPSK is that only the occupied subcarriers of the underlying OFDM-MFSK can be used. For modulation in frequency direction, as in the example in Fig. 3.2, the phase of the subcarrier in group one serves as a reference phase in each OFDM symbol and can be chosen to $\theta_1(k') = 0^\circ$. The phase difference $\Delta \theta_n(k') = \theta_n(k') - \theta_{n-1}(k')$ between the occupied subcarriers in neighboring groups is now used to transmit additional data by using a DPSK alphabet. Assuming $A = 1$,
the example transmit vector in (3.8) can be rewritten as

\[
x(k') = \begin{pmatrix} 0, 1, 0, 0, 0, 0, e^{j\Delta \theta_2(k')} \ldots, 0, 0, 0, e^{j\sum_{n=2}^{N} \Delta \theta_n(k')} \end{pmatrix}^T.
\]  

(3.10)

Alternatively, the additional DPSK modulation can be done in time direction. In this case, all the phases of the occupied subcarriers of the first OFDM symbol in a transmission block are used as reference symbols, for example \( \theta_n(1) = 0^\circ \). The information is transmitted using the phase differences \( \Delta \theta_n(k') = \theta_n(k') - \theta_n(k' - 1) \) of the subcarriers within one group between subsequent OFDM-symbols.

\[
x(1) = \begin{pmatrix} 0, 0, 1, 0, 1, 0, 0, \ldots, 0, 0, 0, 1 \end{pmatrix}^T.
\]

\[
x(2) = \begin{pmatrix} 0, e^{j\Delta \theta_1(2)}, 0, 0, e^{j\Delta \theta_2(2)}, 0, 0, \ldots, e^{j\sum_{i=2}^{N} \Delta \theta_N(2)} \end{pmatrix}^T.
\]

\[\vdots\]

\[
x(k') = \begin{pmatrix} e^{j\sum_{i=2}^{k'} \Delta \theta_1(i)}, 0, 0, 0, e^{j\sum_{i=2}^{k'} \Delta \theta_2(i)}, 0, 0, \ldots, 0, 0, 0, e^{j\sum_{i=2}^{k'} \Delta \theta_N(i)} \end{pmatrix}^T.
\]

(3.11)

The choice for differential modulation in time or frequency direction depends on the physical channel. If the coherence time is small compared to the coherence bandwidth, i.e., \( \Delta t_c/((N + N_G) \cdot T) \ll \Delta f_c/f_\Delta \), it is advantageous to use differential modulation in frequency direction. In the opposite case of high frequency selectivity and strong correlation in time direction, differential modulation in time direction is the better choice. However, for OFDM-MFSK-based hybrid modulation schemes, high frequency selectivity also degrades the differential modulation in time direction because in subsequent OFDM symbols the occupied subcarriers within one group can change.

The total bandwidth efficiency for an uncoded OFDM-MFSK-DPSK transmission is given as

\[
\eta = \log_2 M + \log_2 M_{\text{DPSK}} \cdot \frac{N}{N + N_G}.
\]

(3.12)
3.2 Iterative Receivers

In the receiver, the optimal solution in terms of BER would be to perform demapping of the receive symbols and decoding of the channel code jointly over a whole code block. In our case of bit interleaved coded modulation (BICM) with a convolutional code,
Fig. 3.4: Principle of iterative demapping and decoding (turbo detection).

a super-trellis could be used that is also accounting for the demapping. However, this would lead to a very high complexity. Especially if an interleaver is used, the complexity of joint demapping and decoding would become unfeasibly high. In many practical receivers, the two tasks are therefore performed separately. The demapper produces estimates for the code bits, possibly including reliability information, and afterwards, the decoder tries to recover the information bits. In general, this method is suboptimal, because the demapper cannot exploit the redundancy of the channel code. The performance of the receiver can be improved by feeding back information from the channel decoder to the demapper, where this information is used to improve the estimates for the code bits. Li and Ritcey [50] proposed such a scheme where the information is iterated between the demapper and the decoder and termed it BICM with iterative detection (BICM-ID).

Iterative techniques have a long history in wireless communications. In the early sixties, Gallager described the iterative decoding of low-density parity-check codes [25]. After the presentation of turbo codes [9], iterative methods became very popular and the turbo principle [32] was recognized as a general method in communication systems.

Fig. 3.4 shows the principle of turbo detection. In the demapper (DEMAP), the receive symbols $\tilde{x}$ are used to calculate reliability information for each bit. These reliability values are deinterleaved ($\Pi^{-1}$) and passed on to the decoder (DEC). The channel decoder uses the code constraints to improve the reliability for each code bit. This information is re-interleaved and fed back to the demapper, where it is used as a priori information. Like this, demapper and decoder can exchange information and iteratively improve the reliability, before hard decisions $\hat{q}$ on the information bits are made in the final step. To avoid a multiple reuse of the same information, only extrinsic information is exchanged during the iterations, i.e., only information that was gained in a device is forwarded by subtracting the input information. The source bit sequence $q(k_q)$ and the corresponding code bit sequence $c(k_c)$ are segmented into blocks. Channel coding is done over each block independently so that we can look at one single code block and write it as a vector $c$. The corresponding information bits are written as vector $q$. The vector $\tilde{x}$ in Fig. 3.4 contains the stacked elements of
all received OFDM symbols $\hat{x}(k')$ in one code block. It is segmented into subblocks $\hat{x}_D$ that contain the receive elements that are demapped jointly. In the case of OFDM-MFSK, these are usually the $M$ elements belonging to one subcarrier group. To simplify the notation, the block indices are left away.

A very useful measure of reliability is the log-likelihood ratio (LLR) or L-value. It is defined as the logarithmic ratio of the probabilities that the bit $c_j$ is a zero or a one, respectively:

$$L(c_j) = \ln \left( \frac{P(c_j = 0)}{P(c_j = 1)} \right). \tag{3.13}$$

Similarly, the a posteriori LLR of one code bit $c_j$, under the condition that $\hat{x}_D$ has been received, can be written as

$$L(c_j | \hat{x}_D) = \ln \left( \frac{P(c_j = 0 | \hat{x}_D)}{P(c_j = 1 | \hat{x}_D)} \right). \tag{3.14}$$

The a posteriori probability in (3.14) for a code bit $c_j$ being zero can also be represented by the summed probability of all transmit symbols $a_i$ in the set $S^0_j$, that contains the symbols having a zero at the corresponding position in the bit mapping. The probability for a code bit $c_j$ being one can be represented correspondingly:

$$P(c_j = 0 | \hat{x}_D) = \sum_{a_i \in S^0_j} P(x_D = a_i | \hat{x})$$

$$P(c_j = 1 | \hat{x}_D) = \sum_{a_i \in S^1_j} P(x_D = a_i | \hat{x}) \tag{3.15}$$

Using (3.15) and Bayes’ rule, (3.14) can be rewritten as

$$L(c_j | \hat{x}_D) = \ln \left( \frac{\sum_{a_i \in S^0_j} p(\hat{x}_D | x_D = a_i) P(x_D = a_i)}{\sum_{a_i \in S^1_j} p(\hat{x}_D | x_D = a_i) P(x_D = a_i)} \right). \tag{3.16}$$

$P(x_D = a_i)$ is the a priori probability that symbol $a_i$ was transmitted. This probability can be calculated using extrinsic feedback from previous decoding steps according to

$$P(x_D = a_i) = \prod_k P(c_k = c_{ka_i}), \tag{3.17}$$

where

$$P(c_k = 0) = \frac{\exp(L^\text{Dec}_k)}{1 + \exp(L^\text{Dec}_k)}$$

and

$$P(c_k = 1) = \frac{1}{1 + \exp(L^\text{Dec}_k)} \tag{3.18}$$
are the probabilities that the bits \( c_k a_i \) corresponding to the symbol \( a_i \) were transmitted. The vector \( \mathbf{L}^{\text{Dec}} \) contains the L-values calculated by the channel decoder in the previous iteration. In the first iteration this feedback information is not available and thus the a priori probability for all symbols is assumed to be equal. The first factor \( p(\tilde{x}_D | x_D = a_i) \) in (3.16) is the PDF of the receive vector \( \tilde{x}_D \) given the symbol \( a_i \) was transmitted. This factor depends on the channel and can be evaluated using the received symbol vector \( \tilde{x}_D \). The exact metric will be discussed in detail for different channel conditions, modulation alphabets, and detection methods in Chapter 4. Now we are able to calculate the L-values \( L(c_j | \tilde{x}_D) \) of all bits in the demapper. After subtraction of the a priori information, the vector of extrinsic L-values is denoted as \( \mathbf{L}^{\text{Dem}} \).

In the channel decoder, the BCJR algorithm [5] is used throughout this work to calculate the improved L-values \( \mathbf{L}^{\text{Dec}} \) that are used as a priori information for the demapper in the next iteration.

**Extrinsic Information Transfer Chart**

When looking at iterative receivers, there are some convenient tools to visualize the iterative behavior of the employed algorithms. The most important one is the extrinsic information transfer (EXIT) chart. It was introduced by ten Brink [81, 82] and it can be used to analyze the iterative exchange of mutual information between two components. For this, each component is simulated separately, using artificial a priori information. Then the characteristics of both components are plotted into one graph, considering the fact that in general the output of one component is the input of the other. Based on this graphical representation, the iterative behavior of the total system can be predicted without running time consuming Monte Carlo simulations.

Let us start with the demapper. It has two inputs: the received symbol vectors \( \tilde{x}_D \) and the a priori LLRs \( L(c_j) \). The mutual information between equiprobable bits \( c_j \) and the corresponding a priori LLR is given as [82]

\[
I_A^{\text{Dem}} = I(c_j, L(c_j)) = \frac{1}{2} \sum_{b \in \{0,1\}} \int_{-\infty}^{\infty} p_L(c_j) (\xi | c_j = b) \log_2 \left( \frac{2 p_L(c_j) (\xi | c_j = b)}{p_L(c_j) (\xi | c_j = 1) + p_L(c_j) (\xi | c_j = 0)} \right) d\xi.
\]

(3.19)

It is assumed that the a priori LLRs are independent and Gaussian distributed. Considering the fact that the a priori LLRs are the output of the channel decoder \( \mathbf{L}^{\text{Dec}} \) in the iterative receiver, the first property is quite well achieved by using a large interleaver. It was shown that also the assumption of Gaussian distributed output LLRs of the channel decoder is appropriate [82]. Therefore, the artificial a priori L-values can be modeled as a Gaussian variable according to

\[
L(c_j) = \frac{\sigma^2}{2} (1 - 2c_j) + n_j,
\]

(3.20)
where \( n_j \) is a zero mean Gaussian random variable with variance \( \sigma^2_A \) [82]. Using (3.19) and the Gaussian PDF for the L-values, the mutual information at the demapper input can be written as

\[
J(\sigma_A) = I_{A}^{\text{Dem}}(\sigma_A) = 1 - \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_A} e^{-\frac{(\xi - \frac{\sigma^2_A}{2})^2}{2\sigma^2_A}} \log_2 \left( 1 + e^{-\xi} \right) d\xi.
\]  

(3.21)

To generate the artificial a priori LLRs for a given mutual information, the inverse function \( \sigma_A = J^{-1}(I_A^{\text{Dem}}) \) of (3.21) is needed. Unfortunately, there is no analytical solution for \( J(\sigma_A) \) and its inverse. However, good numerical approximations [12, 83] can be used.

The mutual information \( I_{E}^{\text{Dem}} \) of the extrinsic L-values at the output of the demapper can be determined similar to (3.19):

\[
I_{E}^{\text{Dem}} = I(c_j, L_{j}^{\text{Dem}}) = \frac{1}{2} \sum_{b \in \{0, 1\}} \int_{-\infty}^{\infty} p_{L_{j}^{\text{Dem}}}(\xi|c_j = b) \log_2 \frac{2 p_{L_{j}^{\text{Dem}}}(\xi|c_j = b)}{p_{L_{j}^{\text{Dem}}}(\xi|c_j = 1) + p_{L_{j}^{\text{Dem}}}(\xi|c_j = 0)} d\xi.
\]  

(3.22)

The two conditional output PDFs in (3.22) can be obtained by Monte Carlo simulations and histogram measurements. We are now able to give the mutual information at the demapper output as a function of the a priori input: \( I_{E}^{\text{Dem}} = f(I_A^{\text{Dem}}) \). In addition, the output mutual information depends on channel parameters such as the SNR and fading properties.

To characterize the decoder, also artificial a priori LLRs according to (3.20) are used as input. The output mutual information \( I_{E}^{\text{Dec}} \) of the decoder is determined equivalently to the demapper using (3.19) and the PDFs of the output LLRs \( L_{j}^{\text{Dec}} \) that are obtained by the histogram method. This leads to \( I_{E}^{\text{Dec}} = f(I_A^{\text{Dec}}) \). Note that the interleavers do not change the mutual information. As the interleaved extrinsic output of the demapper is the a priori input of the decoder and vice versa, the two functions can be plotted into one chart.

Fig. 3.5 shows the EXIT chart for OFDM-4FSK with noncoherent detection for an AWGN channel as an example. The solid lines with markers represent the demapper characteristics for different values of \( E_b/N_0 \). From the positive slopes of the demapper characteristics, it can be seen that additional a priori information \( I_A^{\text{Dem}} \) at the demapper improves the extrinsic information at the output \( I_E^{\text{Dem}} \). The dashed line is the result of the channel decoder. In this case, the input of the decoder \( I_A^{\text{Dec}} \) is plotted on the vertical axis and the extrinsic output \( I_E^{\text{Dec}} \) on the horizontal axis. The standard convolutional code from Fig. 2.10 was used here. While for low a priori information \( I_A^{\text{Dec}} < 0.3 \) the output information is close to zero, the output information is close to one (full mutual information) for a priori information \( I_A^{\text{Dec}} > 0.8 \) with a quite abrupt transition in between. The longer the code memory \( m_C \), the more abrupt this transition. This
reflects also the property of convolutional codes regarding their error correction capability. While suffering from error propagation for very bad channels and thus even increasing the error ratio in the decoded sequence, most of the errors can be corrected for channels with less transmit errors. The higher the code memory, the steeper the transition from high to low BER if plotted versus $E_b/N_0$.

Also included in Fig. 3.5 is the trajectory, plotted as solid lines for the three different values of $E_b/N_0$. The trajectory visualizes the evolution of mutual information during the iterations. It is obtained by evaluating the mutual information at the output of both the demapper and the decoder using histograms and (3.19) during the iterative detection process. For this, the two components are not simulated separately with artificial a priori information, but really exchange extrinsic information in the iterative receiver. The plot shows that the trajectories follow the transfer curve of the decoder and the corresponding demapper curve quite well. From the EXIT chart it can be predicted if the iterative detection will improve the performance significantly. A tunnel between the decoder and demapper curve has to be open to allow the trajectory to move to the right side in several steps. In the example in Fig. 3.5, this is not the case for $E_b/N_0 = 5$ dB, where an early intersection point of the demapper and decoder curve causes the trajectory to be stuck after the first step. For $E_b/N_0 = 6$ dB however, it can be seen that the trajectory moves to the right side in several steps. For $E_b/N_0 \geq 7$ dB the trajectory almost reaches the right side after the second step. The number of steps of the trajectory in the EXIT chart determines the number of necessary iterations of the detector. The performance of iterative receivers for different modulation schemes and detection methods will be discussed in detail in Chapter 4.
3.3 Model for Coded Transmissions

Finally, we are able to give a detailed model of the total transmission system. Fig. 3.6 shows the overall vector-valued transmission model. Firstly, the source bit vector \( q_{\text{MFSK}} \), representing one information block, is encoded using a terminated convolutional code according to Section 2.4. Afterwards, the bits are interleaved (\( \Pi_{\text{FSK}} \)) and mapped to several OFDM transmit symbol vectors \( x_{\text{MFSK}}(k') \) using OFDM-MFSK, where \( k' \) denotes the OFDM symbol index. Optionally, we can use hybrid modulation schemes where a part of the source bits in \( q \) is transmitted using DPSK. These bits \( q_{\text{DPSK}} \) are also encoded and interleaved prior to the mapping to symbol vectors \( x_{\text{DPSK}}(k') \). The DPSK and MFSK symbol vectors are then combined to form the OFDM transmit symbol vectors \( x(k') \) as explained in Section 3.1.3. The vectors are transmitted to the receiver using an OFDM transmission which is represented by the vector valued model from Section 2.2. The receive vectors \( \tilde{x}(k') \) are then processed in the iterative receiver described in the previous section. In the case of hybrid modulation schemes, where two separate encoders have been used in the transmitter for the MFSK and DPSK components, also two decoders have to be used in the receiver. However, the demapping can be done jointly for both parts. The components of the iterative receiver exchange only extrinsic information. The necessary subtraction of the input L-values as shown in Fig. 3.4 is included in the demapper (DEMAP) and decoders (DEC) in Fig. 3.6.

In our transmission model, we assume a standard OFDM implementation where the OFDM symbols are cut out of the stream of time domain samples using a rectangular
window at the receiver. The rectangular window in time domain leads to a spectrum which has a $\sin(x)/x$ shape and is thus decaying quite slowly in frequency direction. This leads to relatively large ICI if the channel introduces a frequency shift or Doppler spread. By applying special window functions at the receive side, the amount of ICI can be reduced. For further information on windowing in the receiver and some example windowing functions, please refer to, e.g., [6, 63, 64] and references therein.
Noncoherent Signal Detection

In this chapter, we discuss the performance of noncoherent signal detection for several OFDM-MFSK-based transmission schemes that have been introduced in Section 3.1. Although the focus will be on coded transmission and iterative receivers, we will also give some analytical results and discuss the performance of uncoded transmission. Firstly, we derive the receive metrics for different channel models based on the PDFs of the receive signal vectors. Thereafter, the iterative detection of OFDM-MFSK [94] will be presented and examined. Several methods to improve the performance will be explained and evaluated. Multiple symbol detection is one of them and only comprises modifications at the receiver side. It will be discussed in Section 4.3 both for OFDM-MFSK and hybrid modulation schemes [96]. Extended mapping, explained in Section 4.5, is a method to adapt the bit mapping to the channel code in order to improve the performance of the iterative detection process. In Section 4.6, an additional recursive convolutional code is used to enhance the transmission scheme [95].

4.1 Receive Metrics for Noncoherent Detection

To derive the receive metric, we use a simplified OFDM channel model. The ICI caused by the time variance is modeled as additional noise as explained at the end of Section 2.2. Therefore, the OFDM channel matrices $H_{\text{OFDM}}(k')$ in (2.37) are diagonal matrices and so the dimensions of the system model can be reduced. For this, the received OFDM symbol vectors $\tilde{x}(k')$ are segmented into smaller vectors $\tilde{x}_D(k'')$ of length $N_D$ that contain the jointly demapped receive elements. The vector $x_D(k'')$ contains the corresponding transmitted elements of $x(k')$ and the diagonal OFDM channel matrices $H_{\text{OFDM}}(k')$ are split up into diagonal submatrices $H_D(k'')$, which leads to the following system model:

$$\tilde{x}_D(k'') = H_D(k'')e^{j\varphi(k'')}x_D(k'') + n_D(k'').$$

(4.1)

The index $k''$ is no longer a time index but a block index that refers to the jointly detected blocks. In addition, the phase $\varphi(k'')$, which is assumed to be unknown for
4 Noncoherent Signal Detection

noncoherent detection but common to all subcarriers in $H_D(k'')$, has been extracted. The noise vectors $n_D(k'')$ with variance $\sigma_{nD}^2$ contain both the thermal noise from $n(k')$ and the ICI, modeled as noise with variance $\sigma_{ICI}^2$ according to (2.43). As the two noise components are independent, we can write $\sigma_{nD}^2 = \sigma_n^2 + \sigma_{ICI}^2$. To simplify the notation, the block index $(k'')$ is left away in the following.

For the complex valued channel model in (4.1), the multivariate PDF of the receive vector $\tilde{x}_D$, given the transmit vector $x_D$ and phase $\varphi$, can be written as [41]

$$p(\tilde{x}_D|x_D, \varphi) = \frac{1}{\pi^{N_D} \det(\Lambda)} \exp \left( - (\tilde{x}_D - \bar{x}_D)^H \Lambda^{-1} (\tilde{x}_D - \bar{x}_D) \right),$$

(4.2)

where $\Lambda$ is the covariance matrix of the received vector $\tilde{x}_D$ and $\bar{x}_D$ is its mean value:

$$\Lambda = E \left\{ (\tilde{x}_D - \bar{x}_D) (\tilde{x}_D - \bar{x}_D)^H \right\}$$

$$\bar{x}_D = \bar{H}_D x_D e^{j\varphi}.$$  

(4.3)

The elements of $\Lambda$ can be calculated according to

$$[\Lambda]_{ij} = [x_D]_i [x_D]^*_j E \left\{ \left( [H_D]_{i,i} - [\bar{H}_D]_{i,i} \right) \left( [H_D]_{j,j} - [\bar{H}_D]_{j,j} \right)^* \right\} + \sigma_{nD}^2 \delta_{ij},$$

(4.4)

where

$$\delta_{ij} = \begin{cases} 1 & \text{for } i = j \\ 0 & \text{for } i \neq j \end{cases}$$

(4.5)

and $[H_D]_{i,i}$ are the diagonal elements of $H_D$ that represent the channel transfer function at the corresponding subcarrier frequency. $[\bar{H}_D]_{i,i}$ are the values on the diagonal of the mean channel matrix $\bar{H}_D$. We can also define a channel covariance matrix $\Lambda_H$, whose elements are determined by

$$[\Lambda_H]_{ij} = E \left\{ \left( [H_D]_{i,i} - [\bar{H}_D]_{i,i} \right) \left( [H_D]_{j,j} - [\bar{H}_D]_{j,j} \right)^* \right\}.$$  

(4.6)

Using this definition, we can write (4.4) in a different way:

$$\Lambda = \text{diag}(x_D) \Lambda_H \text{diag}(x_D)^H + \sigma_{nD}^2 I_{N_D}.$$  

(4.7)

We follow the derivations in [20], where a single carrier system was considered, and make the necessary adoptions to the case of OFDM. We evaluate (4.2) under the assumption of a uniform distribution of the unknown phase $\varphi$. This leads to

$$p(\tilde{x}_D|x_D) = \int_{-\pi}^{\pi} \frac{1}{2\pi} p(\tilde{x}_D|x_D, \varphi) d\varphi$$

$$= \frac{1}{\pi^{N_D} \det(\Lambda)} \exp \left( - \tilde{x}_D^H \Lambda^{-1} \tilde{x}_D - \tilde{x}_D^H \bar{H}_D \Lambda^{-1} \bar{H}_D \tilde{x}_D \right)$$

$$\cdot I_0 \left( 2 \tilde{x}_D^H \Lambda^{-1} \bar{H}_D x_D \right),$$

(4.8)
4.1 Receive Metrics for Noncoherent Detection

where \( I_0(\cdot) \) denotes the zero order modified Bessel function of the first kind. For an uncoded transmission with noncoherent detection and equiprobable transmit symbol vectors, \((4.8)\) can be used to make maximum likelihood (ML) decisions on the transmitted symbol vectors:

\[
\hat{x}_D = \arg \max_{a_i} p(\tilde{x}_D | x_D = a_i).
\]

(4.9)

The PDF in \((4.8)\) holds for general Gaussian channels whose properties can be described by \( \bar{H}_D \) and the channel covariance matrix \( \Lambda_H \). If we specify the channel conditions more precisely, the expression can be simplified. This will be done for different standard channels in the following subsections. Although noncoherent detection in principle assumes no channel knowledge, the applicability of the following receive metrics presumes that the corresponding channel conditions are met.

4.1.1 AWGN Channel

For the AWGN channel, the channel matrix \( H_D \) is fixed and equal to its mean:

\[
H_D = \bar{H}_D = I_{ND}.
\]

(4.10)

Therefore, we obtain \( \Lambda_H = 0_{ND \times ND} \), which leads to \( \Lambda = \sigma^2_{ND} I_{ND} \). Inserting these values into \((4.8)\), we obtain

\[
p(\tilde{x}_D | x_D) = \frac{1}{\pi^{ND} \sigma^2_{ND}} \exp \left( -\frac{\tilde{x}_D^H \tilde{x}_D}{\sigma^2_{ND}} - \frac{x_D^H x_D}{\sigma^2_{ND}} \right) I_0 \left( \frac{2}{\sigma^2_{ND}} |\tilde{x}_D^H x_D| \right).
\]

(4.11)

In the case of transmit vectors \( x_D = a_i \) with equal energy, i.e., \( a_i^H a_i = ||a_i||^2 = \text{const.} \forall i \), the first factors in \((4.11)\) are independent of the transmit vector \( x_D = a_i \) and we can write

\[
p(\tilde{x}_D | x_D) \propto I_0 \left( \frac{2}{\sigma^2_{ND}} |\tilde{x}_D^H x_D| \right)
\]

(4.12)

In addition, \( I_o(\cdot) \) is a monotonically increasing function of its argument, so that the ML decision in \((4.9)\) reduces to

\[
\hat{x}_D = \arg \max_{a_i} |\tilde{x}_D^H a_i|.
\]

(4.13)

4.1.2 Rayleigh Channel

In the case of a Rayleigh flat fading channel, the channel matrix has zero mean: \( \bar{H}_D = 0_{ND \times ND} \). The fading coefficients on all subcarriers are fully correlated, so that the channel covariance matrix is given as \( \Lambda_H = \sigma^2_h 1_{ND \times ND} \), where \( \sigma^2_h = E \{ |h(k)|^2 \} = 1 \)
4 Noncoherent Signal Detection

is the variance of the Rayleigh coefficient that is normalized according to (2.17). This leads to

$$\Lambda = x_D x_D^H + \sigma_n^2 I_{N_D},$$  (4.14)

where $x_D x_D^H$ is the dyadic product of the transmit vectors. If we use the Sherman-Morrison-Woodbury formula [31] for matrix inversion of $\Lambda$, (4.8) simplifies to

$$p(\tilde{x}_D|x_D) = \frac{1}{\pi^{N_D} \sigma_n^{2 N_D} \left( 1 + \frac{x_D^H x_D}{\sigma_n^2} \right)} \cdot \exp \left( -\frac{x_D^H \tilde{x}_D}{\sigma_n^2} + \frac{1}{\sigma_n^4 \left( 1 + \frac{x_D^H x_D}{\sigma_n^2} \right)} x_D^H x_D x_D^H \tilde{x}_D \right).$$  (4.15)

Assuming again the case of transmit vectors with equal energy, we can write

$$p(\tilde{x}_D|x_D) \propto \exp \left( c |\tilde{x}_D^H x_D|^2 \right),$$  (4.16)

with the constant factor

$$c = \frac{1}{\sigma_n^4 \left( 1 + \frac{x_D^H x_D}{\sigma_n^2} \right)}.$$  (4.17)

The ML decision rule for a Rayleigh flat fading channel is equivalent to the one for an AWGN channel:

$$\hat{x}_D = \arg \max_{a_i} |\tilde{x}_D^H a_i|^2 = \arg \max_{a_i} |\tilde{x}_D^H a_i|.$$  (4.18)

4.1.3 WSSUS Channel

The WSSUS channel model, introduced in Section 2.1.3, actually includes the above Rayleigh channel as a special case of one resolvable propagation path. Therefore, the following results can be seen as an extension from flat fading to frequency selective channels. Although strictly seen, the uncorrelated scattering assumption is in general not fulfilled for fast time-variant channels with limited bandwidth, we still assume this property for our derivation, as it is a good approximation if the Doppler spread is small compared to the bandwidth of the receive filter, i.e., $B_d/B << 1$ [75].

Like for the Rayleigh model, the channel matrix for the WSSUS model has zero mean: $H_D = 0_{N_D \times N_D}$. The channel covariance matrix can be calculated according to

$$\Lambda_H = NF_L \Lambda_h F_L^H,$$  (4.19)
where $\mathbf{F}_L$ is a matrix of size $N_D \times L$, comprising the first $N_D$ rows and $L$ columns of the DFT matrix $\mathbf{F}$. $\Lambda_h$ is the time domain channel covariance matrix. For a WSSUS channel, it is a diagonal matrix and given as

$$\Lambda_h = \begin{pmatrix} 
\varphi \varphi(0 \cdot T) & 0 \\
0 & \ddots \\
0 & \varphi \varphi((L-1) \cdot T) 
\end{pmatrix},$$

(4.20)

which is defined by the PDP from (2.22) and normalized to $\sum_{l=0}^{L-1} \varphi \varphi(l \cdot T) = 1$. In this case, the receive PDF is given as

$$p(\tilde{x}_D| x_D) = \frac{1}{\pi N_D \det (\Lambda)} \exp \left(-\tilde{x}_D^H \Lambda^{-1} \tilde{x}_D \right).$$

(4.21)

Inserting (4.20) and (4.19) into (4.7), the receive PDF can be evaluated. However, without specifying the transmit symbol vectors $x_D = a_i$ more closely, no further simplifications are possible.

### 4.2 Single Symbol Detection

In the previous section we have derived the PDF of the receive vector and the corresponding ML detection rule for different channel constraints. Now we will apply these results to the modulation schemes we have introduced in Sections 3.1.1 and 3.1.3. Let us start with OFDM-MFSK, where the detection is implemented based on single MFSK symbols. An OFDM-MFSK symbol vector comprises $M$ elements that correspond to the subcarriers of one group, as specified in Section 3.1.1. The set of possible transmit symbol vectors also contains $M$ elements $a_1 \ldots a_M$. Table 4.1 shows the transmit symbol vectors for $M = 4$. Due to the noncoherent detection, the absolute phase of the subcarrier elements is unknown in the receiver. Without influencing the detection process, it has been chosen to $0^\circ$, so that $a_1$ represents an occupied subcarrier. Table 4.1 also includes two possible bit mappings. For Gray mapping, the bit label of transmit symbols that occupy neighboring subcarriers, differs in only one bit. In the case of anti-Gray mapping, neighboring transmit symbols differ in a maximum number of bits. In the case of time-invariant channels, the orthogonality of all subcarriers is maintained. Of course, the mapping has no influence on the BER in this case. However, if the orthogonality of the subcarriers is lost (e.g. due to a time-variant channel), wrong decisions on symbols occupying neighboring subcarriers are more probable and so the mapping influences the BER.
4.2.1 Performance of OFDM-MFSK

Uncoded Transmission

For an uncoded transmission over an AWGN channel using OFDM-MFSK and ML detection according to the metric derived in Section 4.1.1, the BER performance is equivalent to conventional orthogonal MFSK. Fig. 4.1 shows the bit error curves for different $M$. This is the general result for $M$-ary transmission with orthogonal signals that can be found in most standard text books (e.g. [53, 66]). As mentioned in Section 3.1.1, there is a trade off between power and bandwidth efficiency. A good compromise is OFDM-4FSK, which has the same bandwidth efficiency as OFDM-2FSK but improved power efficiency.

In Section 3.1 we also mentioned OOK as a suited modulation scheme for noncoherent transmission. Its performance for the AWGN channel is also included in Fig. 4.1. Due to the fact that the possible transmit symbols have unequal energy, we have to use the general PDF (4.11) in the ML decision metric (4.9). Inserting the symbol alphabet $a_i \in \{0, e^{j\theta}\}$ leads to the following AWGN decision rule for OOK:

$$\exp \left( -\frac{1}{\sigma_{nD}^2} \right) I_0 \left( \frac{2}{\sigma_{nD}^2} |\hat{x}_D| \right) < 1,$$

(4.22)

where the receiver decides for $\hat{x}_D = 0$ if (4.22) is true and $\hat{x}_D = e^{j\theta}$ otherwise. This decision rule is equivalent to the optimal metric derived in [46] for equiprobable transmit symbols. The advantage of OOK compared to MFSK is its higher bandwidth efficiency of one bit per subcarrier. However, Fig. 4.1 shows the reduced power efficiency compared to MFSK for $M > 2$. A second drawback is the necessity of estimating some channel parameters in order to determine the decision threshold or to normalize the channel as in (4.22), respectively.

For comparison, the result for binary phase shift keying (BPSK) has been included in Fig. 4.1. However, we have to keep in mind that this is a modulation scheme that needs coherent detection. If an OFDM system with cyclic prefix is used, there is a shift in $E_b/N_0$ according to the additional energy that is used for the cyclic prefix.

Table 4.1: Transmit symbol alphabet for OFDM-4FSK with Gray and anti-Gray mapping.
4.2 Single Symbol Detection

Fig. 4.1: BER vs. $E_b/N_0$ for uncoded OFDM-MFSK with noncoherent detection and transmission over the AWGN channel. $N_G = 0$ (no cyclic prefix). OOK assumes noncoherent detection and perfect knowledge of the signal and noise energies. The BPSK reference curve assumes coherent detection.

Coded Transmission

Fig. 4.2 shows the performance for coded OFDM-MFSK transmission using the convolutional code from Fig. 2.10. For all simulations, one code block comprises 100 OFDM symbols, each containing $N = 256$ subcarrier elements. A cyclic prefix with relative length $N_G/N = 0.25$ is used to achieve comparability with the results in other sections. This causes a shift of the error curves of $(N + N_G)/N = 1.25 \triangleq 0.97$ dB. In the receiver, L-values are calculated according to (3.16), where the receive PDF $p(\tilde{x}_D|x_D)$ is evaluated using the AWGN metric from (4.12). The a priori probabilities for all symbol vectors are assumed to be equal and thus cancel out in (3.16) so that we obtain

$$L(c_j|\tilde{x}_D) = \ln \left( \frac{\sum_{a_i \in S_j^0} p(\tilde{x}_D|x_D = a_i)}{\sum_{a_i \in S_j^1} p(\tilde{x}_D|x_D = a_i)} \right)$$

$$= \ln \left( \frac{\sum_{a_i \in S_j^0} I_0 \left( \frac{2}{\sigma_{nD}} |\tilde{x}_D a_i| \right)}{\sum_{a_i \in S_j^1} I_0 \left( \frac{2}{\sigma_{nD}} |\tilde{x}_D a_i| \right)} \right). \quad (4.23)$$

For high SNR values, the Bessel functions in (4.23) can take very large values, leading to numerical problems. This can be prevented by approximating $I_0(\xi)$ by $\frac{1}{\sqrt{2\pi\xi}} \exp(\xi)$
4 Noncoherent Signal Detection

Fig. 4.2: BER vs. $E_b/N_0$ for coded OFDM-MFSK with noncoherent detection and transmission over the AWGN channel. $N_G/N = 0.25$; convolutional code: $r = 1/2$, $m_C = 6$. The BPSK reference curve assumes coherent detection.

Using the equivalence

$$\ln \left( \sum_i e^{\xi_i} \right) = \max_i \{\xi_i\} + \ln \left( \sum_n e^{-|\xi_n - \max_i \{\xi_i\}|} \right) \approx \max_i \{\xi_i\}. \tag{4.24}$$

The last approximation can be made for high SNR because the exponential function is a fast growing function of its argument. It is often called the max-log approximation.

With this, (4.23) can be written as

$$L(c_j | \bar{x}_D) \approx \max_{a_i \in S_j^0} \left\{ \frac{2}{\sigma_n^2} |\bar{x}_D^H a_i| - \frac{1}{2} \ln |\bar{x}_D^H a_i| \right\} \max_{a_i \in S_j^1} \left\{ \frac{2}{\sigma_n^2} |\bar{x}_D^H a_i| - \frac{1}{2} \ln |\bar{x}_D^H a_i| \right\}$$

$$\approx \max_{a_i \in S_j^0} \{|\bar{x}_D^H a_i|\} - \max_{a_i \in S_j^1} \{|\bar{x}_D^H a_i|\} \tag{4.25}$$

Although the calculation of the LLRs actually would require the knowledge of the SNR at the receiver, the common factor $\frac{2}{\sigma_n^2}$ has been left away in the second approximation of (4.25), as simulations have shown only little influence on the detection result. This confirms similar findings in [90].

The most interesting scenario for noncoherent transmission schemes is the transmission over a time-variant channel. Fig. 4.3 shows the performance of coded OFDM-4FSK and OFDM-8FSK for transmission over the two-path channel with equal path amplitudes as defined in Section 2.1.3. The results are given for the time-invariant case where the normalized Doppler spread is $B_d/f_\Delta = 0$ and a time-variant channel with $B_d/f_\Delta = 0.135$. We can see that OFDM-MFSK is quite robust against time variance.
4.2 Single Symbol Detection

![Graph](image)

Fig. 4.3: BER vs. $E_b/N_0$ for coded OFDM-MFSK with noncoherent detection and transmission over a two-path channel. Convolutional code: $r = 1/2$, $m_C = 6$; $N_G/N = 0.25$; Gray mapping.

and the robustness is increasing with modulation order $M$. As introduced in (2.44), the ICI noise $\sigma^2_{ICL}$ can be approximately bounded by

$$\sigma^2_{ICL} \leq \frac{E_s}{12 \cdot M} \left(2\pi f_{D\text{max}}/f_\Delta\right)^2 \quad (4.26)$$

in the case of OFDM-MFSK, if we assume that the disturbing ICI noise is $M$ times smaller compared to an OFDM system where all subcarriers are occupied. This leads to a signal to interference ratio (SIR) of

$$\frac{E_b}{\sigma^2_{ICL}} \geq \frac{E_s}{\log_2 M \cdot \frac{N+N_G}{N}} \cdot \left(2\pi (B_d/f_\Delta) / 2\right)^2 \quad (4.27)$$

For OFDM-4FSK we obtain $\frac{E_b}{\sigma^2_{ICL}} \geq 25.2$ dB and for OFDM-8FSK $\frac{E_b}{\sigma^2_{ICL}} \geq 26.5$ dB for the two-path channel with $B_d/f_\Delta = 0.135$. This confirms the simulation results in Fig. 4.3 which indicate that the ICI noise is not dominating the BER for medium $E_b/N_0$.

In Fig 4.3, we can observe a reduced slope of the BER curves even for the time-invariant case. An explanation for this effect is the PDF of the receive amplitudes of the two-path channel. In contrast to Rayleigh or WSSUS fading, where the fading amplitudes are Rayleigh distributed, the fading amplitude distribution of the two-path channel has a shape similar to the Jakes spectrum (cf. Fig. 2.6) with a nonzero probability for deep fades (i.e. amplitude zero).
Due to the nonzero delay difference $\Delta \tau$, the two-path channel is frequency selective. However, the delay difference, and therefore the coherence bandwidth $\Delta f_c \approx 1/\Delta \tau$, does not influence the performance for single symbol detection, because only one subcarrier of each symbol is occupied. This is also confirmed by the fact that the PDFs for Rayleigh flat fading (4.16) and WSSUS channels (4.21) coincide in this case. Following similar steps as for the AWGN channel, the simplified metric to calculate the LLRs can be written as

$$L(c_j|\tilde{x}_D) \approx \max_{a_i \in S_j^0} \{|\tilde{x}_D^H a_i|^2\} - \max_{a_i \in S_j^1} \{|\tilde{x}_D^H a_i|^2\}. \quad (4.28)$$

Because we assume no channel knowledge at the receiver, this metric was also used to obtain the results in Fig. 4.3.

### 4.2.2 Iterative Detection

The basics of iterative detection have been introduced in Section 3.2. The a priori probabilities to calculate the LLRs in the demapper are obtained according to (3.17) using the extrinsic information that is fed back from the decoder. In contrast to the derivations in the previous section, the a priori probabilities in (3.16) are not equal after the first decoding step and thus do not cancel out. For the AWGN channel, we can use the PDF from (4.12) so that the metric for OFDM-MFSK considering the a priori information is given as

$$L(c_j|\tilde{x}_D) = \ln \left( \frac{\sum_{a_i \in S_j^0} I_0 \left( \frac{2}{\sigma_{nD}^2} |\tilde{x}_D^H a_i| \right) P(x_D = a_i)}{\sum_{a_i \in S_j^1} I_0 \left( \frac{2}{\sigma_{nD}^2} |\tilde{x}_D^H a_i| \right) P(x_D = a_i)} \right)$$

$$= \ln \left( \frac{\sum_{a_i \in S_j^0} I_0 \left( \frac{2}{\sigma_{nD}^2} |\tilde{x}_D^H a_i| \right) \prod_{k=1}^{\log_2 M} \exp \left( L_{Dec}^k (1 - c_k a_i) \right)}{\sum_{a_i \in S_j^1} I_0 \left( \frac{2}{\sigma_{nD}^2} |\tilde{x}_D^H a_i| \right) \prod_{k=1}^{\log_2 M} \exp \left( L_{Dec}^k (1 - c_k a_i) \right)} \right). \quad (4.29)$$

For $M = 2$, the product terms in (4.29) contain only one element, which is determined by the a priori L-value of the considered bit itself. However, due to the fact that only extrinsic information is passed to the decoder, this a priori information is subtracted at the output of the demapper according to Fig. 3.4 and iterative detection cannot improve the performance. This is also the intuitive result, because a 2FSK symbol is only determined by one bit. Knowledge about the other bits does not improve the reliability in the demapper. For MFSK with $M > 2$, more than one code bit is assigned to each MFSK symbol. In this case, the redundancy introduced by channel coding can be exploited in the demapper. In [87,88] a similar iterative detection scheme for noncoherent single carrier MFSK is presented for a system using turbo codes.
4.2 Single Symbol Detection

For Rayleigh fading channels, the L-value can be calculated using the corresponding PDF from (4.16) and we obtain

\[
L(c_j | \tilde{x}_D) = \ln \left( \frac{\sum_{a_i \in S_j^0} \exp \left( c|\tilde{x}_D^H a_i|^2 \right) \prod_{k=1}^{\log_2 M} \exp \left( L_{\text{Dec}}^k \left( 1 - c_k a_i \right) \right)}{\sum_{a_i \in S_j^1} \exp \left( c|\tilde{x}_D^H a_i|^2 \right) \prod_{k=1}^{\log_2 M} \exp \left( L_{\text{Dec}}^k \left( 1 - c_k a_i \right) \right)} \right),
\]

with the constant \( c \) as defined in (4.17).

From (4.29) and (4.30) we can see that the receiver needs an estimate of the noise variance \( \sigma^2_{nD} \) to calculate the L-values and the average receive signal power to normalize the channel. Simulations have shown, that the BER performance is quite insensitive to errors in estimating these values. In our simulations we therefore estimated these values by taking the average of the strongest subcarrier in each group as the signal power and the average energy on all other subcarriers represents noise. The averaging is done over the complete code block.

**Performance of Iterative Detection for the AWGN Channel**

Fig. 4.4 shows the BER curves of OFDM-4FSK and OFDM-8FSK for different numbers of iterations in the receiver for transmission over an AWGN channel. The dashed curves represent the result after the first decoding step which is equivalent to the result for non iterative detection from Fig. 4.2. It can be seen that for OFDM-4FSK for high \( E_b/N_0 \) after one iteration an improvement of about 0.7 dB can be achieved. A further
increase in the number of iterations only improves the performance in the region of medium $E_b/N_0$. Similar observations can be made for OFDM-8FSK. However, the performance gain for high $E_b/N_0$ is almost 1.5 dB in this case and the region where more than one iteration can reduce the number of errors is larger.

**EXIT Chart Analysis for the AWGN Channel**

Let us use the EXIT chart tool that was introduced in Section 3.2 to investigate the behavior of the iterative detection process. The EXIT charts for OFDM-4FSK and OFDM-8FSK are plotted in Fig. 4.5. The chart for OFDM-4FSK was already used as an example in Section 3.2. Now we can use it to explain the BER curves of Fig. 4.4. We can clearly see the reason why the BER of OFDM-4FSK for $E_b/N_0 \leq 5 \text{ dB}$ is almost the same for all iterations: There is no open tunnel between the demapper and the decoder curve, and therefore, the trajectory gets stuck after the first decoding step at a very low value for the output information. The intersection point of the demapper and the decoder curve marks the maximum of mutual information that can be achieved with iterative detection. In the region $5 \text{ dB} < E_b/N_0 < 7 \text{ dB}$, the BER of OFDM-4FSK can be reduced using several iterations. This region is sometimes called the waterfall region, because for a sufficient number of iterations, a small increase in $E_b/N_0$ leads to a large drop in the error rate. The corresponding behavior in the EXIT chart is represented by the trajectory for $E_b/N_0 = 6 \text{ dB}$. Now a narrow tunnel between the demapper and the decoder curve is open and the trajectory moves from the left to the right in
several steps. By counting the number of steps we can also estimate the necessary number of iterations. In the case of $E_b/N_0 = 6$ dB, the mutual information does not significantly increase any more after two iterations. For high $E_b/N_0$, the achievable gain is obtained after only one iteration. The tunnel in the EXIT chart is now very wide so that the trajectory terminates at the intersection point of the demapper and the decoder curve after only two steps, i.e., one iteration. Looking at the EXIT chart for OFDM-8FSK in Fig. 4.5b, we can see that the slopes of the demapper curves are larger than for OFDM-4FSK. This explains the larger achievable iterative gain for high $E_b/N_0$. The demapper for OFDM-8FSK can benefit from the available a priori information to a greater extent than for OFDM-4FSK. The larger slope is also the reason for an increased range of $E_b/N_0$, where more than one iteration improves the mutual information. The slopes of the demapper and the decoder curve for OFDM-8FSK are very similar over a wide interval of mutual information. This leads to a narrow tunnel for $4 \text{ dB} < E_b/N_0 < 5 \text{ dB}$, where the trajectory can move to the intersection point on the right side of the EXIT chart in many steps. This is also reflected in the BER plot, where the waterfall region is more pronounced and improvements in terms of BER are still observable for the third iteration at $E_b/N_0 = 4.5 \text{ dB}$.
4 Noncoherent Signal Detection

Fig. 4.7: EXIT chart (demapper characteristics) for OFDM-8FSK for the two-path channel with \( E_b/N_0 = 10 \text{ dB} \) and different bit mapping schemes; \( N_G/N = 0.25 \).

Performance of Iterative Detection for the Two-Path Channel

Let us have a look at the performance of the iterative receiver for the two-path channel in Fig. 4.6. The actual channel realization is assumed to be unknown, so the Rayleigh fading metric (4.30) is used. Fig. 4.6a compares the BER of the non iterative receiver from Fig. 4.3 with the results for turbo detection. We can see that the gain of the iterative receiver for high SNR is similar to the gain for transmission over the AWGN channel. It should be mentioned that in the case of time-variant channels the bit to symbol mapping does have an influence on the performance, because the orthogonality of the subcarriers is lost. While the performance for non-iterative detection is best for Gray mapping, it might be advantageous to apply non-Gray mapping in the case of iterative detection [84]. If anti-Gray mapping is used for example, neighboring carriers differ in a maximum number of bits for the MFSK symbols. As this phenomenon is more pronounced for higher order modulation, we look at OFDM-8FSK. From Fig. 4.6b we can see that while for non-iterative detection the sensitivity to Doppler spread is increased using anti-Gray mapping, the robustness for iterative detection is higher. This phenomenon can be explained using the demapper characteristics in the EXIT chart in Fig. 4.7. Due to the ICI, the demapper curves for both mappings move downwards. While for low a priori information, anti-Gray mapping experiences a larger loss than Gray mapping, the opposite is the case for high a priori information. This leads to a larger slope for anti-Gray mapping, which leads to a larger possible gain for the iterative receiver.
4.2 Single Symbol Detection

4.2.3 Serial Detection of Hybrid Modulation Schemes

Hybrid modulation schemes, where the subcarriers that are occupied by the OFDM-MFSK modulation are used to transmit additional information, have been introduced in Section 3.1.3. A straightforward receive method is to use a serial detection of MFSK and DPSK symbols as shown in the block diagram in Fig. 4.8. The (de-)interleavers are now included in the (de-)coding blocks. At first, the OFDM-MFSK symbols are detected. Because this detection can be done noncoherently, it is not influenced by the differentially modulated subcarrier phases. After demapping and decoding of the MFSK bits, these bits are encoded again to obtain knowledge about the occupied subcarriers while benefiting from the error correction capability of the MFSK channel code. Subsequently, the DPSK symbols can be detected using the phase differences of the occupied subcarriers. Please note that this serial detection is different from the approach of a joint detection of the MFSK and DPSK information as shown in Fig. 3.6, which will be discussed in detail in Section 4.3.

Fig. 4.9 shows the performance of two hybrid modulation schemes that combine OFDM-4FSK with 2DPSK and 4DPSK, respectively. For transmission over the AWGN channel and using serial detection, both hybrid modulation schemes outperform plain OFDM-4FSK not only in terms of power efficiency, but also in terms of bandwidth efficiency. Both the 4FSK bits $q_{\text{4FSK}}$ and the DPSK bits $q_{\text{DPSK}}$ are encoded using the same convolutional code. As a reference, we also included the BPSK curve again. For high $E_b/N_0$ the performance of OFDM-4FSK-4DPSK is similar to BPSK that has equal bandwidth efficiency but requires coherent detection.
Fig. 4.9: Overall BER for coded OFDM-4FSK combined with DPSK and transmission over the AWGN channel. Convolutional code: \( r = 1/2, m_C = 6; N_G/N = 0.25; \) serial detection.

Fig. 4.10: Overall BER for coded hybrid modulation schemes with 4DPSK and transmission over the two-path channel with different path delays. Convolutional code: \( r = 1/2, m_C = 6; N_G/N = 0.25, B_d/f_\Delta = 0.135; \) serial detection. For 2FSK-4DPSK, a fixed subcarrier (SC) is used between two adjacent subcarrier groups (cf. Section 3.1.3).
4.2 Single Symbol Detection

To get a feeling for the performance of the hybrid modulation schemes for time-variant multipath channels, the overall BER of OFDM-4FSK-4DPSK for a transmission over the two-path channel is plotted in Fig. 4.10. We can see the main drawback of the hybrid modulation schemes here: The information contained in the phase differences of the occupied subcarriers is quite sensitive to frequency selectivity. The reason for this is that the occupied subcarriers may have a large distance in frequency direction due to the MFSK modulation (cf. Fig. 3.2). For the channel with a path delay of $\Delta \tau / (TN) = 0.009$ relative to the OFDM symbol duration, the hybrid modulation scheme achieves a gain of around 3 dB compared to OFDM-4FSK for low BER, which is comparable to the gain for the AWGN channel. However, for the more frequency selective channel with $\Delta \tau / (TN) = 0.023$ the degradation is severe. This degradation is solely due to errors in the DPSK component. The robustness of the underlying OFDM-MFSK against frequency selectivity and time variance is not affected. Plotting a separate BER curve for the bits transmitted in the OFDM-4FSK part would lead to the dashed curve of plain OFDM-4FSK, taken from Fig. 4.3.

There are several ways to improve the robustness of the hybrid modulation schemes against frequency selectivity. The most obvious one is to reduce the order of the MFSK modulation to avoid large gaps of unoccupied subcarriers. However, the robustness of the underlying MFSK against noise and Doppler spread is reduced in this case. Another possibility already presented in Section 3.1.3 is to use a fixed carrier between two adjacent subcarrier groups. The BER results for OFDM-2FSK-2DPSK with fixed subcarriers are included in Fig. 4.10. The curves show that the scheme with fixed subcarriers improves the robustness of the DPSK component and therefore the overall BER against frequency selectivity. The degradation for the longer delay difference is almost negligible. This is because of the denser packing of occupied subcarriers, which also increases the overall bandwidth efficiency. However, the robustness of the MFSK component against noise and time variance is reduced and the bandwidth efficiency of the underlying OFDM-MFSK is lower. Generally speaking, there is a trade-off between the robustness of the underlying OFDM-MFSK against noise and time variance and the robustness of the DPSK component against frequency selectivity.

In Section 4.2.2 we proposed turbo detection to improve the performance of OFDM-MFSK. Of course, it is also possible to apply such an iterative scheme to both the FSK and DPSK symbol detection of the hybrid modulation schemes. However, we will not go into more detail about this for serial detection. Depending on the channel conditions, the expected gain for the FSK part is equal to the gain observable in Fig. 4.4 and Fig. 4.6. A more promising approach is to detect the MFSK and DPSK symbols jointly in an iterative way. The following section will investigate such methods in detail.
4.3 Multiple Symbol Detection

Multiple symbol detection is a well known method for noncoherent transmission to recover some of the loss compared to coherent detection. The basic idea behind multiple symbol detection (MSD) is to exploit the correlation of the channel among several receive symbols without the need for an explicit channel estimation. This is achieved by extending the detection window over several receive symbols and processing all receive symbols within the detection window jointly. The performance gain of using MSD for DPSK has been pointed out in [19]. MSD can also be applied to MFSK and hybrid modulation schemes. In [13], the authors examined the performance of MSD for uncoded “NFSK-LDPSK”, which is a modulation scheme similar to our hybrid modulation, but based on single carrier transmission.

We would like to point out, that MSD does not influence the signal generation at the transmitter. Despite the fact that multiple symbols, possibly containing FSK and DPSK information, are detected jointly at the receiver, the components for hybrid modulation schemes can be generated independently at the transmitter as shown in the system model in Fig. 3.6.

4.3.1 Multiple Symbol Detection for OFDM-MFSK

Let us recall the simplified OFDM system model from (4.1) and the corresponding receive metric based on the PDF of the receive symbols \( \tilde{\mathbf{x}}_D \) (4.8). We apply MSD in frequency direction and therefore, the detection window spans several MFSK symbols that are transmitted using neighboring subcarrier groups. Because MSD exploits the phase correlation of neighboring subcarriers, the phase relation of the subcarrier phases at the transmitter has to be known to the receiver. For example, if the subcarrier phases have been adapted to achieve a reduced PAPR (see Section 3.1.2), knowledge about the applied phase vector has to be transmitted as side information and compensated at the receiver.

Let us assume that \( N_{MSD} \) MFSK symbols are detected jointly. The receive vectors \( \tilde{\mathbf{x}}_D \) and transmit vectors \( \mathbf{x}_D \) consist of \( M \cdot N_{MSD} \) elements. We can look at the possible transmit vectors \( \mathbf{x}_D = \mathbf{a}_i \) as super-symbols containing \( N_{MSD} \log_2 M \) bits of information. Accordingly, the set of possible transmit vectors contains \( M^{N_{MSD}} \) elements. As an example, we give the possible transmit vectors for OFDM-2FSK with double symbol detection, that are used as test vectors in (4.9) or for coded transmission in (3.16), respectively:

\[
\mathbf{a}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{a}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \quad \mathbf{a}_3 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad \mathbf{a}_4 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}.
\] (4.31)
4.3 Multiple Symbol Detection

Theoretical Bit Error Probability for the AWGN Channel

The bit error probability of OFDM-MFSK with MSD can be estimated by using the union bound [53], which states that the probability that at least one of the events $E_i$ happens is not higher than the sum of the individual event probabilities. It is given as

$$\text{Prob} \left\{ \bigcup_i E_i \right\} \leq \sum_i \text{Prob} \{ E_i \}.$$  \hfill (4.32)

This leads to the following upper bound for the uncoded bit error probability:

$$P_b \leq \sum_{a_i} \frac{1}{M^{N_{\text{MSD}}}} \sum_{a_j \neq a_i} \frac{d_H(c_i, c_j)}{N_{\text{MSD}} \log_2 M} \cdot \text{Prob} \{ p(\tilde{x}_D | x_D = a_i) < p(\tilde{x}_D | x_D = a_j) | x_D = a_i \},$$  \hfill (4.33)

where $d_H(c_i, c_j)$ denotes the Hamming distance between the two bit vectors corresponding to the transmit symbols $a_i$ and $a_j$ and the last factor stands for the pairwise error probability (PEP), that the transmitted symbol $a_i$ is erroneously detected as $a_j$. The outer sum in (4.33) is taking the average over all elements $a_i$ of the transmit symbol alphabet.

For the AWGN channel, using the decision metric (4.13), the PEP can be written as

$$\text{PEP} = \text{Prob} \{ p(\tilde{x}_D | x_D = a_i) < p(\tilde{x}_D | x_D = a_j) | x_D = a_i \}$$

$$= \text{Prob} \left\{ |\tilde{x}_D a_i| < |\tilde{x}_D a_j| \right\}$$

$$= \text{Prob} \left\{ |\tilde{x}_D a_i|^2 < |\tilde{x}_D a_j|^2 \right\}.$$  \hfill (4.34)

Using the derivations in Appendix A, the PEP is given as

$$\text{PEP} = \frac{1}{2} \left( 1 - Q \left( \sqrt{b}, \sqrt{a} \right) + Q \left( \sqrt{a}, \sqrt{b} \right) \right),$$  \hfill (4.35)

with

$$\begin{bmatrix} a \\ b \end{bmatrix} = \frac{1}{2\sigma_n^2} \left( N_{\text{MSD}} + \sqrt{N_{\text{MSD}}^2 - \delta_{a_i,a_j}^2} \right),$$  \hfill (4.36)

where

$$\delta_{a_i,a_j} = \sum_{a_i^{(l)} = a_j^{(l)}} 1$$  \hfill (4.37)
is the number of subcarriers that are occupied in both $a_i$ and $a_j$. $Q(x, y)$ in (4.35) is Marcum’s Q function [66]. Because (4.33) is independent of which input vector is actually chosen, it can be simplified to

$$P_b \leq \sum_{a_j \neq a_1} \frac{d_H(c_1, c_j)}{N_{\text{MSD}} \log_2 M} \text{PEP}. \quad (4.38)$$

For low noise, we can approximate the PEP as (see Appendix A.1)

$$\text{PEP} \approx \frac{1}{2} \text{erfc} \left( \sqrt{\frac{1}{2\sigma_n^2} (N_{\text{MSD}} - \delta_{a_i,a_j})} \right) \sqrt{\frac{N_{\text{MSD}} + \delta_{a_i,a_j}}{2\delta_{a_i,a_j}}}. \quad (4.39)$$

In this case, neighboring symbol vectors that differ in only one MFSK symbol dominate the BER. For these symbols we have $\delta_{a_i,a_j} = N_{\text{MSD}} - 1$. Additionally, the Hamming distance is independent of the position of the MFSK symbol in the detection window, so we can confine the sum in (4.38) to all $M - 1$ joint symbol vectors $a_j$, that differ in the first MFSK symbol. Assuming that these are the symbol vectors $a_2 \ldots a_M$ we get the approximated upper bound

$$P_b \leq \sum_{j=2 \ldots M} \frac{d_H(c_1, c_j)}{\log_2 M} \frac{1}{2} \text{erfc} \left( \sqrt{\frac{1}{2\sigma_n^2}} \right) \frac{\sqrt{2N_{\text{MSD}} - 1}}{\sqrt{2(N_{\text{MSD}} - 1)}}. \quad (4.40)$$

For large detection windows ($N_{\text{MSD}} \to \infty$) we obtain

$$P_b \leq \sum_{j=2 \ldots M} \frac{d_H(c_1, c_j)}{\log_2 M} \frac{1}{2} \text{erfc} \left( \sqrt{\frac{1}{2\sigma_n^2}} \right), \quad (4.41)$$

which is equivalent to the union bound result for the bit error probability of coherent M-ary orthogonal modulation [66].

**Simulation Results for Uncoded Transmission**

Fig. 4.11 shows the simulation results for uncoded OFDM-MFSK with MSD when transmitting over the AWGN channel. From Fig. 4.11a we can see that extending the detection window to $N_{\text{MSD}} = 2$ is sufficient to obtain a performance similar to coherent detection for high $E_b/N_0$. Only for low $E_b/N_0$, increasing the detection window to $N_{\text{MSD}} = 4$ gives a significant additional gain. However, the complexity of the ML detector grows exponentially with $N_{\text{MSD}}$. For OFDM-4FSK the union bound (4.38) has been included in the plot for $N_{\text{MSD}} = 2$ and $N_{\text{MSD}} = 4$ using square markers. We can see that for high $E_b/N_0$, the union bound fits quite well to the simulation results. Fig. 4.11a also shows the approximated union bound for $N_{\text{MSD}} \to \infty$ for OFDM-4FSK. This approximated union bound is very close to the results for coherent detection, allowing the conclusion, that MSD can recover the loss of noncoherent detection compared to coherent detection if the detection window is large enough. As all curves
4.3 Multiple Symbol Detection

Fig. 4.11: BER vs. $E_b/N_0$ for uncoded OFDM-MFSK with multiple symbol detection. AWGN channel, $N_G/N = 0$.

with MSD approach the coherent curve for high $E_b/N_0$, detecting only a few symbols jointly is sufficient to recover the loss in this case. Fig. 4.11b shows the simulation results for OFDM-2FSK and OFDM-8FSK. Basically, the conclusion that detecting only a few symbols jointly allows to approach the performance of coherent detection also holds in this case.

Simulation Results for Iterative Detection

For the joint iterative detection of multiple MFSK symbols, the same metric as for single symbol detection is used to calculate the LLRs in the demapper. In the case of an AWGN channel, this is Equation (4.29). The only difference is that the receive symbol vector $\tilde{x}_D$ and the test vectors $a_i$ now comprise several MFSK symbols.

Looking at the simulation results for OFDM-4FSK in Fig. 4.12a, we can observe that jointly detecting two 4FSK symbols results in a gain of approximately 1 dB for the non iterative detection. While the iterative gain for three iterations is about 0.7 dB for high $E_b/N_0$ and single symbol detection, the iterative gain in the case of two jointly detected symbols is more than 1 dB. So for the AWGN channel, MSD results in a significant gain for the non iterative detection and in addition, the achievable gain of the iterative receiver is increased. The reason for the increased iterative gain is that the a priori knowledge about the bits of a detected symbol also improves the LLR of a jointly detected neighbor symbol because the phase reference becomes more reliable. Further simulations have shown that for $N_{MSD} \geq 2$ the increased iterative
Fig. 4.12: BER vs. $E_b/N_0$ for coded OFDM-MFSK with iterative multiple symbol detection and transmission over the AWGN channel. $N_G/N = 0.25$; convolutional code: $r = 1/2$, $m_C = 6$.

Gain is reduced again because the reliability of the phase reference without a priori knowledge is already quite high. The performance limit for iterative MSD is the BER curve of iterative coherent detection, which is included in Fig. 4.12a. The gap between $N_{MSD} = 2$ and coherent detection is less than 1 dB for three iterations so that the potential improvement for a further increase of $N_{MSD}$ is quite limited. The effect of an increased gain for iterative MSD is even more obvious in the case of OFDM-2FSK in Fig. 4.12b. Because the OFDM-2FSK symbols are determined by one single bit in the case of single symbol detection, there is no iterative gain at all. The curves for all iterations coincide with the solid line. However, for $N_{MSD} = 2$ there is a small iterative gain of about 0.3 dB. In addition, there is a gain of 1.5 dB that is also achieved for the non iterative receiver due to the increased detection window. For coherent detection the phase reference is perfect so that there is no iterative gain again.

To examine the robustness of MSD in time-variant and frequency selective environments, the WSSUS channel model is used. We assume that the channel has a uniform power delay profile with maximum delay $T_m = LT$. Each of the $L$ uncorrelated paths has a Jakes Doppler spectrum according to (2.23). The only knowledge about the channel at the receive side is the average signal and noise power. Simulations have shown, that the performance is quite insensitive to estimation errors of these parameters. They can be easily estimated by averaging over many OFDM symbols. Because the channel is unknown at the receive side, the metric for flat Rayleigh fading channels (4.16) was used to obtain the simulation results in Fig. 4.13. The results for channel
4.3 Multiple Symbol Detection

Fig. 4.13: BER for coded OFDM-4FSK with iterative multiple symbol detection and transmission over WSSUS channels with uniform power delay profile and different multipath spread. Three receiver iterations, Rayleigh fading metric; channel C1: $L/N = 0.039$, C2: $L/N = 0.078$; $B_d/f_\Delta = 0.135$, $N_G/N = 0.25$; convolutional code: $r = 1/2$, $m_C = 6$.

C1 show that for channels with low frequency selectivity, improvements are possible by detecting two symbols jointly. The relative multipath spread of C1 compared to the OFDM symbol duration is $L/N = 0.039$. This means that the coherence bandwidth is in the order of $\Delta f_c/f_\Delta \approx N/L = 26$ subcarriers. This value is large enough to achieve a significant performance gain by using a metric that assumes full correlation of the jointly detected subcarriers. Channel C2 has a larger multipath spread of $L/N = 0.078$ which leads to $\Delta f_c/f_\Delta \approx N/L = 13$ subcarriers. In this case, the assumption of full correlation of neighboring occupied subcarriers is strongly violated and therefore the joint detection of $N_{MSD} = 2$ symbols leads to a significant degradation. Other simulations confirmed the intuitive result that increasing the detection window to $N_{MSD} > 2$ makes the detection even more sensitive against frequency selectivity while the achievable additional gain is quite limited.

We have seen that overestimating the frequency correlation for MSD can lead to large degradations compared to single symbol detection. Therefore, we examine the BER for the channels C1 and C2 when the WSSUS receive metric (4.21) for $N_{MSD} = 2$ is applied. The exact channel is still unknown, however some assumptions about the statistical properties of the channel are made at the receiver. Fig. 4.14 shows the results for channel C1 and C2 when the receiver assumes a uniform PDP according to C1 or C2, respectively. In the case that the receiver uses the correct metric, i.e., it has perfect knowledge about the frequency correlation of the channel, the correlation of the subcarriers can be exploited and the performance is increased compared to $N_{MSD} = 1$ in Fig. 4.13. As the correlation for C1 is larger than for C2, the achievable
gain is also larger. In addition, the curves for an erroneous assumption about the frequency correlation are included in Fig. 4.14. When transmitting over channel C2 and assuming the PDP of C1, i.e., overestimating the frequency correlation, this leads to a degradation of about 1 dB. Also in the case of underestimating the frequency correlation by assuming the statistics of C2 when transmitting over channel C1 leads to a degradation. However, underestimating the frequency correlation can never lead to a result that is worse than single symbol detection \((N_{\text{MSD}} = 1)\). The reason for this is that the PDF for WSSUS with \(L = N\) is given as

\[
p(\tilde{x}_D | x_D) \propto \exp \left( \frac{1}{\sigma_{nD}^2} \tilde{x}_D^H \text{diag} (x_D) \left( \sigma_{nD}^2 \mathbf{I}_N + \text{diag} (x_D)^H \text{diag} (x_D) \right)^{-1} \text{diag} (x_D)^H \tilde{x}_D \right),
\]

which means that for calculating the L-values, the elements of the receive vector \(\tilde{x}_D\) are squared independently and weighted according to the test vector. In the case of OFDM-MFSK this is basically equivalent to single symbol detection.

### 4.3.2 Joint Multiple Symbol Detection of Hybrid Modulation Schemes

In Section 4.2.3 a serial detection scheme was presented, where the MFSK and DPSK components of the hybrid modulation schemes are detected independently. However,
4.3 Multiple Symbol Detection

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Table 4.2: Set of test vectors $a_i$ for OFDM-2FSK-2DPSK and $N_{\text{MSD}} = 2$.

<table>
<thead>
<tr>
<th>$N_{\text{MSD}}$</th>
<th>4FSK</th>
<th>2FSK-2DPSK</th>
<th>4FSK-2DPSK</th>
<th>4FSK-4DPSK</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{\text{MSD}} = 2$</td>
<td>16</td>
<td>8</td>
<td>32</td>
<td>64</td>
</tr>
<tr>
<td>$N_{\text{MSD}} = 3$</td>
<td>64</td>
<td>32</td>
<td>256</td>
<td>1024</td>
</tr>
<tr>
<td>$N_{\text{MSD}} = 4$</td>
<td>256</td>
<td>128</td>
<td>2048</td>
<td>16384</td>
</tr>
</tbody>
</table>

Table 4.3: Number of test vectors $N_{a_i}$ for MSD.

as already indicated in the block diagram of the system model in Fig. 3.6, a joint detection of both components combined with turbo detection promises superior results and is therefore examined in this section.

The PDF from (4.8) that is used to calculate the LLRs is of course also valid for the detection of hybrid modulation schemes. The only difference is the set of test vectors. To give an example, we extend the set of test vectors given in (4.31) for OFDM-2FSK with $N_{\text{MSD}} = 2$. Table 4.2 shows all possible test vectors $a_i$ for the combination of OFDM-2FSK with 2DPSK and $N_{\text{MSD}} = 2$. The number of test vectors $N_{a_i}$ is given as

$$N_{a_i} = M^{N_{\text{MSD}}} \cdot (M_{\text{DPSK}})^{(N_{\text{MSD}} - 1)}, \tag{4.43}$$

where $M_{\text{DPSK}}$ denotes the modulation order of the DPSK alphabet. Due to the fact that the DPSK information is contained in the phase difference of neighboring subcarrier groups, the detection windows have to overlap by one group, i.e., $M$ subcarriers.

The complexity of the detection depends on the number of test vectors $N_{a_i}$ that are used to calculate the L-value corresponding to one bit using (3.16). Table 4.3 shows the number of test vectors according to (4.43) that are needed in a full search. Due to the exponential growth of $N_{a_i}$ with $N_{\text{MSD}}$, it is obvious that only small number of symbols can be detected jointly.

Performance for the AWGN Channel

Fig. 4.15 shows the simulation results for hybrid modulation schemes, where OFDM-4FSK is combined with 4DPSK (‘H4’) and with 2DPSK (‘H2’), respectively. The AWGN channel model is used and in the receiver the corresponding AWGN metric is applied.
For comparison, the results for plain OFDM-4FSK with iterative detection as well as the BER curve for coherently detected BPSK are included. The dashed curves with markers show the previous result for the total BER of the hybrid modulation schemes with serial detection from Section 4.2.3. Let us have a look at the results for OFDM-4FSK-4DPSK in Fig. 4.15a first. We can see a performance gain for joint iterative MSD compared to serial detection. Three iteration loops are used for joint MSD. OFDM-4FSK-4DPSK also outperforms BPSK which has equal bandwidth efficiency but assumes a perfect estimate of the channel phase.

In Fig. 4.15b the performance of OFDM-4FSK-2DPSK is plotted. Also in this case, we observe a large gain compared to serial detection and the hybrid modulation scheme outperforms BPSK in terms of power efficiency. Further simulations have shown, that the bit errors are mainly caused by the FSK part for the combination of 4FSK and 2DPSK when the same code with $r = 1/2$ is used for both components. Therefore, also a weaker channel code with rate $r_{DPSK} = 3/4$ for the DPSK component was applied, which was obtained from the original code by puncturing. The results show an additional improvement in power efficiency while the bandwidth efficiency is improved at the same time so that it is coming close to the bandwidth efficiency of BPSK. A comparison of the bandwidth efficiency $\eta$ of all considered modulation schemes can be found in Table 4.4. The fact that one occupied subcarrier per OFDM symbol is needed as a reference for the differential symbols has only a small influence on $\eta$ and was therefore
4.3 Multiple Symbol Detection

<table>
<thead>
<tr>
<th></th>
<th>4FSK</th>
<th>4FSK-2DPSK</th>
<th>4FSK-2DPSK</th>
<th>4FSK-4DPSK</th>
<th>BPSK  (coh.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r(r_{\text{DPSK}}) )</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{2} / \frac{1}{2} )</td>
<td>( \frac{1}{2} / \frac{3}{4} )</td>
<td>( \frac{1}{2} / \frac{1}{2} )</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>( \eta ) [bits ( T_{\text{Hz}} )]</td>
<td>0.2</td>
<td>0.3</td>
<td>0.35</td>
<td>0.4</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Table 4.4: Bandwidth efficiencies of different modulation schemes in combination with OFDM; \( N_{G}/N = 0.25 \).

neglected.

Both for OFDM-4FSK with 2DPSK and with 4DPSK we can see a substantial gain if the detection window is increased from \( N_{\text{MSD}} = 2 \) to \( N_{\text{MSD}} = 3 \). The reason for this is that especially the additional DPSK component can profit from the fact that two differential phase symbols are included in the joint detection instead of one. This gain has to be traded off with the complexity that is growing exponentially with \( N_{\text{MSD}} \) as shown in Table 4.3. To reduce the complexity, suboptimal strategies such as sphere detection [4, 62] could be used, but will not be examined here.

EXIT Chart Analysis

We use the EXIT chart again to get a better insight into the exchange of information in the iterative receiver. For the hybrid modulation schemes, there are four quantities that can be observed in the receiver (see Fig. 3.6): the extrinsic outputs of the demapper corresponding to the FSK bits \( I_{E,\text{Dem,FSK}}^{\text{FSK}} \) and to the DPSK bits \( I_{E,\text{Dem,DPSK}}^{\text{DPSK}} \) as well as the extrinsic outputs of the two channel decoders belonging to the FSK component \( I_{E,\text{Dec,FSK}}^{\text{FSK}} \) and to the DPSK component \( I_{E,\text{Dec,DPSK}}^{\text{DPSK}} \), respectively. The joint observation of all four quantities requires a four-dimensional EXIT chart, which is impossible to draw. However, it is possible to split this up into two three-dimensional EXIT charts - one for the FSK output of the demapper and one for the DPSK output. Fig. 4.16 shows the EXIT charts for OFDM-4FSK-2DPSK with \( N_{\text{MSD}} = 2 \) and transmission over the AWGN channel. In Fig. 4.16a, the extrinsic information \( I_{E,\text{Dem,FSK}}^{\text{FSK}} \) corresponding to the FSK bits at the output of the demapper is plotted versus the a priori information about the FSK bits \( I_{A,\text{Dem,FSK}}^{\text{FSK}} \) and the a priori information about the DPSK bits \( I_{A,\text{Dem,DPSK}}^{\text{DPSK}} \). The result is a tilted plane where both an increase in \( I_{A,\text{Dem,FSK}}^{\text{FSK}} \) and \( I_{A,\text{Dem,DPSK}}^{\text{DPSK}} \) lead to an increased extrinsic output information for the FSK component. The decoder characteristics for the FSK channel decoder are again included in the plot and result in an S-shaped surface, that is independent of the DPSK information. In a similar way, the plot in Fig. 4.16b is created that shows the extrinsic information corresponding to the DPSK bits. The demapper characteristics are again represented by a tilted plane. In this case of 2DPSK and \( N_{\text{MSD}} = 2 \), we can observe that \( I_{A,\text{Dem,DPSK}}^{\text{DPSK}} \) is not influencing the output information of the DPSK component. This is obvious, because only one differential symbol containing one DPSK bit is contained in the detection window. However, the availability of a
4 Noncoherent Signal Detection

Fig. 4.16: EXIT charts for jointly detected OFDM-4FSK-2DPSK. AWGN channel, $E_b/N_0 = 4$ dB, $N_G/N = 0.25$, $N_{\text{MSD}} = 2$, $r = r_{\text{DPSK}} = 1/2$, $m_C = 6$.

priori information about the FSK bits $I_A^{\text{Dem,FSK}}$ improves the output information of the DPSK component significantly. The trajectory represents the evolution of the extrinsic information during the receiver iterations. It is limited by the intersection line of the demapper and decoder surfaces, which is marked with a bold line in Fig. 4.16. In addition, the intersection line in the FSK EXIT chart is projected onto the demapper surface of the DPSK EXIT chart and vice versa so that we obtain two lines, limiting the trajectories in both EXIT charts. These two lines in turn define a tunnel, similar to the case of two-dimensional EXIT charts and their intersection point defines the point of convergence. In Fig. 4.16, the tunnel is open and the intersection point of the two lines is close to the point $(I_{\text{Dec,FSK}}^E = 1, I_{\text{Dec,DPSK}}^E = 1)$, which corresponds to a very low BER for both components. By counting the steps of the trajectory, we can obtain the necessary number of iterations in the receiver. For the parameters in Fig. 4.16, two to three receiver iterations are sufficient.

Another important task where the EXIT chart is very helpful is the choice of the proper code rates for the two components. As already mentioned in the previous subsection, the total BER for OFDM-4FSK-2DPSK is dominated by errors in the FSK component when using the same code with $r = 1/2$ for both components. This fact is also visible in the EXIT charts of Fig. 4.16. In Fig. 4.16b, there is a large distance between the demapper and the decoder curves for the DPSK component (at least for high FSK a priori information). This means that a weaker code with higher code
4.3 Multiple Symbol Detection

(a) Extrinsic information for 4FSK component; (b) Extrinsic information for 2DPSK component; 
\[ r = \frac{1}{2} \]
\[ r_{DPSK} = \frac{3}{4} \]

Fig. 4.17: EXIT charts for jointly detected OFDM-4FSK-2DPSK. AWGN channel, \( E_b/N_0 = 4 \text{ dB} \), \( N_G/N = 0.25 \), \( N_{MSD} = 2 \), \( r = \frac{1}{2} \), \( r_{DPSK} = \frac{3}{4} \), \( m_C = 6 \).

rate would be sufficient for the DPSK component without significantly changing the intersection point of the two lines that are defining the tunnel. Fig. 4.17 shows the EXIT charts corresponding to the BER curves of Fig. 4.15, where puncturing was used to obtain a channel code with rate \( r_{DPSK} = \frac{3}{4} \) for the 2DPSK component. Compared to 4.16 the tunnel is narrower but the intersection point is still very close to the back corner of the EXIT charts so that convergence to a low BER is possible.

Performance for the WSSUS Channel

Fig. 4.18 shows the performance of OFDM-4FSK-2DPSK for different WSSUS channels, when the statistics of the channel are unknown at the receiver and the Rayleigh fading metric is used. In addition to the channels C1 and C2, a third channel C0 with uniform PDP and a normalized multipath spread of \( L/N = 0.020 \) is examined. The results show that the joint iterative detection of two MFSK symbols (including one differential symbol) gives a gain in power efficiency of over 2 dB compared to serial detection for the channel C0, which has a low frequency selectivity. Looking at the results for channel C1, we can see a degradation for the multiple symbol detector that is due to the increased frequency selectivity. Although the degradation for serial detection is still very small, the iterative detector still outperforms serial detection. For C2, where we have high frequency selectivity, all schemes have an error floor which does not
allow data transmission. In this case, the assumption of fully correlated phases within the detection window is severely violated and the subcarrier phases cannot be used for signal detection. However, the robustness of iterative single symbol detection for the OFDM-MFSK part is not reduced and can always be applied if MSD or the joint detection of the hybrid schemes fail. The performance of the OFDM-MFSK part is then equal to the results for $N_{\text{MSD}} = 1$ from Fig. 4.13.

4.4 Transinformation for OFDM-MFSK-based modulation schemes

We have discussed several modulation schemes based on OFDM-MFSK. By using MSD in the receiver, the performance can be improved if the channel coefficients of the jointly detected subcarriers are highly correlated. To bring the considered modulation schemes into relation to the theoretical channel capacity, we look at the transinformation for some OFDM-MFSK-based modulation schemes. This transinformation is sometimes also called constellation constrained capacity. It can serve as an upper bound for the performance if powerful channel coding is applied. The transinformation is defined as the average mutual information between the transmit symbol vectors $\mathbf{x}$ and the received vector $\tilde{\mathbf{x}}$. The averaging is done over all possible transmit symbol vectors $\mathbf{a}_i$ and can be written as

$$T(\tilde{X}; X) = E_{\tilde{X},A} \left\{ \log_2 \frac{p(\tilde{\mathbf{x}}|\mathbf{a}_i)}{p(\tilde{\mathbf{x}})} \right\},$$

(4.44)
where we used capital letters to denote the random variable of the corresponding vector symbol. To evaluate (4.44), the PDF of the receive symbols $\tilde{x}_D$, given in Section 4.1 for different channel conditions, can be used. The transinformation has to be normalized to the average number of subcarriers $N_{av}$ on which the information is transmitted so that we get the average number of bits per channel use:

$$T' = \frac{1}{N_{av}} T(\tilde{X}; X).$$ (4.45)

For plain OFDM-MFSK we obtain

$$T' = \frac{1}{MN_{MSD}} T(\tilde{X}; X) = \frac{1}{MN_{MSD}} T(\tilde{X}; C_0, \ldots, C_{(N_{MSD} \log_2 M) - 1}).$$ (4.46)

In (4.46) we have written the transinformation also in terms of the information about the transmitted bits $c_0, \ldots, c_{(N_{MSD} \log_2 M) - 1}$ contained in each symbol vector. It can be seen that the dependencies between the bits have to be considered in the receiver. Using standard BICM, these dependencies are neglected and the transinformation can be written as [24]

$$T'_{BICM} = \frac{1}{N_{av}} \sum_i T(\tilde{X}; C_i) \leq \frac{1}{N_{av}} T(\tilde{X}; C_0, \ldots, C_{(N_{MSD} \log_2 M) - 1}).$$ (4.47)

In general, BICM is not sufficient to approach the maximum transinformation for a given modulation scheme. Instead, multilevel coding (MLC) with multistage decoding [24] or, which is our approach, BICM-ID has to be used.

The above findings also hold for the hybrid modulation schemes. However, the calculation of $N_{av}$ has to consider the fact, that the detection windows are overlapping, but the overlapping MFSK information must not be counted twice:

$$N_{av} = \frac{M (N_{MSD} \log_2 M + (N_{MSD} - 1) \log_2 M_{DPSK})}{\log_2 M + \log_2 M_{DPSK}}.$$ (4.48)

Figure 4.19 shows the transinformation according to (4.46) for several OFDM-MFSK-based modulation schemes using MSD with $N_{MSD} = 2$ for transmission over the AWGN channel. As a reference, the transinformation for BPSK modulation is also included in the plot. For high SNR the transinformation is limited by the number of possible transmit symbol vectors for all modulation schemes. We can see, that although the OFDM-MFSK-based schemes do not come very close to the channel capacity according to Shannon, the transinformation of OFDM-4FSK-4DPSK comes close to the curve for BPSK for medium to high SNR. This is also confirmed by the simulation results in the previous section. Note that for calculating $T'$, the absolute phase of the channels was assumed to be known for BPSK, whereas it was assumed to be unknown for all OFDM-MFSK-based schemes.
4 Noncoherent Signal Detection

4.5 Extended Mapping

The mapping and the channel code in a system with BICM can be seen as the serial concatenation of two codes. The EXIT chart analyses in the previous sections have revealed that the matching of the symbol mapping (inner code) and the channel code (outer code) is an essential design criterion for the performance of BICM-ID. Up to now, we followed the usual approach of assigning $m = \log_2 M$ bits to each symbol of an alphabet of size $M$. This leads to a non-ambiguous mapping where each possible transmit symbol has a distinct bit label. In [15, 35], the authors proposed to use mappings for QAM and PSK alphabets, where more than one label is assigned to the same constellation point. This leads to an ambiguous mapping with $m_E > m$ bits per label. While the ambiguous mapping causes an increased error rate, the additional bits allow the use of a lower rate channel code without changing the overall bandwidth efficiency. The task is to find the right parameters so that the second effect predominates, where the stronger channel code can resolve the ambiguity and improve the total performance. In this work, we will apply extended mapping to OFDM-MFSK in order to improve the performance of the iterative turbo detector.

4.5.1 Choice of the Mapping

A good mapping should provide maximum mutual information $I_{Dem}^{E}$ for full extrinsic a priori information ($I_{Dem}^{A} = 1$). This means, the demapper curve in the EXIT chart should be as high as possible on the right side to allow a good point of convergence and therefore a low error rate in combination with an iterative receiver. At the same time, it is desirable to get an early waterfall region, i.e., a tunnel in the EXIT chart.
should be open at low $E_b/N_0$. For the demapper characteristics this means that $I_{E}^{\text{Dem}}$ without a priori information ($I_{A}^{\text{Dem}} = 0$) should be sufficiently high.

Let us explain the way to find a good mapping according to these criteria at the example of OFDM-4FSK. Figure 4.20 shows different mappings for OFDM-4FSK. The bold arrows indicate an example for the FSK frequencies that are selected if the corresponding data bits below shall be transmitted. The first row shows anti-Gray mapping for $m=2$, which we have used before for iterative detection. The rows below contain extended mappings with one additional bit, i.e., $m_E = 3$. Now, $2^{m_E-m} = 2$ bit labels are assigned to each of the four possible signals in a group. For mapping A, the labels for each signal differ in two bits, while only one bit is different in mapping B, and all bits are inverted for the second label in mapping C. Let us compare the three extended mappings regarding the properties of good mappings from above. For full a priori information, i.e., all other bits are assumed to be known except for the bit that shall be decided, mapping A is non-ambiguous. As the labels differ in two bits, only one label per frequency is possible if all bits but one are known. Mapping A achieves the maximum mutual information for full a priori information. The same holds for mapping C. If two bits are known, the mapping is non-ambiguous. This is not the case for mapping B. We can see that the last two bits are equal for each signal. So even with full a priori information, the signals contain no information about the first bit. This is actually equivalent to puncturing (see Section 2.4), where some code bits are simply not transmitted. The second criterion is the mutual information without a priori information. It is obvious, that the mutual information without a priori information is $I_{E}^{\text{Dem}} = 0$ for mapping C, because all three bits are different in each label,
so that no information on any bit can be extracted by deciding on a received signal. For mapping A, decisions without a priori information can be made about the second bit, because this is equal in both labels of one signal. Mapping B allows decisions on the last two bits in this case, which leads to the maximum mutual information of all mappings without a priori information.

The calculation of the L-values is carried out in the same way as for unambiguous mappings by using Equation (3.16). The only difference is the sets $S_j^0$ and $S_j^1$ that have to be adapted according to the mapping.

To predict the behavior of all possible mappings, the extrinsic mutual information of the demapper is plotted in Fig. 4.21. Although the channel code does not influence the curves directly, a code rate of $r = 1/3$ is assumed so that the overall bandwidth efficiency is equal to non extended mapping and the $E_b/N_0$ is comparable. In addition to the three mappings A, B, and C, the characteristics of all other possible mappings have been included using dashed lines. Of all mappings that reach maximum mutual information for $I_{Dem}^A = 1$, mapping A is the best choice as it has the highest $I_{Dem}^E$ for $I_{Dem}^A = 0$. Note that the mapping does not influence the constellation constrained capacity and therefore the area under the demapper curves is the same for all mappings [17]. The described brute force optimization strategy can also be used for other modulation schemes and extended mappings with more than one additional bit. However, the complexity is increasing significantly for an increasing number of labels.

In this work, the extrinsic mutual information is used to find a suitable mapping as shown in the example in Fig. 4.21. Another method would be to use a distance spectrum as in [35, 73], that counts the number of labels that differ in the considered bit position and have a certain Euclidean distance. Although the Euclidean distance
4.5 Extended Mapping

Fig. 4.22: Decoder characteristics of convolutional codes with rate $r = 1/3$ and different memory $m_C$.

The measure is better suited for QAM alphabets, it can also be used in the case of non-coherent orthogonal signal detection. In this case, a binary switching algorithm with lower complexity could be used to optimize the mapping [72, 73].

4.5.2 Choice of the Channel Code

In the previous section we have looked at the parameters that influence the shape of the demapper characteristics. Now let us look at the second component, the channel code, that should be matched to the mapping. The examples for extended mappings assumed a channel code with rate $r = 1/3$ so we will also use such an example here. Fig. 4.22 shows the decoder characteristics of convolutional codes with rate $r = 1/3$ and different memory $m_C$. We can see that codes with longer memory have a more abrupt change in the output extrinsic mutual information if the a priori information is increased. The area under the decoder curves is equal for all $m_C$ and corresponds to the rate of the convolutional code [73]. Obviously, the parameter $m_C$ can be used to influence the convergence of our iterative receiver. While the tunnel in the EXIT chart starts to open for lower SNR for codes with short memory, the final point of conversion lies further to the right for larger $m_C$.

The total EXIT chart for OFDM-4FSK with extended mapping ($m_E = 3$) is plotted in Fig. 4.23 for the AWGN channel. For the reasons stated in the previous section, mapping A is used as extended mapping scheme. The convolutional code with rate $r = 1/3$ and memory $m_C = 3$ is a good compromise between an early tunnel opening and high mutual information in the converged state. In addition, the non-extended mapping scheme with $m = \log_2 M = 2$ and our standard convolutional code with
rate $r = 1/2$ and memory $m_C = 6$ is also included in the plot. Both combinations have the same bandwidth efficiency because the code rate for the extended mapping scheme is chosen such that it compensates the increased number of bits per symbol. Due to the low starting point for $I_{Dem}^A = 0$ of the demapper characteristics for extended mapping, the necessary $E_b/N_0$ for an open tunnel will be higher as for non extended mapping. The depicted $E_b/N_0$ of 7.5 dB is high enough so that the tunnel is open for both mappings. Due to the stronger channel code, the intersection point of the demapper and decoder curves is slightly further to the right for extended mapping. Additionally, the trajectories obtained in simulations of the complete iterative receiver are included in Fig. 4.23. We can see, that more iterations are needed for the system with extended mapping to reach the final point at the intersection of the demapper and the decoder curve.

### 4.5.3 Numerical Results

Fig. 4.24 shows the simulated BER vs. $E_b/N_0$ for the two combinations of coding and mapping that we already analyzed in the EXIT chart. It compares anti-Gray mapping and the extended mapping A. For an AWGN channel (Fig. 4.24a) we can see the behavior predicted in the EXIT chart. The system with extended mapping needs a higher $E_b/N_0$ before the iterative detector can achieve significant improvements. However, for $E_b/N_0 \geq 7.5$ dB the error rate of the extended mapping scheme is below the curve for standard mapping so that we obtain an improved power efficiency for $BER \leq 10^{-5}$. Below a BER of $10^{-6}$, the steepness of the BER curve decreases again, which leads to an error shoulder (not shown in Fig. 4.24). This is due to the fact that the demapper curve
4.6 Precoding

In order to improve the convergence characteristics of iterative receivers, it is desirable, that the demapper curve ends in the point \((I_{Dem}^{A} = 1, I_{Dem}^{E} = 1)\) in the EXIT chart and does not intersect with the decoder curve. Under these circumstances, we could obtain a very low bit error rate for an open tunnel and a sufficient number of receiver
Fig. 4.25: Structure of the intermediate recursive convolutional code with rate $r_{\text{PC}} = 1$ and memory $m_{\text{PC}} = 1$. Transfer function: $G(D) = 1/(1 + D)$.

Fig. 4.26: System model for an OFDM transmission with additional intermediate code (precode) and iterative detection.

iterations. There would no error shoulder for high SNR and we could use a channel code with low memory.

This goal can be achieved by using an additional intermediate code which acts as a precoder for the mapper. As in [12, 18, 73, 79], we use a recursive intermediate convolutional code with rate $r_{\text{PC}} = 1$ and memory $m_{\text{PC}} = 1$. The structure of our code is given in Fig. 4.25. It does not add any additional redundancy but introduces dependencies among all bits of one codeword, which leads to a large interleaving gain [18].

A block diagram of the system using precoding and iterative detection is depicted in Fig. 4.26. The source bits of one code block contained in the vector $q$ are encoded by the outer channel code ($\text{COD}_C$), using a convolutional code with rate $r_0 < 1$. The encoded bits are then interleaved by an outer interleaver $\Pi_O$ before they are again encoded by the precoder ($\text{COD}_{\text{PC}}$). The output of the precoder is interleaved by the inner interleaver $\Pi_I$. Also in this case, both interleavers are implemented as pseudo random interleavers. The encoded bits are then mapped to OFDM-MFSK symbol vectors $x(k')$, which are transmitted using our standard OFDM transmission system. After the OFDM receiver, the receive symbol vectors $\tilde{x}(k')$ are used in the demapper to calculate soft values for the received code bits in the usual way (cf. Section 4.1), that are passed on to the intermediate decoder. The intermediate and outer decoder, as well as the demapper, can exchange information so that the output of each device can
be improved by using a priori information as shown in Fig. 4.26. Like this, we obtain a system of three serially concatenated codes, namely the channel code (CODC), the code of the precoder (CODPC), and the symbol mapper (MAP). The intermediate code (precode) can process both a priori information for its code and information bits. In order to avoid confusion between the mutual information in the outer and the inner loop of the iterative receiver, we slightly changed our notation. The a priori information for the code bits of the precode are denoted as $A_I^{PC}$, because the information comes from the inner loop. The a priori information for the information bits of the precode that is coming from the outer loop and is denoted as $A_O^{PC}$. The notation for the extrinsic information is analogous.

### 4.6.1 Three-Dimensional EXIT Chart Analysis

We have used the EXIT chart to visualize the exchange of extrinsic mutual information between the components of an iterative receiver with two components. If we look at the block diagram of our system in Fig. 4.26, we can see that for three serially concatenated components, there are four variables that have to be observed. These four variables are represented by the four connections of the intermediate decoder. The EXIT chart for this system has four dimensions. In general, the EXIT chart of an iterative receiver with $n$ serially connected component codes has a dimension of $2(n - 1)$ [12]. Fortunately, it is possible to split up the four-dimensional chart into two three-dimensional EXIT charts [12], so that we are able to plot them. One chart is representing the inner loop and the other is representing the outer loop.

Fig. 4.27 shows the EXIT charts with precoder for OFDM-4FSK modulation and transmission over the AWGN channel. A precoder according to Fig. 4.25 and an outer channel code with rate $r = 1/2$ and memory $m_C = 1$ is used. Both in the EXIT chart for the outer and for the inner loop, the horizontal axes are the two inputs of the intermediate decoder. One is the a priori information from the inner loop ($A_I^{PC}$) and the other from the outer loop ($A_O^{PC}$).

Let us start with the EXIT chart in Fig. 4.27a, which represents the inner loop. The vertical axis denotes the extrinsic output of the intermediate decoder for its code bits $E_{PC}^I$, i.e., the output for the inner loop. The extrinsic output $E_{PC}^I$ of the intermediate decoder depending on the a priori inputs $A_I^{PC}$ and $A_O^{PC}$ is represented by the curved surface. The second component in the inner loop is the demapper, so we can plot its characteristics into the same EXIT chart, regarding that the extrinsic output of the demapper is connected to the a priori input of the intermediate decoder ($A_M = E_M$) and vice versa ($A_M = E_{PC}^I$). We obtain a surface that is depending on $E_b/N_0$. Fig. 4.27 shows the result for $E_b/N_0 = 6.2$ dB. Due to the fact that the demapper is independent of the outer a priori input of the intermediate decoder $A_O^{PC}$, the surface is constant in this dimension. The intersection line of the two surfaces represents the points of convergence between the demapper and the intermediate decoder for different values of $A_O^{PC}$.

The EXIT chart for the outer loop is depicted in Fig. 4.27b. Now the vertical axis
denotes the extrinsic output of the intermediate decoder for its information bits $E_{PC}^O$. The outer loop includes the channel decoder which is represented by the S-shaped surface in the EXIT chart. Again, we obtain an intersection line that represents the points of convergence between the two decoders in the outer loop. An interesting criterion for convergence becomes visible if we project the intersection line of the inner EXIT chart onto the surface of the intermediate decoder in the outer EXIT chart. This is done by identifying the values of $E_{PC}^O$ that have the same coordinates ($A_{PC}^I, A_{PC}^O$) as the inner intersection line. If this projected line and the intersection line of the two surfaces in the outer EXIT chart form an open tunnel, i.e., they do not touch or intersect, the trajectory can move through this tunnel and proceed to a point close to $E_C = 1$, which corresponds to a very low BER.

When designing an iterative receiver for more than two concatenated components, it is necessary to determine the order in which the components are activated. Under the assumption that all extrinsic mutual information are monotonically increasing for increasing a priori information, the activation order does not influence the convergence point and therefore the performance in terms of BER, if a sufficient number of iterations is allowed. However, by optimizing the activation order, the number of necessary iterations can be minimized [12]. In our work, we did not optimize the activation order but used the order DEMAP - DEC$_{PC}$ - DEC$_C$ - DEC$_{PC}$ - DEC$_{PC}$ - DEMAP - · · · and allowed a sufficient number of iterations to obtain convergence.
Fig. 4.28: Projection of the tunnel of the outer loop 3D EXIT chart onto two dimensions. The demapper characteristics without precoding are included for comparison. AWGN channel, $E_b/N_0 = 6.2$ dB, $N_G/N = 0.25$.

**Projection on 2D EXIT Chart**

It is possible to simplify the interpretation of the three-dimensional EXIT charts by a projection onto an EXIT chart with two dimensions as in Fig. 4.28, which basically represents the outer loop. For this, the characteristics of the demapper and the intermediate decoder are regarded jointly. Their characteristics in the outer loop are represented by the projected line in Fig. 4.27b. This line is now projected onto the two-dimensional EXIT chart by setting the $A_{PC}$ coordinate to 0, so that we obtain the solid line in Fig. 4.28. The dashed line represents the decoder characteristics, so that the tunnel that is formed is equivalent to the tunnel in the outer loop 3D EXIT chart. In the 2D EXIT chart we can see even better that a tunnel is open for $E_b/N_0 = 6.2$ dB and we expect convergence to very low BER. In order to come close to the point $(I_{A_{Dem}}^A = 1, I_{Dem}^E = 1)$ we expect a high number of iterations to be necessary. It becomes also obvious that an outer channel code with small memory should be used to avoid an early intersection with the demapper/precoder curve. The characteristics of the demapper without precode is also included in Fig. 4.28. It is clear that in this case, the early intersection point with the decoder curve would lead to a high BER.

Another conclusion, that can be drawn by looking at the 2D EXIT chart, is that we cannot expect large improvements by combining extended mapping and precoding. The precoder makes sure, that the joint demapper and decoder characteristics have a high extrinsic mutual information for high $A_{PC}^O$. However, the extrinsic mutual information of the joint demapper and decoder curve for low a priori information is largely influenced by the demapper characteristics. As this point should be as high as possible, the optimum extended mapping is equivalent to puncturing.
4 Noncoherent Signal Detection

Fig. 4.29: BER vs. $E_b/N_0$ for iteratively detected OFDM-4FSK with precoding. The results for extended mapping from Section 4.5 and the original approach without extended mapping or precoding are included for comparison. $N_G/N = 0.25$. $B_d/f_\Delta = 0.135$.

### 4.6.2 Numerical Results

According to the EXIT charts in Figs. 4.27 and 4.28, we can expect a low BER for $E_b/N_0 = 6.2$ dB for the AWGN channel when using OFDM-4FSK and precoding. This result is confirmed by the BER curve in Fig. 4.29a. For precoding with the recursive intermediate code from Fig. 4.25 and an outer channel code with rate $r = 1/2$ and memory $m_C = 1$, we can see that there is a sudden drop in the BER curve around $E_b/N_0 = 6.2$ dB. If the noise level is low enough so that a tunnel in the EXIT chart is open, the BER drops basically to zero. At a BER of $10^{-5}$ precoding outperforms both extended mapping and the conventional approach with anti-Gray mapping by about 1 dB. In the two-path transmission scenario, this gain is even larger and the performance gain compared to anti-Gray mapping at BER = $10^{-5}$ is almost 4 dB. This indicates that we can obtain significant gains by using precoding also in time-variant and frequency selective channels.
In recent years, noncoherent communication has obtained attention, especially for MIMO transmission [30, 36, 86, 102]. Due to the large number of channel coefficients in systems with multiple transmit and receive antennas, it is even more difficult to obtain a reliable estimate of the channel than for single antenna systems. When using noncoherent communication, no attempt is made to estimate the channel coefficients. Instead, the transmit signals are designed such that the receiver can distinguish them without explicit channel knowledge. Unitary space-time modulation [36] has been introduced for transmission over noncoherent MIMO channels. It was soon recognized that this modulation can be regarded as transmitting information by using subspaces. While the transmitted signal itself is changed by the unknown channel, the subspace that is spanned by the signal remains unchanged at the receiver. This means that the transmitted information is not contained in the orientation of the received signal vector (or matrix) but in the subspace of the received signal [30].

In this chapter, we show that it is also possible to find such a subspace interpretation for the OFDM-MFSK-based modulation schemes that were discussed in the previous chapters. We do not make an attempt to find optimal signal constellations here. The intention is merely to fit these modulation schemes into the commonly used framework of subspace-based information transmission.

### 5.1 Subspace Representation of Noncoherent MIMO Transmission

Before we deal with OFDM-MFSK-based modulation schemes, we present the subspace representation of a noncoherent MIMO transmission similar to, e.g., [102]. Let us assume the following model of a general MIMO system

\[ Y = H_{\text{MIMO}} X + N, \]  

(5.1)
where $H_{\text{MIMO}}$ denotes the MIMO channel matrix with dimension $N_{\text{out}} \times N_{\text{in}}$, $X$ is the $N_{\text{in}} \times N_{\text{col}}$ transmit matrix, $Y$ the receive matrix with size $N_{\text{out}} \times N_{\text{col}}$ and $N$ denotes the corresponding AWGN matrix. From (5.1) it becomes obvious that the MIMO channel matrix $H_{\text{MIMO}}$ acts in the same way on all columns of the transmit matrix $X$. If this model is used for the description of space-time modulation (STM) (e.g. [30, 36]), this implies that the channel matrix can be considered constant during a certain number of time slots that are represented by the columns of $X$. In this case, the entries of the channel matrix represent the coefficients of the flat fading channel that connects the $N_{\text{in}}$ transmit antennas to the $N_{\text{out}}$ receive antennas.

Looking at the general MIMO model (5.1) again, we can observe that the channel matrix carries out $N_{\text{out}}$ linear combinations of the rows of the transmit matrix $X$. This means, if we consider the rows of $X$ to be the basis vectors of an $N_{\text{in}}$-dimensional linear subspace of the $N_{\text{col}}$-dimensional space, the rows of the receive matrix $Y$ lie in the same subspace in the noiseless case. In other words, the left-multiplication by the channel matrix does not change the subspace of the transmitted signal. Only the additive noise can lead to receive matrices outside the transmitted subspace. This becomes clear by expanding (5.1) for the example of $N_{\text{in}} = N_{\text{out}} = 2$:

$$
Y = \begin{pmatrix}
[H_{\text{MIMO}}]_{1,1} & [H_{\text{MIMO}}]_{1,2} \\
[H_{\text{MIMO}}]_{2,1} & [H_{\text{MIMO}}]_{2,2}
\end{pmatrix}
\begin{pmatrix}
x_{r1} \\
x_{r2}
\end{pmatrix}
+ N = 
\begin{pmatrix}
[H_{\text{MIMO}}]_{1,1} x_{r1} + [H_{\text{MIMO}}]_{1,2} x_{r2} \\
[H_{\text{MIMO}}]_{2,1} x_{r1} + [H_{\text{MIMO}}]_{2,2} x_{r2}
\end{pmatrix}
+ N,
$$

(5.2)

where $x_{ri}$ denotes the $i$-th row of $X$.

Due to the fact that the rows of the receive matrix are linear combinations of the rows of the transmit matrix, the number of dimensions of the receive subspace cannot be larger than the number of dimensions of the transmit subspace. This holds also for $N_{\text{out}} > N_{\text{in}}$. In this case, the rows of $Y$ are linearly dependent. In the opposite case of $N_{\text{out}} < N_{\text{in}}$ the receive subspace is only a sub-subspace of the transmitted subspace.

The ML detection rule for noncoherent reception is given as

$$
\hat{X}_{\text{ML}} = \arg \max_{A_i} p(Y|X = A_i),
$$

(5.3)

where $A_i$ is taken from the set of possible transmit matrices. Assuming an independent identically distributed (i.i.d.) Rayleigh channel matrix $H_{\text{MIMO}}$, the PDF of the receive matrix is given as [36]

$$
p(Y|X) = \frac{\exp(-\text{tr}(Y\Lambda^{-1}Y^H))}{\pi^{N_{\text{col}}N_{\text{out}}} \det N_{\text{out}} \Lambda},
$$

(5.4)

where $\Lambda$ denotes the covariance matrix of the receive symbols. For unitary transmit
5.2 Stiefel and Grassmann Manifolds

symbols, i.e., \( A_i A_i^H = I_{N_{in}} \), this leads to (see Appendix B.1)

\[
\hat{X}_{\text{ML}} = \arg \max_{A_i} \text{tr} \left( Y A_i^H A_i Y^H \right) \\
= \arg \max_{A_i} \| Y A_i^H \|_F^2.
\] (5.5)

In (5.5) the notation \( \| \cdot \|_F^2 \) stands for the squared Frobenius norm which is defined as the sum of the squared absolute values of all matrix elements.

Another metric is the generalized likelihood ratio test (GLRT). It is particularly suited for noncoherent signal detection, because it does not even need knowledge about the fading or noise statistics. The GLRT computes the joint ML estimate of the transmit symbol and the channel matrix. It is defined as \([7, 91]\)

\[
\hat{X}_{\text{GLRT}} = \arg \max_{A_i} \sup_{H_{\text{MIMO}}} p(Y | X = A_i, H_{\text{MIMO}}) \\
= \arg \max_{A_i} \text{tr} \left( Y A_i^H \left( A_i A_i^H \right)^{-1} A Y^H \right). \tag{5.6}
\]

The derivation of the GLRT metric (5.6) is given Appendix B.2. For a unitary symbol alphabet and i.i.d. fading the ML and the GLRT metrics are equivalent. Both the GLRT and ML metric project the received symbol on the different subspaces that are spanned by the possible transmit symbols \( A_i \). The decision is made in favor of the projection with the maximum energy.

5.2 Stiefel and Grassmann Manifolds

When we talk about information transmission by subspaces, it is useful to have a look at the Grassmann and the Stiefel manifolds because the employed codewords can be regarded as elements of these manifolds \([30, 86, 102]\). In general, manifolds are spaces that locally look like the Euclidean space, i.e., each point on a manifold of dimension \( n \) has a neighborhood which is homeomorphic to a \( n \)-dimensional open ball in the complex (or real) Euclidean space \( \mathbb{C}^n \) (\( \mathbb{R}^n \)) \([10]\).

The two manifolds that are of particular interest for our application are the Stiefel and the Grassmann manifold. The Stiefel manifold \( V(A, B) \) is defined as the set of all \( B \times A \)-dimensional unitary matrices in the complex vector space \( \mathbb{C}^{B \times A} \)

\[
V(A, B) := \{ U \in \mathbb{C}^{B \times A} | U U^H = I_B \}. \tag{5.7}
\]

As a simple example we can take the case of \( B = 1 \). Now, the unitary matrices are actually unit vectors and the Stiefel manifold is equivalent to the unit sphere in \( \mathbb{C}_A \). Note that the unitary transmit symbols \( A_i \) can be identified as points in the Stiefel manifold.
The definition of the Grassmann manifold $G(A, B)$ is based on the Stiefel manifold and can be written as:

$$G(A, B) := \{ \langle U \rangle \mid U \in V(A, B) \}, \quad (5.8)$$

where $\langle U \rangle$ stands for the subspace spanned by the column vectors of $U$. In other words, the Grassmann manifold is the set of all $B$-dimensional vector subspaces in the $A$-dimensional vector space, i.e. the subspaces spanned by the elements of $V(A, B)$.

As explained in the previous section, the transmission over a noiseless MIMO channel obeying (5.1) can only lead to receive signals that lie in the same subspace as the transmitted symbols. This means, the information that is transmitted is contained in the subspace, i.e., the transmit signal can be identified with a point on the Grassmann manifold, which in turn is defined by a point on the Stiefel manifold.

If we want to characterize a symbol alphabet, we need a distance metric to measure the separation between two symbols. While a subspace can be represented by many different orthonormal bases, we need a measure that gives a unique distance between two subspaces. So the task is to calculate the distance between two points on the Grassmann manifold $G(A, B)$ that represent the two subspaces $\langle U_1 \rangle$ and $\langle U_2 \rangle$. The most useful measure on the Grassmannian space is the chordal distance [16], which is defined as

$$d_c(\langle U_1 \rangle, \langle U_2 \rangle) = \sqrt{\sum_{i'} \sin^2 \theta_p^{(i')}}. \quad (5.9)$$

$\theta_p^{(i')}$ denote the principal angles between the two subspaces. They can be obtained via the singular value decomposition of $U_1 U_2^H$ [31]. The singular value decomposition yields

$$U_1 U_2^H = W \Sigma Z^H, \quad (5.10)$$

where $W$ and $Z$ are unitary matrices and $\Sigma$ is a diagonal matrix with the real non-negative singular values $\sigma_{i'}$ on its main diagonal. The rows of the matrices $U_1$ and $U_2$ have to be an orthonormal basis of the subspaces they represent. In this case, the relationship between the singular values and the principal angles is given as

$$\sigma_{i'} = \cos \left( \theta_p^{(i')} \right) \quad i' = 1, \ldots, B. \quad (5.11)$$

By inserting (5.11) into (5.9), we can also relate the chordal distance directly to the singular values of $U_1 U_2^H$ by

$$d_c(\langle U_1 \rangle, \langle U_2 \rangle) = \sqrt{\sum_{i'} \left( 1 - \cos^2 \theta_p^{(i')} \right)} = \sqrt{\sum_{i'} \left( 1 - \sigma_{i'}^2 \right)}. \quad (5.12)$$
5.3 Subspace Representation of OFDM-MFSK-Based Modulation Schemes

The measures and properties connected to manifolds that have been presented in this section are sufficient to describe the subspace representation of the modulation schemes considered in this work. Therefore, we will not go further into the details about manifolds and refer the interested reader to [10] for additional information.

5.3 Subspace Representation of OFDM-MFSK-Based Modulation Schemes

In this section, we show that it is possible to interpret our OFDM-MFSK-based modulation schemes as information transmission using subspaces similar to the MIMO subspace model presented in Section 5.1. Depending on the specific modulation scheme, certain channel conditions have to be fulfilled in order to make the model applicable.

5.3.1 OFDM-MFSK

Let us begin with the subspace representation of OFDM-MFSK. We assume the OFDM model from (4.1) where the ISCI caused by time variance is regarded as additional noise and the diagonality of the OFDM channel matrix allows a separate consideration of the OFDM-MFSK subcarrier groups:

$$\tilde{x}_D = \mathbf{H}_D x_D + n_D. \quad (5.13)$$

The common phase $\varphi$ from (4.1) has been considered to be part of the channel matrix here. For single symbol detection, the OFDM-MFSK transmit vector $x_D$ contains only one nonzero element (cf. Table 4.1 for OFDM-4FSK). Therefore, only one diagonal element of $\mathbf{H}_D$ will have an influence on the received vector $\tilde{x}_D$ and we can also write (5.13) with a single unknown channel coefficient as

$$\tilde{x}_D = h x_D + n_D. \quad (5.14)$$

In order to obtain a similar notation as in Section 5.1, we rewrite (5.14) as

$$\tilde{x}_D^H = h^* x_D^H + n_D^H, \quad (5.15)$$

so that the transmit and receive symbols are represented by row vectors. If we compare this model to the commonly used model for STM (e.g. [30, 36]) the frequency direction in OFDM-MFSK corresponds to the time direction in STM. For OFDM-MFSK, the transmit symbol alphabet fulfills $a_i^H a_i = 1$, i.e., we have unitary codewords. Using the GLRT metric, we obtain

$$\hat{x}_D = \arg \max_{a_i} \text{tr} \left( \tilde{x}_D^H a_i a_i^H \tilde{x}_D \right)$$

$$= \arg \max_{a_i} |\tilde{x}_D^H a_i|^2,$$  

$$ = \arg \max_{a_i} |\hat{x}_D a_i|^2, \quad (5.16)$$
Fig. 5.1: Subspace representation of OFDM-2FSK without noise. The markers represent the two possible transmit symbols. The corresponding receive symbols can take any place on the dashed/dash-dotted line.

which is equivalent to the metrics derived for OFDM-MFSK in Section 4.1. This means that noncoherent data transmission using an OFDM-MFSK modulation scheme can be regarded as information transmission using subspaces. The $M$ transmit symbols define points on the Grassmann manifold $G(M, 1)$, i.e., they span one-dimensional complex subspaces in the $M$-dimensional space. The channel can only scale and rotate the transmitted symbol vectors within the subspace but cannot lead to points outside the subspace.

As an illustrative example, we plot the signal vectors for OFDM-2FSK in Fig. 5.1. In order to be able to visualize the vectors, we assume real-valued signals. In general, the vector components can have an arbitrary phase for noncoherent transmission. The two possible 2FSK transmit vectors $a_1 = [1, 0]^T$ and $a_2 = [0, 1]^T$ are represented by the round and square markers in Fig. 5.1. The unknown channel can scale the transmitted vector, so that the receive vector is located on the dashed line corresponding to the transmitted signal point. The two lines coincide with the axes the two-dimensional vector space and they represent the two one-dimensional subspaces that are used for information transmission.

We are representing the transmit and receive signals of orthogonal MFSK schemes, so any two different subspaces are orthogonal to each other and the principal angle between the two subspaces is $\theta_p = 90^\circ$:

$$\sigma_1 = |a_i^H a_j| = 0 \quad \text{for} \quad i \neq j$$
$$\theta_p = \arccos (\sigma_1) = 90^\circ. \quad (5.17)$$

The chordal distance between any two different subspaces $\langle a_i \rangle$ and $\langle a_j \rangle$ according to
5.3 Subspace Representation of OFDM-MFSK-Based Modulation Schemes

(5.9) is

\[ d_c (\langle a_i \rangle, \langle a_j \rangle) = 1. \]  

(5.18)

A second possibility to represent OFDM-MFSK transmission is to use all \( N \) OFDM subcarriers as the dimensions of our vector space. Again, we consider the ISCI as additional noise so that the matrix \( H_{\text{OFDM}} \) representing the channel is a diagonal matrix. For OFDM-MFSK we can rewrite the OFDM model (2.37) in the following way to obtain a representation similar to (5.15):

\[ \tilde{x}^H = h^H \cdot X_{\text{FSK}}^H + n^H, \]  

(5.19)

where

\[ X_{\text{FSK}}^H \in \mathbb{C}^{(N/M) \times N} \]
\[ h^H \in \mathbb{C}^{1 \times (N/M)}. \]

The transmit matrix \( X_{\text{FSK}}^H \) represents one single OFDM symbol and has a special structure: In each of the \( N/M \) rows there is only one nonzero element. In particular, the \( n \)-th row contains a nonzero entry corresponding to the occupied subcarrier of the \( n \)-th MFSK group. Because of this structure, we can model an arbitrary diagonal channel matrix \( H_{\text{OFDM}} \) by \( N/M \) coefficients using the vector \( h^H \). The example for OFDM-2FSK in (5.20) shows the structure of a possible transmit matrix. The vertical lines indicate the borders of the subcarrier groups.

\[ X_{\text{FSK}}^H = \begin{pmatrix}
    e^{j \theta_1} & 0 & 0 & 0 & 0 & \cdots \\
    0 & 0 & e^{j \theta_2} & 0 & 0 & \cdots \\
    0 & 0 & 0 & e^{j \theta_3} & 0 & \cdots \\
    \vdots & \vdots & \vdots & \vdots & \ddots & \ddots
  \end{pmatrix}. \]  

(5.20)

Using this representation, the OFDM-MFSK transmit symbols can be interpreted as a basis of \( N/M \)-dimensional subspaces of the \( N \)-dimensional space. The symbols of the transmit alphabet \( A_i^H \) are unitary matrices and represent points on the Grassmann manifold \( G(N, N/M) \):

\[ A_i^H A_i = I_{N/M} \]
\[ \langle A_i^H \rangle \in G(N, N/M) \]  

(5.21)

The receiver performs a projection of the received vector onto the possible subspaces according to

\[ \hat{X}_{\text{FSK}} = \arg \max_{A_i} \text{tr} \left( \tilde{x}^H A_i A_i^H \tilde{x} \right) = \arg \max_{A_i} \| \tilde{x}^H A_i \|_F^2 \]  

(5.22)

and decides for the maximum projection.
The minimum distance in the symbol alphabet is obtained for symbols differing in only one subcarrier group. In this case, $A_i^H A_j$ yields a diagonal matrix with one zero and otherwise ones on the diagonal. The principal angles $\theta_p^{(i')}$ ($i' = 1, \ldots, N/M$) are all $0^\circ$ except for one, which is $90^\circ$. The minimum chordal distance is therefore

$$d_{c,\text{min}} = \sqrt{\sum_{i'} \left(1 - \cos^2 \theta_p^{(i')}\right)} = 1. \quad (5.23)$$

The channel coefficients of different subcarrier groups are assumed to be uncorrelated in (5.15) and (5.19). Therefore, the two models presented in this subsection are suited to describe OFDM-MFSK with single symbol detection, where correlations are not considered in the receiver. However, the general subspace model can also be applied to MSD under certain conditions. Because there is a close connection to hybrid modulation schemes, we will treat these topics jointly in the next section.

### 5.3.2 Hybrid Modulation Schemes

The hybrid modulation schemes presented in Section 3.1.3 can also be interpreted as subspace-based transmission. In contrast to the subspace interpretation of plain OFDM-MFSK that holds for arbitrary diagonal channel matrices $H_{\text{OFDM}}$, we assume a channel model where $N_{\text{MSD}}$ neighboring subcarrier groups are multiplied by the same channel coefficient. This means that the channel has to be sufficiently flat. Under this condition, the model is almost the same as (5.15) for OFDM-MFSK. The only difference is that the transmit and receive symbol vectors contain several MFSK symbols. We can write

$$\tilde{x}_D^H = h^* x_D^H + n_D^H, \quad (5.24)$$

where the transmit symbols $x_D = a_i$ have the following properties:

$$a_i^H \in \mathbb{C}^{1 \times M \cdot N_{\text{MSD}}} \quad (5.25)$$

$$a_i^H a_i = N_{\text{MSD}}. \quad (5.26)$$

After multiplication with the unknown channel coefficient (5.24), the received symbols can occupy any point on a line in the $(M \cdot N_{\text{MSD}})$-dimensional space. The information is thus contained in the one-dimensional subspace defined by the transmit symbol:

$$\langle a_i^H \rangle \in G(M \cdot N_{\text{MSD}}, 1). \quad (5.27)$$

The receiver executes the projection of the receive symbols on the subspaces spanned by the transmit symbols:

$$\hat{x}_D = \arg \max_{a_i} |\tilde{x}_D^H a_i|^2. \quad (5.28)$$
5.3 Subspace Representation of OFDM-MFSK-Based Modulation Schemes

(a) Four possible transmit symbols.

(b) Possible receive symbols (real part) that were subject to fading and include noise.

Fig. 5.2: Subspace representation of OFDM-2FSK-2DPSK.
5 Noncoherent Communication Based on Subspaces

This is equivalent to the metrics derived for OFDM-MFSK in Section 4.1.

As an example, we use the symbol alphabet for hybrid OFDM-2FSK-2DPSK modulation and MSD with \( N_{\text{MSD}} = 2 \) that was given in Table 4.2. The minimum principal angle in this case is

\[
\theta_{\text{p}, \text{min}} = \min_{i \neq j} \arccos \left( \frac{|a_i^H a_j|}{N_{\text{MSD}}} \right) = 60^\circ.
\]  

As we are only able to visualize three dimensions, we take only the symbols where the first component is \( a_{i,1} = 0 \) and plot the other three components. Fig. 5.2a shows the corresponding part of the transmit symbol alphabet. The real parts of the receive symbols are plotted in Fig. 5.2b. The channel leads to a scaling and complex rotation of the symbols inside their subspaces. This is clearly visible in Fig. 5.2b as all the possible receive symbols belonging to a certain transmit symbol are located along a line. Only the noise leads to receive symbols outside these lines. This figure shows also that not the Euclidean distance between the transmit vectors is the appropriate distance measure but the separation of the lines, i.e., the principal angles between the subspaces.

We would like to point out that the interpretation for hybrid modulation schemes given in this subsection is also valid for plain OFDM-MFSK with MSD but without additional phase modulation. The only difference is the set of transmit vectors \( x_D = a_i \).

5.3.3 Multitone FSK

In Section 3.1.1 we mentioned that multitone FSK can be used in combination with OFDM to increase the bandwidth efficiency compared to OFDM-MFSK [54]. Although we didn’t go into more detail about this method and its performance in this work, we want to show that it is possible to describe multitone FSK as a subspace-based information transmission.

In our original OFDM model (4.1), the multitone FSK transmit vectors \( x_D \) consist of \( M \) elements where \( N_{\text{MT}} \) elements are assigned a certain nonzero value and the others are zero. In order to apply the subspace model, it is advantageous to modify the model slightly. Similar to (5.19) we can rewrite the model so that the transmit symbols can be written as matrices that contain only one nonzero element per row and we obtain

\[
\tilde{x}_D^H = h^H \cdot X_{\text{MT-FSK}}^H + n^H,
\]

where

\[
X_{\text{MT-FSK}}^H \in \mathbb{C}^{N_{\text{MT}} \times M}
\]

\[
h^H \in \mathbb{C}^{1 \times N_{\text{MT}}}
\]

The model contains one fading coefficient per occupied subcarrier so that we can represent arbitrary OFDM channel matrices similar to (5.19). The transmit matrix \( X_{\text{MT-FSK}}^H \)
represents one multitone FSK symbol that uses \( M \) subcarriers, i.e., we consider only a subgroup of the OFDM subcarriers. An example transmit matrix for \( N_{\text{MT}} = 2 \) and \( M = 4 \) is

\[
X^H_{\text{MT-FSK}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}.
\] (5.31)

The symbol alphabet of all possible transmit matrices \( X^H_{\text{MT-FSK}} = A^H_i \) consists of unitary matrices and each matrix spans an \( N_{\text{MT}} \) dimensional subspace of the \( M \) dimensional space:

\[
A^H_i A_i = I_{N_{\text{MT}}}, \quad \langle A^H_i \rangle \in G(M, N_{\text{MT}}). \tag{5.32}
\]

Fig. 5.3 shows the subspace model of multitone OFDM-FSK with \( M = 3 \) and \( N_{\text{MT}} = 2 \). Although this is not an example of practical relevance, it is the only example for multitone FSK that can be visualized in a three-dimensional plot. The figure shows the three possible subspaces that are spanned by the rows of the transmit matrices. The two basis vectors spanning \( \langle A^H_i \rangle \) are included in the plot for the example

\[
A^H_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.
\] (5.33)

The principal angles between any two subspaces are \( \theta^{(1)}_p = 90^\circ \) and \( \theta^{(2)}_p = 0^\circ \) which leads to a chordal distance of \( d_c = 1 \) between all subspaces.

Again, the noncoherent GLRT metric

\[
\hat{X}_{\text{MT-FSK}} = \arg \max_{A_i} \| \tilde{x}_D^H A_i \|^2_F
\] (5.34)

calculates the projection of the receive vectors on all possible subspaces and is equivalent to summing up the energies on all subcarriers according to the test symbol.

### 5.4 General Remarks

The previous sections show that is is possible to interpret OFDM-MFSK-based modulation schemes as information transmission using vector subspaces. The subspace representation reveals, that the distance of the transmitted subspaces is by no means optimum for all considered modulation schemes. For example, some subspace pairs for OFDM-2FSK-2DPSK have a principal angle of \( \theta^{(1)}_p = 60^\circ \) while other pairs have \( \theta^{(1)}_p = 90^\circ \). This means that the distribution of the eight one-dimensional subspaces in the four-dimensional space is not optimum. In [36], the authors propose an algorithm that can be used to reduce the largest pairwise correlation between the signals in a constellation by iteratively rotating the two vectors with the largest correlation away from each other.
Fig. 5.3: Subspace representation of multitone OFDM-FSK with $M = 3$ and $N_{MT} = 2$ without noise. The marked vectors represent the basis vectors that span the subspace $\langle A_1^H \rangle$, which is a plane. The corresponding receive symbols can take any point in this plane.

from each other. This is equivalent to maximizing the minimum angle between the corresponding two subspaces. However, by rotating the vectors, our OFDM-MFSK-based modulation schemes lose their special structure so that a single symbol detection of the MFSK component is not possible any more if the channel turns out to be too frequency selective to recover the DPSK component.
Summary and Conclusions

In this thesis, we examined transmission schemes that are suited for reliable transmission over time-variant multipath channels. Because there might be many channel parameters that are changing with time, it can be quite hard to estimate them with the required precision at all times. However, exact channel knowledge is essential for coherent signal detection. An alternative is to use modulation schemes that allow non-coherent symbol detection at the receiver so that an estimation of the channel is not necessary. In this work, we focused on the second approach. Furthermore, all considered schemes are based on OFDM so that ISI can be avoided and there is no need for a complex equalizer in the receiver.

Chapter 2 presented the necessary fundamentals for this thesis. Besides the introduction of some basic channel models, the chapter dealt with the general description of time-variant and frequency selective channels. We recapitulated the description of deterministic and stochastic channel models by the corresponding system functions. Furthermore, we presented the vector transmission model for OFDM that we used throughout this thesis including the case of time-variant channels. Finally, the capacity for channels under certain constraints was given in order to see the theoretical limits for digital transmissions. The focus was on channels without a priori knowledge at the transmit and the receive side.

In Chapter 3, we introduced several new OFDM-based modulation schemes that are suited for reliable data transmission under challenging channel conditions. The proposed modulation schemes are based on a combination of OFDM and MFSK. The use of OFDM in conjunction with channel coding and a cyclic prefix leads to a scheme that can cope with multipath propagation without complex equalization. In addition, the noncoherent detectability of the MFSK symbols allows a simple receiver that does not need precise channel knowledge. Therefore, it is very robust against rapid channel variations. Plain OFDM-MFSK has a quite low bandwidth efficiency. Therefore, we proposed some modulation schemes that employ the unused phases of the MFSK symbols to transmit additional data. In this case, the phases of neighboring MFSK symbols within an OFDM symbol are differentially encoded using DPSK. Because there are two
Summary and Conclusions

components that are combined to form the transmitted OFDM symbols, we called these schemes \textit{hybrid} modulation schemes. The detection of these hybrid OFDM-FSK-DPSK schemes can still take place in a noncoherent way, i.e., explicit channel knowledge is not necessary. Chapter 3 also presented the complete system model that was used to model the transmission including channel coding and hybrid modulation. We proposed the use of an iterative detector that employs a joint demapping of the MFSK and DPSK component contained the receive vectors.

Chapter 4 addressed the noncoherent signal detection for OFDM-MFSK-based modulation schemes. By adapting known results from the literature, a general receive metric was derived for our OFDM-MFSK-based modulation schemes and its applicability when using hybrid modulation schemes was pointed out. The general metric was further specified for different channel models. We showed that the performance of OFDM-MFSK can be improved by using an iterative receiver both for the case of an AWGN channel and time-variant and frequency selective fading channels. The iterative receiver uses feedback information from the channel decoder in order to improve the reliability of the demapper output. In addition, we proposed multiple symbol detection for OFDM-MFSK. For this, several neighboring MFSK symbols within an OFDM symbol are detected jointly in order to exploit correlations of the channel transfer function at neighboring subcarrier frequencies. Our results indicate that the BER can be improved significantly for channels with low frequency selectivity or if the statistical properties of the channel are known at the receiver. For the AWGN channel it was shown that jointly detecting two MFSK symbols is enough to come close to the BER of coherent detection for high SNR.

For hybrid modulation schemes we examined a serial detection method where the MFSK information is extracted from the receive vectors independently of the subsequent detection of the DPSK symbols. This has the advantage that the performance of the MFSK transmission is not degraded by a failed detection of the DPSK component. However, for channels with low frequency selectivity our simulation results revealed that it is advantageous to employ a joint detection of super-symbols containing several MFSK and DPSK symbols. We showed that in this way the receiver can benefit from correlations of the channel within the detection window. Also for the joint detection it is beneficial to use an iterative receiver. To analyze the behavior of the joint iterative detector for hybrid modulation schemes, we used three-dimensional EXIT charts that visualize the exchange of mutual information among the channel decoders and the demapper output for the MFSK and DPSK component, respectively.

The EXIT chart analyses in Chapter 4 revealed that the matching of the symbol mapping and the channel code has an essential influence on the performance of the iterative detector for OFDM-MFSK-based modulation schemes. Therefore, we proposed the use of two methods that can be applied to adapt the bit mapping of OFDM-MFSK to the channel code. The first one, extended mapping, leads to an ambiguous mapping where several bit labels are assigned to the same symbol of the transmit alphabet. Due to the increased number of transmitted bits we can use a lower rate channel code without reducing the overall bandwidth efficiency. We showed how to find optimized
mappings and matching channel codes by using EXIT charts. We also presented simulation results that confirmed the improved performance of extended mapping for high SNR. The second method is the use of an additional recursive convolutional code with rate one that serves as an intermediate code in order to improve the convergence characteristics of the iterative receiver. This precoder introduces dependencies among all bits of one codeword prior to the mapping to transmit symbols. The iterative receiver can make use of these dependencies so that we were able to obtain an improved BER both for AWGN and fading channels. The mapper, precoder, and channel decoder form a system of three serially concatenated codes. Also in this case we used three-dimensional EXIT charts to obtain an insight into the iterative behavior of the overall detector.

In Chapter 5 we introduced a new interpretation of noncoherent communication with OFDM-MFSK-based modulation schemes. We showed that it is possible to interpret these schemes as a digital transmission using subspaces, similar to unitary space-time modulation for multiple antenna systems. The transmitted information is contained in the subspace that is spanned by the transmit vector or matrix, respectively. If certain channel conditions are fulfilled, the channel cannot lead to receive vectors outside the subspace that is spanned by the transmit symbol. Only the additive noise can lead to receive symbols outside the transmitted subspace. The receive metric projects the received vector onto the subspaces and decides for the maximum projection. The metric is equivalent to the receive metric derived for the original OFDM model in Chapter 4.

One advantage of OFDM-MFSK-based transmission is the simplicity inherent to all modulation schemes that allow noncoherent detection because there is no need for channel estimation and equalization. This independence of precise channel knowledge is especially advantageous in time-varying environments, where the estimated channel is outdated very quickly. Due to the use of OFDM with a cyclic prefix, we can avoid ISI also for multipath propagation. Maybe the biggest advantage of the proposed hybrid modulation schemes is their flexibility in terms of data rate and robustness against time-variant and frequency selective channels. As long as the frequency selectivity of the channel is low, we can jointly detect the MFSK and DPSK symbols and improve the error probability by using iterative MSD. If the joint detection fails because the frequency selectivity of the channel is too high, it is still possible to extract the information contained in the MFSK component. Without the need for retransmission we can simply use the saved receive vector and apply iterative single symbol detection of the MFSK symbols, which is not affected by the frequency selectivity of the channel. This fact makes the transmission scheme particularly suited for applications where a part of the data has to be transmitted with high reliability also if the channel is difficult. In addition, the differential phase component can be used to transmit low priority data where a loss of data is less critical.
The PEP for the AWGN channel has been derived for different modulation schemes in [13, 19, 20, 77]. We follow these derivations and adapt them to our application. The PEP for OFDM-MFSK with MSD is given as

\[
\text{Prob} \left\{ |\bar{x}_D^H a_i|^2 < |\bar{x}_D^H a_j|^2 | a_i \right\} = \frac{1}{2} \left( 1 - Q \left( \sqrt{b}, \sqrt{a} \right) + Q \left( \sqrt{a}, \sqrt{b} \right) \right),
\]

(A.1)

with

\[
\left\{ \begin{array}{c}
a \\ b
\end{array} \right\} = \frac{1}{2N_S} \left( \frac{S_1 + S_2 - 2|\rho|\sqrt{S_1 S_2}}{1 - |\rho|^2} + \frac{S_1 - S_2}{\sqrt{1 - |\rho|^2}} \right),
\]

(A.2)

using the notation of [19], where

\[
S_1 = |\bar{x}_D^H a_i|^2 = N_{\text{MSD}}^2,
\]

(A.3)

\[
S_2 = |\bar{x}_D^H a_j|^2 = \delta_{a_i a_j}^2, \quad \text{where} \quad \delta_{a_i a_j} = \sum_{a_i^{(l)} = a_j^{(l)}} 1,
\]

(A.4)

\[
N_S = |\bar{x}_D^H a_i - \bar{x}_D^H a_j|^2 = |\bar{x}_D^H a_j - \bar{x}_D^H a_i|^2
\]

\[
= |n_D^H a_i|^2 = N_{\text{MSD}} \sigma_n^2,
\]

(A.5)
and
\[
\rho = \frac{1}{N_S} \left( \tilde{x}_D^H a_i - \tilde{x}_D^H a_j \right)^* \left( \tilde{x}_D^H a_j - \tilde{x}_D^H a_j \right)
= \frac{1}{N_{\text{MSD}}} \frac{1}{\sigma_n^2} \tilde{n}_D^H a_i a_j
= \frac{1}{N_{\text{MSD}}} \delta_{a_i a_j}. \tag{A.6}
\]

This leads to
\[
\begin{align*}
\left\{ \begin{array}{c}
a \\
b \\
\end{array} \right\} &= \frac{1}{2N_{\text{MSD}} \sigma_n^2} \left( \frac{N_{\text{MSD}}^2 + \delta_{a_i a_j}^2 - \frac{2\delta_{a_i a_j}}{N_{\text{MSD}}} \sqrt{N_{\text{MSD}}^2 \delta_{a_i a_j}^2}}{1 - \left( \frac{\delta_{a_i a_j}}{N_{\text{MSD}}} \right)^2} \right) + \frac{N_{\text{MSD}}^2 - \delta_{a_i a_j}^2}{\sqrt{1 - \left( \frac{\delta_{a_i a_j}}{N_{\text{MSD}}} \right)^2}}, \\
&= \frac{1}{2N_{\text{MSD}} \sigma_n^2} \left( \frac{N_{\text{MSD}}^2 - \delta_{a_i a_j}^2}{1 - \left( \frac{\delta_{a_i a_j}}{N_{\text{MSD}}} \right)^2} \right) + \frac{N_{\text{MSD}} \left( N_{\text{MSD}}^2 - \delta_{a_i a_j}^2 \right)}{\sqrt{N_{\text{MSD}}^2 - \delta_{a_i a_j}^2}}, \\
&= \frac{1}{2\sigma_n^2} \left( N_{\text{MSD}}^2 \pm \sqrt{N_{\text{MSD}}^2 - \delta_{a_i a_j}^2} \right). \tag{A.7}
\end{align*}
\]

### A.1 Approximation of the PEP for OFDM-MFSK with MSD for High SNR

For low noise, we can use the following approximation in (4.35) [19]
\[
\frac{1}{2} \left( 1 - Q \left( \sqrt{b}, \sqrt{a} \right) + Q \left( \sqrt{a}, \sqrt{b} \right) \right) \\
\approx \frac{1}{2} \exp \left( - \left( \sqrt{b} - \sqrt{a} \right)^2 / 2 \right) \left( \sqrt{a} \sqrt{b} + \sqrt{b} \sqrt{a} \right). \tag{A.8}
\]

The PEP for OFDM-MFSK with MSD can therefore be approximated by
\[
\text{PEP} \approx \frac{1}{2} \exp \left( - \frac{1}{2\sigma_n^2} \left( N_{\text{MSD}} - \delta_{a_i a_j} \right) \right) \sqrt{N_{\text{MSD}} + \delta_{a_i a_j}}. \tag{A.9}
\]

With
\[
\text{erfc}(x) \approx \frac{1}{\sqrt{\pi x}} \exp(-x^2) \tag{A.10}
\]

104
we can further approximate the PEP, leading to:

$$\text{PEP} \approx \frac{1}{2} \text{erfc} \left( \sqrt{\frac{1}{2\sigma_r^2}} (N_{\text{MSD}} - \delta_{a_i,a_j}) \right) \frac{\sqrt{N_{\text{MSD}} + \delta_{a_i,a_j}}}{\sqrt{2\delta_{a_i,a_j}}} \right). \quad (A.11)$$
Metrics for noncoherent MIMO channels

B.1 ML Metric

We assume a channel matrix $H_{\text{MIMO}}$ with dimension $N_{\text{out}} \times N_{\text{in}}$ and i.i.d. Rayleigh coefficients. Because the channel coefficients and therefore also the rows $y_{rn}$ of the receive matrix are statistically independent, the conditional PDF of the receive matrix can be determined as

$$p(Y|X) = \prod_{n} p(y_{rn}|X), \quad (B.1)$$

where $y_{rn}$ denotes the $n$-th row of the receive matrix $Y$. Similar to (4.2), the PDF of $y_{rn}$ is given as [41]

$$p(y_{rn}|X) = \frac{\exp \left( -y_{rn} \Lambda^{-1} y_{rn}^H \right)}{\pi^{N_{\text{col}}} \det \Lambda}, \quad (B.2)$$

which leads to the result of [36]

$$p(Y|X) = \frac{\exp \left( -\text{tr} \left\{ Y \Lambda^{-1} Y^H \right\} \right)}{\pi^{N_{\text{col}} N_{\text{out}}} \det N_{\text{out}} \Lambda}. \quad (B.3)$$

The covariance matrix of the receive symbols can be determined as

$$\Lambda = E \left\{ (y_{rn} - \bar{y}_{rn})^H (y_{rn} - \bar{y}_{rn}) \right\} = E \left\{ (h_{rn}X + n_{rn})^H (h_{rn}X + n_{rn}) \right\} = X^H \Lambda_{\text{MIMO}} X + \sigma_{rn}^2 I_{N_{\text{col}}}, \quad (B.4)$$

where we use the fact that the receive vectors have zero mean and denote the $n$-th row of the channel matrix as $h_{rn}$. According to our assumption, the coefficients of the
channel matrix are uncorrelated so that $\Lambda_{\text{MIMO}} = I_{N_{\text{in}}}$. Taking the inverse of $\Lambda$ yields
\[
\Lambda^{-1} = \left( X^H X + \sigma_n^2 I_{N_{\text{col}}} \right)^{-1} \\
= \frac{1}{\sigma_n^2} I_{N_{\text{col}}} - \frac{1}{\sigma_n^2} X^H \left( 1 + \frac{1}{\sigma_n^2} X X^H \right)^{-1} X \frac{1}{\sigma_n^2} \\
= \frac{1}{\sigma_n^2} I_{N_{\text{col}}} - \frac{1}{\sigma_n^2} \left( 1 + \frac{1}{\sigma_n^2} \right)^{-1} X^H X.
\]
(B.5)

The last step in (B.5) uses the assumption of unitary transmit symbols so that $XX^H = I_{N_{\text{in}}}$. Inserting (B.5) into (B.3) leads to
\[
p(Y|X) = \exp \left( -\text{tr} \left\{ Y \left( \frac{1}{\sigma_n^2} I_{N_{\text{col}}} - \frac{1}{\sigma_n^2} \left( 1 + \frac{1}{\sigma_n^2} \right)^{-1} X^H X \right) Y^H \right\} \right) \\
\quad \pi_{N_{\text{col}} N_{\text{out}}} (1 + \sigma_n^2)^{N_{\text{col}} N_{\text{out}}}.
\]
(B.6)

Ignoring the terms that are independent of the transmit symbols, the ML decision reduces to a maximization of the conditional PDF with respect to all possible transmit symbols $A_i$ using
\[
\hat{X}_{\text{ML}} = \arg \max_{A_i} p(Y|X = A_i) \\
= \arg \max_{A_i} \text{tr} \left\{ YA_i^H A_i Y^H \right\} \\
= \arg \max_{A_i} \|YA_i^H\|^2_{F}.
\]
(B.7)

### B.2 GLRT Metric

From the definition of the GLRT we get
\[
\hat{X}_{\text{GLRT}} = \arg \max_{A_i} \sup_{H_{\text{MIMO}}} p(Y|X = A_i, H_{\text{MIMO}}) \\
= \arg \min_{A_i} \inf_{H_{\text{MIMO}}} \|Y - H_{\text{MIMO}} A_i\|^2_F.
\]
(B.8)

The best estimate for the channel is given by [91]
\[
\hat{H}_{\text{MIMO}} = YA_i^H \left( A_i A_i^H \right)^{-1}.
\]
(B.9)
Inserting the estimate for the channel matrix (B.9) into (B.8) we obtain

\[
\hat{X}_{\text{GLRT}} = \arg \min_{\mathbf{A}_i} \text{tr} \left\{ \left( \mathbf{Y} - \mathbf{Y} \mathbf{A}_i^H \left( \mathbf{A}_i \mathbf{A}_i^H \right)^{-1} \mathbf{A}_i \right) \left( \mathbf{Y} - \mathbf{Y} \mathbf{A}_i^H \left( \mathbf{A}_i \mathbf{A}_i^H \right)^{-1} \mathbf{A}_i \right)^H \right\}
\]

\[
= \arg \min_{\mathbf{A}_i} \text{tr} \left\{ \mathbf{Y} \mathbf{Y}^H - \mathbf{Y} \mathbf{A}_i^H \left( \mathbf{A}_i \mathbf{A}_i^H \right)^{-1} \mathbf{A}_i \mathbf{Y}^H \right\}
\]

\[
= \arg \max_{\mathbf{A}_i} \text{tr} \left\{ \mathbf{Y} \mathbf{A}_i^H \left( \mathbf{A}_i \mathbf{A}_i^H \right)^{-1} \mathbf{A}_i \mathbf{Y}^H \right\}. \quad (B.10)
\]

In contrast to the ML metric (B.7), which assumed an i.i.d. channel matrix and uncorrelated noise, the GLRT metric does not need any information about the fading or noise statistics.
### List of Frequently Used Operators, Symbols, and Acronyms

#### Operators

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A^*$</td>
<td>complex conjugate of matrix $A$</td>
</tr>
<tr>
<td>$A^H$</td>
<td>complex conjugate transposition of $A$</td>
</tr>
<tr>
<td>$A^T$</td>
<td>transposition of $A$</td>
</tr>
<tr>
<td>$A^{-1}$</td>
<td>inverse of $A$</td>
</tr>
<tr>
<td>$\bar{A}$</td>
<td>mean of $A$</td>
</tr>
<tr>
<td>$[A]_{m,n}$</td>
<td>$(m, n)$-th entry of $A$</td>
</tr>
<tr>
<td>$[a]_m$</td>
<td>$m$-th entry of vector $a$</td>
</tr>
<tr>
<td>$E{\cdot}$</td>
<td>expected value</td>
</tr>
<tr>
<td>$\text{Re}{\cdot}$, $\text{Im}{\cdot}$</td>
<td>real/imaginary part of a complex quantity</td>
</tr>
<tr>
<td>$\text{tr}(A)$</td>
<td>trace of matrix $A$</td>
</tr>
<tr>
<td>$\lfloor \cdot \rfloor$</td>
<td>floor function; returns largest integer, smaller than the argument</td>
</tr>
<tr>
<td>$\max(\cdot)$</td>
<td>maximum value of the arguments</td>
</tr>
<tr>
<td>$\arg \max_i f(i)$</td>
<td>value of $i$ that maximizes $f(i)$</td>
</tr>
<tr>
<td>$\sup(\cdot)$</td>
<td>supremum; smallest upper bound</td>
</tr>
<tr>
<td>$\angle(\cdot)$</td>
<td>angle of the argument</td>
</tr>
<tr>
<td>$\text{Prob}(\cdot)$</td>
<td>probability of the argument</td>
</tr>
<tr>
<td>$\text{erfc}(\cdot)$</td>
<td>complementary error function</td>
</tr>
<tr>
<td>$Q(x, y)$</td>
<td>Marcum's Q function</td>
</tr>
<tr>
<td>$\text{diag}(a)$</td>
<td>forms a diagonal matrix with the vector $a$ on its diagonal</td>
</tr>
<tr>
<td>$|A|_F^2$</td>
<td>squared Frobenius norm (sum of the squared absolute values of all elements of $A$)</td>
</tr>
<tr>
<td>$\mathbb{C}^n$</td>
<td>complex Euclidean space of dimension $n$</td>
</tr>
<tr>
<td>$\mathbb{R}^n$</td>
<td>real Euclidean space of dimension $n$</td>
</tr>
<tr>
<td>$G(A, B)$</td>
<td>Grassmann manifold ($B$-dimensional subspaces in $\mathbb{C}^A / \mathbb{R}^A$)</td>
</tr>
</tbody>
</table>
V(A, B) \quad \text{Stiefel manifold (set of } B \times A \text{ unitary matrices)}

\langle A \rangle \quad \text{subspace spanned by the column vectors of } A

**List of Frequently Used Symbols**

\begin{align*}
0_{N \times M} & \quad \text{all zero matrix of size } N \times M \\
1_{N \times M} & \quad \text{all ones matrix of size } N \times M \\
a_i & \quad \text{element of the transmit vector alphabet} \\
B & \quad \text{bandwidth of the transmitted signal} \\
B_d & \quad \text{Doppler spread} \\
C & \quad \text{channel capacity} \\
C_\varepsilon & \quad \text{channel capacity with outage probability } \varepsilon \\
E_b & \quad \text{average energy per transmitted information bit} \\
f & \quad \text{frequency} \\
f_c & \quad \text{carrier frequency} \\
f_\Delta & \quad \text{subcarrier spacing} \\
\Delta f_c & \quad \text{coherence bandwidth} \\
f_D & \quad \text{Doppler frequency} \\
F & \quad \text{DFT matrix} \\
g(t) & \quad \text{receive signal} \\
g(k) & \quad \text{discrete-time receive signal} \\
h(\tau, t) & \quad \text{time-variant channel impulse response} \\
h(l, k) & \quad \text{discrete-time channel impulse response} \\
H_{\text{OFDM}}(k') & \quad \text{time-variant OFDM channel matrix} \\
H_D & \quad \text{OFDM channel matrix corresponding to } \tilde{x}_D \\
I_N & \quad \text{identity matrix of size } N \times N \\
I(x, y) & \quad \text{mutual information between } x \text{ and } y \\
L & \quad \text{length of discrete-time CIR} \\
L(c_j) & \quad \text{log-likelihood ratio corresponding to bit } c_j \\
m & \quad \text{number of bits per MFSK symbol} \\
m_E & \quad \text{number of bits per MFSK symbol for extended mapping} \\
m_C & \quad \text{memory of a convolutional code} \\
m_{\text{PC}} & \quad \text{memory of a convolutional code used as precode} \\
M & \quad \text{size of MFSK alphabet} \\
M_{\text{DPSK}} & \quad \text{size of DPSK alphabet} \\
n(t) & \quad \text{sample function of the noise process} \\
n(k) & \quad \text{noise samples} \\
n(k') & \quad \text{noise sample vector} \\
n_D & \quad \text{noise sample vector corresponding to } \tilde{x}_D \\
N & \quad \text{number of subcarriers} \\
N_0 & \quad \text{power spectral density of white Gaussian noise} \\
N_{a_i} & \quad \text{number of test vectors } a_i
\end{align*}
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_D$</td>
<td>jointly detected elements of the OFDM receive vector</td>
</tr>
<tr>
<td>$N_G$</td>
<td>length of the guard interval</td>
</tr>
<tr>
<td>$N_{MSD}$</td>
<td>number of jointly detected MFSK symbols</td>
</tr>
<tr>
<td>$P_b$</td>
<td>bit error probability</td>
</tr>
<tr>
<td>$r$</td>
<td>code rate</td>
</tr>
<tr>
<td>$r_{PC}$</td>
<td>code rate of the precode</td>
</tr>
<tr>
<td>$s(t)$</td>
<td>transmit signal</td>
</tr>
<tr>
<td>$s(k)$</td>
<td>discrete-time transmit signal</td>
</tr>
<tr>
<td>$t$</td>
<td>absolute time</td>
</tr>
<tr>
<td>$T$</td>
<td>sampling period</td>
</tr>
<tr>
<td>$T_m$</td>
<td>multipath spread</td>
</tr>
<tr>
<td>$\Delta t_c$</td>
<td>coherence time</td>
</tr>
<tr>
<td>$\mathbf{x}(k')$</td>
<td>OFDM transmit vector</td>
</tr>
<tr>
<td>$\tilde{\mathbf{x}}(k')$</td>
<td>OFDM receive vector</td>
</tr>
<tr>
<td>$\mathbf{x}_D$</td>
<td>vector of jointly detected elements of $\tilde{\mathbf{x}}$</td>
</tr>
<tr>
<td>$\mathbf{x}_D$</td>
<td>transmit vector corresponding to $\tilde{\mathbf{x}}_D$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>spectral efficiency</td>
</tr>
<tr>
<td>$\tau$</td>
<td>delay time</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>covariance matrix of the receive vector</td>
</tr>
<tr>
<td>$\Lambda_h$</td>
<td>time domain channel covariance matrix</td>
</tr>
<tr>
<td>$\Lambda_H$</td>
<td>frequency domain channel covariance matrix</td>
</tr>
<tr>
<td>$\sigma_n^2$</td>
<td>noise variance</td>
</tr>
<tr>
<td>$\theta_p$</td>
<td>principal angle</td>
</tr>
</tbody>
</table>
List of Acronyms

AWGN  additive white Gaussian noise
BER  bit error ratio
BICM  bit interleaved coded modulation
BICM-ID  BICM with iterative detection
BPSK  binary phase shift keying
CIR  channel impulse response
CSI  channel state information
DFT  discrete Fourier transform
DAB  digital audio broadcasting
DAPSK  differential amplitude and phase shift keying
DPSK  differential phase shift keying
DVB-T  digital video broadcasting - terrestrial
EXIT  extrinsic information transfer
FFT  fast Fourier transform
FSK  frequency shift keying
GLRT  generalized likelihood ratio test
GSM  global system for mobile communications
ICI  inter carrier interference
IDFT  inverse discrete Fourier transform
i.i.d.  independent identically distributed
ISCI  inter subchannel interference
ISI  inter symbol interference
JFPM  joint frequency-phase modulation
LLR  log-likelihood ratio
LOS  line of sight
LTE  long term evolution
LTI  linear time invariant
MFSK  M-ary frequency shift keying
MIMO  multiple input multiple output
ML  maximum likelihood
MLC  multilevel coding
MMSE  minimum mean square error
MSD  multiple symbol detection
OFDM  orthogonal frequency division multiplexing
OOK  on-off keying
PAPR  peak to average power ratio
PEP  pairwise error probability
PDF  probability density function
PDP  power delay profile
PSK  phase shift keying
QAM  quadrature amplitude modulation
SINR  signal to interference plus noise ratio
SIR  signal to interference ratio
SNR  signal to noise ratio
STBC  space-time block codes
STM  space-time modulation
UMTS  universal mobile telecommunications system
WSSUS  wide sense stationary uncorrelated scattering
WiMAX  worldwide interoperability for microwave access
WLAN  wireless local area network


Bibliography


