Behavioral Aspects of Product Design and Demand in Retirement Savings

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Research Context and Summary of Research Papers

1 Field of Research

This cumulative thesis contributes to the field of behavioral economics and decision making in the context of retirement savings.

Traditional economic theory considers utility maximizing rational decision makers with well-defined and time-consistent preferences, who process all relevant information. In recent years, economics increasingly focuses on explaining discrepancies between predictions of traditional economic theory and actual human decision making, cf. Rabin (2002), DellaVigna (2009) and Barberis (2013). Findings from the field of psychology show that heuristics can lead to systematic cognitive biases which can massively influence human decision making, cf. also Slovic (1987) and Gilovich et al. (2002). The field of behavioral economics incorporates these findings to identify, describe and analyze the impact of behavioral aspects in this context. The resulting descriptive economic models attempt to explain and predict human behavior, cf. Kahneman and Tversky (1979), Thaler (1980), Tversky and Kahneman (1992) and Machina (1995). Ideally, a better understanding of the drivers of decisions can be used to develop tools helping humans to overcome errors due to cognitive biases, cf. Cronqvist and Thaler (2004), Thaler and Benartzi (2004), Beshears et al. (2009), Hershfield et al. (2011) and Johnson et al. (2012).

The vast scholarly literature on retirement savings covers aspects of mandatory state pension systems, occupational pension insurance as well as private pension insurance. These three layers form the foundation of the German pension system (as well as of many other countries). In face of an aging society, occupational and private pension insurance become increasingly important, cf. Wilke (2009) and Gough and Niza (2011). The movement towards more occupational and private pension insurance results in more choices but also in a higher complexity and self-responsibility. In this context, individuals are typically confronted with two fundamental decisions: Firstly, the decision on how to invest in the accumulation phase. Secondly, the decision on how to decumulate savings after approaching retirement age in order to maintain a desired standard of living in old age. Both decisions are very complex, contain a high degree
of uncertainty, e.g., stock market fluctuations and mortality, and are often viewed as one-shot decisions. Consequently, these decisions are particularly prone to cognitive biases. Many studies show that traditional economic theory and even common behavioral economic models face problems in explaining the decision making of individuals in this context as well as the resulting demand for many popular retirement savings products, cf. Davidoff et al. (2005), Hu and Scott (2007), Benartzi and Thaler (2007), Døskeland and Nordahl (2008), Mitchell et al. (2009), Benartzi et al. (2011) and Ebert et al. (2012).

The research papers included in this cumulative thesis contribute to the field by modeling and analyzing various behavioral aspects of decision making in the context of retirement savings. We propose new models to describe actual human decision making in the accumulation phase (investment decision) as well as in the decumulation phase (annuitization decision). The results provide new helpful insights into the role of various behavioral aspects on these fundamental decisions and point out promising directions for future research.

2 Motivation and Objectives

For many years, the market for private pension insurance in Germany (as well as in other countries) was dominated by traditional participating life insurances (TPLI). Life insurers manage TPLI contracts within a heterogeneous insurance portfolio and pool their assets and liabilities. This allows for return smoothing and risk sharing elements, which provide rather stable returns for the policyholders. Additionally, TPLI contracts are typically equipped with a cliquet-style (year-to-year) guarantee. In Germany, after the deregulation of the life insurance market in 1994, life insurers have started to develop new product designs with various different embedded features. In addition, the current low-interest environment and rather restrictive solvency requirements have accelerated the trend towards capital efficient\(^1\) versions of TPLI contracts as well as unit-linked products, which have been very successful in Anglo-Saxon markets, e.g., in the context of variable annuities.\(^2\) In particular, unit-linked products allow for an increased participation in the capital market. The most popular unit-linked products limit the downside risk in some kind by means of certain guarantees or specific investment strategies.

There exists a wide stream of literature examining the optimal design of pension insurance products, e.g., Merton (1971), Dybvig (1988), Black and Perold (1992), Huang et al. (2008) and Branger et al. (2010), to name but a few. Overall, the results of this stream of literature show that under standard Expected Utility Theory (EUT) guarantee features result in a suboptimal terminal payoff. Consequently, typical EUT-investors would not invest in pension insurance

\(^1\)Capital efficiency can be described in terms of profitability in relation to capital requirement, cf. Reuß et al. (2015).

\(^2\)Variable annuities are deferred annuities that are unit-linked during the deferment period.
products equipped with guarantee features. This discrepancy between theoretically optimal and observed demand has been analyzed by several authors, cf. Døskeland and Nordahl (2008), Dierkes et al. (2010), Dichtl and Drobetz (2011), and Ebert et al. (2012). These papers focus on explaining the observed demand by applying Cumulative Prospect Theory (CPT), which was introduced by Tversky and Kahneman (1992) and has become one of the most prominent descriptive theories of decision making. A main result of these papers is that CPT can explain the demand for products with guarantees at maturity. The main reason is that CPT considers gains and losses with respect to a reference point (sum of contributions) instead of the total wealth. Furthermore, it assumes that investors are more sensitive to losses than to gains (loss aversion) and overweight tail-events with small probabilities (probability distortion). However, the results also show that even CPT fails to explain the demand for many popular pension insurance products equipped with more complex guarantees, particularly annual lock-in features such as (year-to-year) cliquet-style or ratchet guarantees. In addition, results of Graf (2017) imply that EUT and CPT also face problems in explaining the popularity of so-called life-cycle strategies reducing risk exposure towards the product’s maturity. In total, these findings cast doubt on whether EUT as well as CPT in its standard forms are appropriate to describe actual decision making of long-term investors.

In both, EUT and CPT, the preferences of the investor only depend on the distribution of the terminal payoff. However, long-term investors regularly receive information on their investment, e.g., annually with the financial statement. A good performance in the past year might increase the investor’s reference point against which losses are evaluated. A subsequent drop in the account value might then be perceived as a loss, even if the overall performance of the product is still positive. This is closely related to the concept of mental accounting introduced by Kahneman and Tversky (1984) and Thaler (1985). This concept describes how investors categorize investments in order to monitor the future performance. Inspired by findings on repeated gambling, cf. Samuelson (1963), Benartzi and Thaler (1995) propose the theory of myopic loss aversion, a combination of loss aversion and frequent investment evaluation. They argue that mental accounting implies that investors tend to evaluate their investment decision on short evaluation periods and therefore prefer to invest only small fractions of their wealth in risky assets. Based on this, Bellemare et al. (2005) find evidence that such interim information alone affects perceived financial well-being (ex post). This suggests that focusing solely on the terminal value is not sufficient to describe actual preferences of long-term investors. Indeed, mental accounting implies that investors tend to take into consideration the potential future changes in the account value already when making an investment decision (ex ante). These findings strongly indicate that an appropriate descriptive model for long-term investment decision making also captures the impact of potential interim changes in the account value. Further, this naturally raises the question whether the consideration of potential interim changes is able to explain the popularity of common pension insurance products.
However, as mentioned above, accumulating wealth is only one aspect of retirement savings. When approaching retirement age, individuals are confronted with the question on how to invest and decumulate savings during the retirement period. Starting with the seminal paper of Yaari (1965), numerous authors have analyzed optimal investment and consumption strategies for the retirement period, cf. Davidoff et al. (2005) and Peijnenburg et al. (2016). The vast majority of literature shows that under traditional economic assumptions it is optimal for many individuals to annuitize a significant fraction of their liquid wealth, that is, buying a life annuity which provides a fixed stream of income for the rest of their life. However, observed voluntary annuitization rates are dramatically lower than suggested by the literature. This discrepancy is known as the “Annuity Puzzle”. Recent studies suggest that the framing-effect, which describes that decisions depend on how choices are presented, has a strong impact on the annuitization decision and is able to explain the annuity puzzle, cf. Hu and Scott (2007) and Gazzale and Walker (2009). Particularly, these studies show that annuities are often considered as a gamble on a long life rather than a protection against longevity risk. By doing so, individuals focus solely on the investment risk and return characteristics on protection against longevity risk. The results of Brown et al. (2008) strongly indicate that annuities are much more appealing when presented under a so-called consumption frame where individuals focus on maintaining an aspired standard of living expressed through consumption. However, Brown et al. (2013) shows that even under the consumption frame, individuals are not making annuitization choices by perfectly maximizing utility over the life-cycle. The drivers of this result and their interaction are not fully clear. More precisely, these findings raise questions on how individuals evaluate annuitization and the impacts of cognitive biases under different frames.

To summarize, the following questions are considered in the present thesis:

1. How do long-term investors perceive and evaluate investments? What are the impacts of potential interim changes in the account value on decisions of long-term investors? How can we model this impact? Is the consideration of potential interim value changes able to explain the observed demand for common products in the segment of retirement savings, e.g., cliquet-style (year-to-year) guaranteed products or life-cycle funds?

2. How attractive are return smoothing and risk sharing elements provided by life insurers for a long-term investor? Are these aspects able to explain the popularity of TPLI contracts?

3. How can we explain the strong impact of framing on the annuitization decision? How can we describe a descriptive approach to model decision making under different frames? What are the impacts of various behavioral aspects under these frames?

These research questions have not been answered in the literature so far and this dissertation aims to close this gap. The objective of the first paper is to motivate and describe a preference function which models actual decision making of long-term investors more accurately than
existing approaches. Particularly, the approach should take into account the subjective utility of potential interim changes in the account value when making the investment decision. Further, we want to investigate the demand for popular guaranteed products under this preference function. The second paper seeks to provide further evidence of the predictive power of the proposed preference function by explaining the popularity of life-cycle funds. The aim of the third paper is to analyze the impact of return smoothing and risk sharing elements provided by life insurers on the preferences of long-term investors. In addition, the paper aims to explain the demand for TPLI contracts which make use of these elements. The objective of the fourth paper is to propose a descriptive model to analyze the impact of framing on the annuitization decision. Particularly, the paper provides new insights into the influence of various behavioral aspects on decision making in this context.

A detailed summary of the four research papers follows in the next Section.

3 Summary of Research Papers

Research Paper 1: Multi Cumulative Prospect Theory and the Demand for Cliquet-Style Guarantees

In the first paper we propose a modification of CPT which assumes that long-term investors tend to take into account the subjective utility of potential interim changes of the account value when making an investment decision. We denote this approach as “Multi Cumulative Prospect Theory” (MCPT). Further, as a first application example of MCPT, we investigate the demand for different guaranteed products and show that in contrast to EUT and CPT, MCPT can explain the demand for cliquet-style (year-to-year) guarantees. This paper is joint work with Jochen Ruß and has been accepted for publication in the Journal of Risk and Insurance. The paper has been presented at the International Congress on Insurance: Mathematics and Economics (2015) in Liverpool, UK, at the World Risk and Insurance Economics Congress (2015) in Munich, Germany, at the CEAR/MRIC Behavioral Insurance Workshop (2016) in Munich, Germany, and at the Annual Meeting of the German Insurance Science Association (2017) in Berlin, Germany. Furthermore, the paper has been awarded the DIA Zukunftspreis 2016 by the German Institute for Retirement Savings (Deutsches Institut für Altersvorsorge).

As described above, there is substantial literature analyzing optimal and observed investment decisions in the context of retirement savings. While existing work focuses on the application of EUT or CPT in their standard form, several studies indicate that the investment decision of long-term investors is not solely driven by the distribution of the terminal payoff, but also
by the distribution of potential interim changes in the account value, cf. Benartzi and Thaler

Consequently, in the first part of the paper, we introduce a model, the so-called MCPT, to
capture the subjective utility of potential interim changes of the account value. The model
assumes that investors adapt their reference point based on the potential evolution of the
investment value. Further, the subjective utility is based on the CPT value generated by
interim changes of the account value. Additionally, we also propose a combination of CPT and
MCPT which considers annual changes and the terminal value relative to a reference point.
This is motivated by the fact that the terminal value still has an outstanding role when making
an investment decision.

We propose that investors who are attracted by specific contract features, like annual lock-in
guarantees, might be particularly inclined to take into account the subjective utility of potential
future fluctuations of the account value when making the investment decisions. Hence, in the
second part of the paper, we have analyzed the demand for guaranteed products which are
common in many markets (roll-up, ratch-up and cliquet) and a contract without guarantee
(constant mix). The roll-up guarantee provides a minimal terminal payoff which is based on
some guaranteed rate. The ratch-up guarantee additionally includes a lock-in feature: The
guaranteed benefit is the higher of a fixed guaranteed amount and the highest investment
account value at any pre-specified lock-in date. Finally, the cliquet guarantee credits in each
period the higher of a guaranteed rate and the performance of the underlying investment. For
the sake of comparability, we use the same model framework as Ebert et al. (2012), that is,
a stock being driven by a geometric Brownian motion and a constant risk-free interest rate.
In this setting we can derive closed-form solutions for the arbitrage free prices of all three
guaranteed products at any valuation date. We use these prices as the basis for the annual
value changes. We then use Monte Carlo simulations to evaluate these products under EUT,
CPT and MCPT.

We are able to replicate the results of Ebert et al. (2012) under EUT and CPT, particularly
the fact that CPT can explain the demand for terminal guarantees but not for more complex
forms of guarantees with lock-in features. For MCPT-investors, that is, investors who take
into account potential annual changes, we derive certainty equivalent returns which describe
the fixed annual return that an investor would regard as equally desirable. The results for
these investors show that the complex guaranteed products are typically preferred over other
products. This is mainly caused by the more right-skewed distribution of the interim changes
in the value of the cliquet contract compared to the other contracts; that is, they include more
relatively high gains with low probabilities. Due to the probability distortion, these outcomes
get overweighted and generate a higher MCPT value. Hence, MCPT is able to explain the
demand for these contracts. Also, if only products without guarantee are considered, under
MCPT, an equity ratio of 0% is often optimal. This explains the demand for very safe assets even for long term investments that can be observed in many countries. Additionally, we have analyzed the contracts under the combined model. Our results show that also in this combined model many investors have a preference for the more complex guaranteed products. This means that the demand for more complex guarantees can even be explained if annual value changes only partly influence the investor’s subjective utility.

In summary, this paper answers most of the first part of the research questions. We have proposed a model to capture the subjective utility of potential interim changes of the account value when making an investment decision. Moreover, we can show that taking into account the subjective utility of interim value fluctuations is able to explain the popularity of guarantees with lock-in features. Hence, the results indicate that MCPT has descriptive power for long-term investment decision making.

**Research Paper 2: As You like It: Explaining the Popularity of Life-Cycle Funds with Multi Cumulative Prospect Theory**

In the second paper, we analyze the popularity of so-called life-cycle funds which typically decrease their risk exposure over time following a (deterministic) glide path. We particularly consider life-cycle and balanced funds with a comparable probability distribution of the terminal wealth. We show that, in contrast to EUT and CPT, MCPT can explain the popularity of life-cycle funds. This paper is joint work with Stefan Graf and Jochen Ruß and has been submitted to a special issue of *Risk Management and Insurance Review* on perspectives on behavioral insurance research. The paper has been presented at the International Congress of Actuaries (2018) in Berlin, Germany.

Life-cycle (or target-date) funds have been very successful in many countries in the segment of old age provision. Particularly, a large number of pension plans selected life-cycle funds as their default allocation option, that is, pension plans allocate their contribution to a life-cycle fund if no other fund is actively chosen by the client, cf. Falkof et al. (2012). Life-cycle funds are also discussed as a default option for the pan-European personal pension product (PEPP), cf. Berardi et al. (2018). In addition, life-cycle funds are also frequently chosen when no default option exists, cf. Mitchell et al. (2009). Hence, for many individuals, life-cycle funds have a significant impact on their retirement wealth and the affordable standard of living in old age. However, in the literature a sound opinion whether a life-cycle fund (with some deterministic glide path) is a good investment choice for this purpose does not exist, cf. Graf (2017) for a discussion.
In a recent paper, Graf (2017) has analyzed life-cycle funds for single and regular contributions and under different asset models. Particularly, his results show that for any life-cycle fund there exists an appropriately calibrated “matching” balanced fund with a similar (often stochastically dominating) distribution of the terminal wealth.\(^3\) His results imply that the very existence of life-cycle funds is challenging when only the terminal wealth is relevant. However, since life-cycle funds and balanced funds – even if they were designed to have a similar distribution of terminal wealth – significantly differ with respect to the distributions of the potential changes in the account value over time, they would create significantly different subjective utility for an MCPT-investor as proposed by Ruß and Schelling (2018).

Consequently, in the first part of the paper, we describe MCPT closely following Ruß and Schelling (2018) and adapt the approach to regular contributions. Next, we describe the model framework. We consider a financial market model with stochastic interest rates and stochastic volatility, both modeled by a Cox-Ingersoll-Ross diffusion process. The balanced funds have a constant portion invested in stocks and the remaining part in a “rolling” investment in zero-coupon bonds. In contrast, the life-cycle funds apply a time-dependent (however, not path-dependent) asset allocation strategy which typically results in a decreasing stock exposure over time.

In the first part of the results we summarize the main results of Graf (2017). Particularly, we derive the “matching” pairs of life-cycle and balanced funds that come with a similar distribution of terminal wealth. Further, we point out that the results of Graf (2017) imply that the popularity of common life-cycle strategies cannot be explained by EUT or CPT (only considering the terminal wealth). In the second part of the results, we show that for an MCPT-investor life-cycle funds are typically more attractive than “matching” balanced funds. Moreover, we find that for very low degrees of loss aversion the investor prefers a pure stock investment, for rather high degrees of loss aversion a pure bond investment and in between a life-cycle fund. It is striking that life-cycle funds are particularly preferred by MCPT-investors with degrees of loss aversion which seem to prevail for most individuals according to empirical studies. Furthermore, we find that life-cycle funds are increasingly attractive for MCPT-investors for longer investment horizons. Finally, we show that the results also hold if potential annual changes only partly influence the investor’s subjective utility (using a combined model). Our findings hold for various degrees of risk preference, different life-cycle glide paths, regular and single premiums as well as under various capital market assumptions.

Summarizing, the paper provides further insights into the first part of the research questions. Based on Graf (2017) we have shown that life-cycle funds, which reduce the potential losses throughout the investment horizon, are generally preferred by MCPT-investors over balanced

\(^3\)Graf (2017) finds first-order stochastic dominance for a single premium investment in a Black-Scholes model and “practical” stochastic dominance, that is, (far) upper and lower tails of the distribution are ignored, for regular contributions as well as under a more complex asset model.
funds. Hence, MCPT is able to explain the observed popularity of life-cycle funds. Combined with the results from Ruß and Schelling (2018), this provides strong evidence that, particularly for long investment horizons, MCPT can explain observed demand better than EUT or CPT.

Research Paper 3: Return Smoothing and Risk Sharing Elements in Life Insurance from a Client Perspective

The third paper investigates how return smoothing and risk sharing elements provided by life insurers can significantly increase the subjective utility for MCPT-investors. Furthermore, we show that for these investors traditional participating life insurance (TPLI) contracts which make use of these elements are more attractive than common unit-linked (guaranteed) products. This paper is joint work with Jochen Ruß.

TPLI contracts have been the core business of life insurers for many years and versions of TPLI contracts are still very popular in the segment of retirement savings. In contrast to individualized products, life insurers pool the assets and liabilities of a heterogeneous portfolio of TPLI contracts which allows for intergenerational risk sharing. In many countries, TPLI contracts are typically equipped with a cliquet-style (year-to-year) guarantee where the guaranteed return must be credited to the policyholder’s individual account at the end of each year. Additionally, TPLI contracts receive a surplus participation being based on the return of a collective investment which is subject to various smoothing elements. Goecke (2013) shows that such smoothing and risk sharing elements can heavily reduce the short-term volatility of returns without significantly affecting the long-term risk-return-profile. However, while smoothing elements can reduce the volatility of returns, they cannot compensate for a long-term decline in the capital market returns like in the current low interest environment. Furthermore, due to insurance portfolios with long-term contracts and rather high guaranteed rates (especially in old contracts) in combination with rather restrictive solvency requirements, the insurer’s asset allocation allows only for low risk taking. For these reasons, life insurers tend to develop capital efficient versions of TPLI contracts with different types of guarantees, cf. Reuß et al. (2015), or offer more (individualized) unit-linked contracts, cf. Graf et al. (2012). However, a complete shift to individualized contracts leads to a loss of intergenerational risk sharing and questions the role of the life insurer in this context. The aim of this paper is to explain the popularity of TPLI contracts and to shed light on how return smoothing and risk sharing elements are perceived by long-term investors using MCPT as proposed by Ruß and Schelling (2018).

In the first part of the paper, we describe the model framework. As we are particularly interested in analyzing the impact of return smoothing and risk sharing elements, we model these elements very detailed by means of a stylized life insurance company. We consider a heterogeneous insurance portfolio and a balance sheet which allows for building up collective reserves on both
sides in good years and dissolving these reserves to compensate for poor returns in bad years. By doing so, we assume that contracts are influenced by intergenerational effects among different cohorts. In particular, the surplus participation is subject to regulation but allows also for some discretion by the insurance company. We assume that one part of the surplus is credited directly to the account value at the end of each year and another is credited at maturity from a terminal bonus fund. Moreover, we also consider stochastic interest rates, different types of charges, and regular premium payments. The initial insurance portfolio is derived based on historical data. In the resulting base case the considered TPLI contract is exposed to an ex ante “collective malus”, that is, the contract is expected to suffer more than profit from intergenerational effects, cf. also Hieber et al. (2016) and Eckert et al. (2018). To separate the impact of smoothing and risk sharing from the impact of systematic intergenerational effects at some point in time, we also consider other contract settings without an ex ante “collective malus”.

The first results of the paper confirm that return smoothing elements based on a collective investment of a life insurer can heavily stabilize annual returns without significantly changing the risk-return characteristics of the distribution of the terminal value compared to an unsmoothed investment in the same assets. Nevertheless, we show that CPT-investors who focus solely on the terminal value prefer an unsmoothed investment (as smoothing comes at a certain cost). In contrast, for MCPT-investors, products with smoothed returns are highly attractive. Furthermore, we show that contracts with smoothed returns are even attractive in the case of a (significant) ex-ante “collective malus” and even if the subjective utility is only partly influenced by potential annual changes (using a combined model). These results show that return smoothing elements based on a collective investment are a key aspect for the attractiveness of TPLI contracts for long-term investors.

In the second part of the paper, we compare TPLI contracts with common unit-linked products with and without guarantees (balanced funds, variable annuity products and constant proportion portfolio insurance products). For unit-linked products we can confirm the result of Ruß and Schelling (2018), that is, products with a cliquet-style (year-to-year) guarantee are preferred by loss-averse MCPT-investors over products without or with a terminal guarantee only. However, by comparing these products with TPLI contracts, we can show that for MCPT-investors, TPLI contracts with smoothing and risk sharing elements are typically even more attractive than common unit-linked products. Further, this is even true if the subjective utility is only partly influenced by potential annual changes and even in the case of a moderate ex ante “collective malus”. Moreover, the results hold also for different assumptions on the asset allocation of the life insurer. Hence, in contrast to standard approaches, MCPT is able to explain the preference of many long-term investors for smoothed returns and the popularity of TPLI contracts.
Summarizing, this paper answers the second part of the research questions. We have shown that return smoothing and risk sharing elements provided by life insurers can heavily increase the attractiveness of long-term investments. Further, comparing TPLI contracts with common unit-linked products under MCPT suggests that these elements are the main reason for the popularity of TPLI contracts. Further, the findings strongly indicate that participating products with lower guarantees (e.g., applied at maturity only) seem very promising in being subjectively attractive for long-term investors while at the same time providing an objectively superior distribution of terminal wealth. Combined with the results from Ruß and Schelling (2018) and Graf et al. (2018), this gives further evidence that MCPT provides a convincing description of the decision making of long-term investors.

**Research Paper 4: When and How Framing Makes Annuitization Appealing: A Model-Based Analysis**

In the fourth paper, we analyze how the annuitization decision depends on framing. We propose preference functions under the investment frame as well as under the consumption frame and study the impact of various behavioral aspects within a theoretical model framework. The paper has been presented at the Research Workshop IVW/ifa (2018) in Ulm, Germany, at the Annual Meeting of the American Risk and Insurance Association (2018) in Chicago, USA, and at the CEAR/MRIC Behavioral Insurance Workshop (2018) in Munich, Germany.

At first, we describe the model framework: We consider an individual at retirement age who deals with the question of annuitization which is assumed to be a one-time decision. The individual can choose between a lifetime annuity and a balanced fund with a constant stock ratio. Focusing on individuals of middle wealth, we assume that future consumption is determined by income and liquid wealth. Furthermore, we assume that the individual receives predefined regular constant lifelong social security benefits. Dependent on the frame, the individual evaluates the annuity under different perspectives.

Next, we propose preference functions that attempt to describe how individuals actually perceive and (possibly subconsciously) evaluate annuitization under the different frames. Under the investment frame, the individual solely focuses on investment return and risk characteristics. Under this frame the individual considers the difference between the accumulated annuity payments and the price of the annuity which serves as a reference point. Further, we assume that the individual evaluates the resulting outcome according to CPT. Under the consumption frame, the individual focuses on maintaining a desired standard of living expressed through consumption. Motivated by several studies, e.g., Koop and Johnson (2012), we assume that the individual considers multiple reference points under this frame: A minimal consumption requirement in order to cover the basic needs (for example for housing, energy, and food). A
minimal consumption goal needed to maintain a desired standard of living (additionally covers expenses for comforts of everyday life, e.g., for a car or leisure activities). And finally, an aspired consumption goal per year that is sufficient to meet further aspirations (for example traveling during the retirement period). We assume that a consumption below the minimal goal (or even the minimal requirement) leads to (harsh) cuts in the standard of living. We apply the Tri-Reference Point Theory and a double S-shaped value function introduced by Wang and Johnson (2012) in a modified version to evaluate the annuitization decision under the consumption frame.

In the numerical analysis part, we analyze the impact of various determinants on the annuitization decision under both frames focusing on behavioral aspects. As we aim to model and analyze actual decision making, we refrain from deriving consumption patterns that maximize the preference function but rather restrict the analysis to several consumption plans based on common recommendations (for example by financial advisers).

Under the investment frame, we show that partial annuitization is only appealing for individuals with a significantly higher subjective life expectancy than the average objective life expectancy and only in combination with a rather low level of loss aversion. Differences between the subjective and the objective average probabilities can arise for objective reasons like a better or worse health condition or due to estimation errors caused by cognitive biases. The results illustrate that annuitization rates will remain low as long as individuals evaluate the annuitization under the investment frame. Other cognitive biases like loss aversion additionally intensify the annuity aversion. These results are in line with previous studies, cf. Hu and Scott (2007).

The main contribution of this work is that we are able to model and disentangle impacts of various determinants on the annuitization decision under the consumption frame. We show that under this frame, most individuals are attracted by partial annuitization if their subjective life expectancy is not significantly shorter than the objective average life expectancy. Furthermore, while the main driver of the higher annuitization rates is the consideration of consumption goals as reference points, we show that other determinants can play a crucial role. We find in almost all cases that especially individuals with social security benefits below the minimal consumption goal prefer high annuitization rates. Moreover, already a low level of loss aversion increases annuitization rates significantly in most cases. The impact of loss aversion is particularly strong for individuals whose basic needs are not fully covered by social security. Overweighting of the small probabilities of reaching old ages increases annuitization rates particularly for individuals with lower levels of initial liquid wealth and whose basic needs are covered by social security benefits. Individuals equipped with a rather high subjective time preference prefer significantly lower annuitization rates - particularly, in case of lower initial liquid wealth.

In total, this paper answers the third part of the research questions. The paper provides a descriptive model for the annuitization decision under an investment frame as well as under a
consumption frame. The main results illustrate that framing can significantly increase voluntary annuitization and provide plausible explanations for this effect. However, cognitive biases can have diverse impacts on the decision. The presented insights improve our understanding of these impacts and their interactions from a theoretical point of view.
References


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1 Multi Cumulative Prospect Theory and the Demand for Cliquet-Style Guarantees

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Multi Cumulative Prospect Theory and the Demand for Cliquet-Style Guarantees

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Abstract

Expected Utility Theory (EUT) and Cumulative Prospect Theory (CPT) face problems explaining preferences of long-term investors. Previous research motivates that the subjective utility of a long-term investment also depends on interim value changes. Therefore we propose an approach that we call Multi Cumulative Prospect Theory. It is based on CPT and considers annual changes in the contract values. As a first application we can show that in contrast to EUT and CPT, this approach is able to explain the demand for guaranteed products with lock-in features, which in this framework generate a higher subjective utility than products without or with simpler guarantees.

Keywords: Behavioral Insurance, Prospect Theory, Guaranteed Products, Myopic Loss Aversion

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1 Introduction

Cumulative Prospect Theory (CPT), introduced by Tversky & Kahneman (1992), has become one of the most prominent behavioral theories in finance, especially as a behavioral counterpart to Expected Utility Theory (EUT). This is due to the fact that CPT can explain behavior that can not be explained by EUT, but is still frequently observed in real life. While complex financial products and long-term investments are well studied under EUT, an analysis of such products under CPT has only recently been in the focus of academic literature. Døskeland & Nordahl (2008) consider different participating life insurance contracts under CPT to explain the demand for guaranteed products. In an empirical work, Dierkes et al. (2010) investigate the preferences of a CPT investor considering different investment strategies and time horizons. Historical and Monte Carlo simulations were used by Dichtl & Drobetz (2011) to analyze portfolio insurance strategies based on simple CPPI (Constant Proportional Portfolio Insurance) strategies. Ebert et al. (2012) determine the “optimal” specification of different guarantee types, where optimality is defined as creating the maximum subjective utility for a CPT investor. A main result of these papers is that, in contrast to EUT, CPT can explain the demand for guarantees. Nevertheless, even CPT is not able to explain the popularity of more complex guaranteed products, such as ratchet or cliquet guarantees, since a CPT investor should prefer a simple guarantee at maturity over more complex guarantees with lock-in features.

In both EUT and CPT, the preferences of the investor only depend on the distribution of the terminal value. In reality, however, investors tend to re-evaluate a financial product regularly, e.g., annually when they receive a financial statement. Information about a good performance in the past year might increase the investors reference point against which losses are evaluated. A subsequent drop of in the product’s value might then be perceived as a loss, even if the overall performance since the start of the product is still positive. This is related to the concept of mental accounting introduced by Thaler (1985). This concept describes how investors categorize investments in order to monitor the future performance. In a later work, Thaler &

\footnote{It is worth noting that certain results can also be achieved under EUT if the underlying assumptions are more realistic. E.g., Chen et al. (2015) recently showed that the consideration of mortality can explain the preferences for simple guarantees at maturity also in a EUT-framework.}
Johnson (1990) studied how prior gains and losses affect decision makers and how they frame such problems under Prospect Theory. Arkes et al. (2008) provide additional evidence that investors mentally account for previous price changes and therefore regularly adapt their reference point. Benartzi & Thaler (1995) propose the theory of myopic loss aversion, a combination of loss aversion and frequent investment evaluation, and show that an annual evaluation can solve the equity premium puzzle. They argue that mental accounting implies that investors tend to evaluate their investment decision on short evaluation periods and therefore prefer to invest only small fractions of their wealth in risky assets. Benartzi & Thaler (1999) give evidence that investors make less risky choices if they are shown one-year rather than long-term rates of return. Barberis et al. (2001) propose a model in terms of asset pricing, in which investors derive utility from annual changes of the value of their financial wealth and Barberis & Xiong (2012) are able to shed light on the disposition effect and other puzzles by introducing the realization utility, which suggest that investors derive utility from interim gains and losses.

If interim changes of the value influence the utility of investors during the investment horizon, it seems only natural that when making the investment decision investors are also affected by potential future interim changes of the value.

Based on these insights, we propose a modification of CPT, which assumes that long-term investors tend to take into account the subjective utility of interim changes of the value of the contract when making an investment decision. We denote this approach “Multi Cumulative Prospect Theory” (MCPT).

As a first application example of MCPT, we investigate the demand for different guaranteed products. For the sake of comparability, we apply this approach to the guaranteed contracts presented by Ebert et al. (2012), that is, we consider three different types of guarantees (roll-up, ratch-up and cliquet) and a product without guarantee. The roll-up guarantee provides a minimal terminal payoff, which is based on some guaranteed rate of interest. The ratch-up guarantee additionally includes a lock-in feature: The guaranteed benefit is the higher of a
fixed guaranteed amount (calculated as in the roll-up case) and the highest investment account value at any pre-specified lock-in date. Finally, the cliquet guarantee credits in each period the higher of a guaranteed rate and the performance of the underlying investment. We analyze these products in a Black-Scholes framework without considering mortality or default risk. In this setting we can derive closed-form solutions for the arbitrage free prices of all three products at any valuation date. We use these prices as the basis for the annual changes of the value. We then use Monte Carlo simulations to evaluate these products under EUT, CPT and MCPT. We are able to replicate the results of Ebert et al. (2012) under EUT and CPT, particularly the fact that CPT can explain the demand for guarantees but not for the more complex forms of guarantees. However, under our new MCPT approach, the complex products typically dominate the simple products: For investors who evaluate utility considering possible future interim changes, the subjective utility is higher for the complex than for the simple guaranteed products. Also, if only products without guarantee are considered, under MCPT, an equity ratio of 0% is often optimal. This result is in line with the findings of Benartzi & Thaler (1995) and explains the demand for very safe assets even for long term investments that can be observed in many countries. Finally, we present a combined CPT and MCPT approach, where both, annual changes and the distribution of the terminal wealth are evaluated by the investor. Our analysis under this combined model shows that the demand for complex guaranteed products can be explained, even if the annual price changes only partially influence the total subjective utility.

The remainder of this paper is organized as follows. Section 2 gives a short introduction to CPT. Moreover, we motivate and present our MCPT approach. In Section 3, we apply MCPT to explain the demand for cliquet type guarantees. We specify the considered products and the model for the financial market and present numerical results, as well as sensitivity analyses. Section 4 summarizes and gives an outlook for future research. Finally, the closed-form arbitrage free prices for the considered contracts are given in the Appendix.
2 Prospect Theory and Extensions

Prospect Theory (PT) introduced by Kahneman & Tversky (1979) has been developed as one possible way of explaining behavior that can be observed in real life but can not be explained by Expected Utility Theory (EUT). In particular its well-known modification, Cumulative Prospect Theory (CPT) has become very popular.

2.1 Cumulative Prospect Theory

Cumulative Prospect Theory is based on the idea, that the subjective utility of an investment $A$ with final outcomes given by a random variable $E$ is described by an S-shaped PT value function $v$ for the gains and losses $X$ corresponding to the outcomes with respect to a given reference point $\chi$. The gains and losses are described by the random variable $X := E - \chi$, and modified by a probability weighting function $w$, that overweights (particularly extreme) events with small probabilities and underweights events with high probabilities. A natural and the most prominent choice for an investor’s reference point is the initial price of the investment, cf. Kahneman & Tversky (1979), which is also called a Status Quo reference point (SQ), that is, $\chi = A_0$ and hence $X = E - A_0$, where $A_0$ denotes the fair value of the investment $A$ at $t = 0$. Now let $\mu_X$ be the probability measure given by the random variable $X$. Then the CPT utility is defined as

$$CPT(X) := \int_{-\infty}^{0} v(x) d\left(w\left(F(x)\right)\right) + \int_{0}^{\infty} v(x) d\left(-w\left(1 - F(x)\right)\right),$$

with $F(s) = \mathbb{P}(X \leq s) = \int_{-\infty}^{s} d\mu_X$. This is a natural generalization (cf. Hens & Rieger (2010)) of the discrete case introduced by Tversky & Kahneman (1992).

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2 A continuous function $v : \mathbb{R} \rightarrow I$ ($I \subset \mathbb{R}$ an interval containing 0) is called PT value function, if $v$ is strictly monotonically increasing, $v(0) = 0$ (reference-point), $v(x)$ is strictly convex for $x < 0$ (decreasing loss sensitivity), $v(x)$ is strictly concave for $x > 0$ (decreasing gain sensitivity) and $|v(-x)| > v(x)$ (loss aversion).

3 A continuous function $w : [0, 1] \rightarrow [0, 1]$ is called probability weighting function, if $w$ is strictly monotonically increasing, $w(0) = 0$ and $w(1) = 1$ and $w(p) > p$ for $0 < p \ll 1$ and $w(p) < p$ for $0 \ll p < 1$. 
2.2 Multi Cumulative Prospect Theory

Especially for long term investments, e.g. retirement savings, studies show that investors regularly evaluate their investment. E.g., Benartzi & Thaler (1995) find that the size of the equity premium is consistent with loss averse investors with annual portfolio evaluation (myopic loss aversion) causing even long term investors to choose their strategies based on short evaluation periods. Arkes et al. (2008) provide evidence that investors adjust their reference point over the investment horizon. Also, a long term investor usually receives an annual report with current information about the investment. Bellemare et al. (2005) find evidence that such interim information alone affects perceived utility (ex post). Mental accounting implies that investors tend to take into account the potential future fluctuation of the contract’s value when making an investment decision (ex ante). This motivates that for long term investors, the initial subjective utility of an investment is not only dependent on the distribution of the terminal wealth, but also on the possible future interim changes.

We therefore propose an extension of CPT, which uses CPT utility with multiple reference points and evaluation periods to measure the subjective utility of the potential interim value changes. We refer to this as Multi Cumulative Prospect Theory (MCPT).

We consider an investor and an investment $A$ with time horizon $[0, T]$, $T \in \mathbb{N}$, at time $t_0 = 0$. Moreover, to simplify notation, we assume future interim evaluations take place annually. Therefore, we have to introduce a measure for the future annual changes of the value of the investment $A$. Since in many countries for fund-linked products the market value of the product has to be communicated to the client on a regular basis, we consider for all $t \in \{1, \cdots, T\}$ the annual gain or loss $X_t := A_t - \chi_t$, where $A_t$ is the fair value of the investment $A$ at time $t$ and $\chi_t$ is the reference point for time $t$. In this setting, the natural SQ reference point choice for each period is given by $\chi_t = A_{t-1}$. Hence $X_t = A_t - A_{t-1}$ represents the annual value change with respect to the SQ. Note that this setting implies that investors use different reference points for different points in time. Based on equation (1) we can evaluate the CPT utility at
\( t_0 = 0 \) of each annual value change \( X_t \) by

\[
CPT(X_t) = \int_{-\infty}^{0} v(x) d\left(w\left(F_t(x)\right)\right) + \int_{0}^{\infty} v(x) d\left(-w\left(1 - F_t(x)\right)\right),
\]

where \( F_t(x) = P(X_t \leq x) \) and \( v \) is the investor’s value-function.

The MCPT utility at time \( t_0 = 0 \) of an investor with investment \( A \) is then given by

\[
MCPT(A) := \sum_{t=1}^{T} \rho^t CPT(X_t)
\]

with a discounting parameter \( \rho \in \mathbb{R}_+ \).

### 2.3 Discussion and Choice of the Functions

#### 2.3.1 MCPT Preferences

Consider two investments \( A \) and \( B \) with the same time horizon \([0, T], \ T \in \mathbb{N}\). We assume an investor who considers the same future interim evaluation periods (e.g., annually) for both investments. Moreover, the investor makes the investment decision at \( t = 0 \) under the assumption that the contract will be held until maturity.\(^4\) Recall that in contrast to dynamic choice models, where typically interim evaluation and decision making go hand in hand, the MCPT evaluation periods are not connected with decisions.\(^5\) An MCPT investor with a given value and probability weighting function prefers \( A \) over \( B \) at time \( t_0 = 0 \) if \( MCPT(A) > MCPT(B) \).

A desired and natural consequence of the MCPT definition is that stochastic dominance in the traditional sense is violated that is, if the terminal value of investment \( A \) stochastically

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\(^4\)Many long-term investors spend a lot of time before they make a decision for a certain product. But then they stay with it, that is, the investor does not question the contract after making the decision. Possible explanations are long-term investors are not willing to regularly spend much time on comparing different investment contracts, and that rather high surrender fees in the “old” contract and new commission payments in the new contract make a change less attractive, etc.

\(^5\)Note that allowing for interim decisions would require a consideration of path-dependent decision rules and a detailed execution of dynamic consistency issues leading to further restrictive assumptions or model adjustments. A discussion of dynamic choices for non-expected utility models is done by Sarin & Wakker (1998) and with focus on changing reference points by Barkan & Busemeyer (1999) and Barkan & Busemeyer (2003).
dominates\(^6\) investment \(B\), investment \(A\) does not necessarily have a higher MCPT utility than investment \(B\). This results from MCTP utility being based on the distributions of all annual changes rather than on the distribution of the final outcome only. Nevertheless, since CPT fulfills the stochastic dominance (cf. Levy (2006)) we can conclude that if the annual changes \(X^A_t\) stochastically dominate \(X^B_t\) for all \(t \in \{1, \cdots, T\}\), then \(MCPT(A) > MCPT(B)\).

### 2.3.2 Reference Point

The choice of the reference point is very important when applying PT or CPT. There are several studies which provide evidence that the SQ plays an outstanding role, cf. Shefrin & Statman (1985) or Spranca et al. (1991). Other reasonable static reference points are given by static guaranteed amounts, which describe a minimum requirement, the payoff of a risk free investment or some other static comparison values which indicate e.g., the investor’s goal, cf. Heath et al. (1999), or an expectation about future outcomes, cf., e.g., Kőszegi & Rabin (2006). Non static variants include the idea that investors adjust their reference point over the investment horizon depending on the evolution, cf. Arkes et al. (2008) or Khuman et al. (2012), which includes the idea that investors wants to retain past gains, or other path dependent outcomes of some benchmark. Other studies suggest the use of multiple reference points, cf. Koop & Johnson (2012) or Wang & Johnson (2012), which include a minimum requirement, SQ, and the investor’s goal captured by a double S-shaped value function. Moreover, studies suggest that investors can simultaneously consider multiple reference points without combining them, cf., e.g., Ordóñez et al. (2000). Knoller (2016) shows that adding a goal that serves as cushion can partially explain the high demand for guarantees in annuity products.

With increasing regulation, more and more countries require that investors receive regular information about the value of their contract. Therefore, one might expect that moving and multiple reference points become more important particularly for long term contracts e.g., in old age provision or retirement planning. MCPT takes this into account by using different

\(^6\)An investment \(A\) is stochastically dominant over an investment \(B\), if for every value \(x\), the probability to obtain more than \(x\) is larger or equal for \(A\) than for \(B\) and there exists at least some value \(x\), such that this probability is strictly larger. Cf. Hens & Rieger (2010).
reference points for different future points in time. But in contrast to a reference point adap-
tation or the use of multiple reference points only, MCPT investors anticipate already in the
investment decision their future annual evaluations (based on potential future reference points
and outcomes).

2.3.3 Value and Probability Weighting Function

There is a variety of literature on the choice of the value function in PT, e.g., Stott (2006).
For the purpose of this paper we focus on the most common PT value function in finance, the
power value function, which is defined as

\[
v(x) := \begin{cases} 
  x^a, & x \geq 0 \\
  -\lambda |x|^b, & x < 0
\end{cases},
\]

where \( \lambda > 0 \) is the loss aversion parameter (in PT typically \( \lambda \approx 2 \)) and \( a \in \mathbb{R}_+ \) and \( b \in \mathbb{R}_+ \) ef-
fect the different sensitivity to losses and gains. In PT, typically \( a, b \leq 1 \) and it is very common
to set \( a = b \), such that \( \lambda \) becomes the only parameter that affects the difference between the
treatment of gains and losses. This assumption is based on several experimental and empiri-
cal results e.g., in Tversky & Kahneman (1992), Camerer & Ho (1994) or Tversky & Fox (1995).

As probability weighting function we use the Tversky Kahneman version:

\[
w(p) := \frac{p^\gamma}{(p^\gamma + (1 - p)^\gamma)^\frac{1}{\gamma}} \text{ with } \gamma \in (0.28, 1],
\]

where the lower boundary for \( \gamma \) is chosen, such that \( w(p) \) is strictly monotonically increasing
for \( p \in [0, 1] \). Similar as for the value function we refrain from a different treatment of gains and
losses with respect to the probability weighting. Note that \( \gamma = 1 \) represents the case without
probability weighting.

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2.4 Combining CPT and MCPT

The MCPT utility reflects the subjective utility created by potential interim changes. Nevertheless, the terminal value plays an outstanding role. Therefore, we propose a combination of CPT and MCPT, where investors consider both, interim value changes \( X_t = A_t - A_{t-1} \) and the terminal value change \( X = A_T - A_0 \). We define this combination by

\[
CPT^{\text{com}}(A) := sMCPT(A) + (1 - s)CPT(X)
\]

with \( s \in [0,1] \) controlling the influence of the interim value changes on the total subjective utility.

3 Application of MCPT: Explaining the Demand for Cliquet-Type Guarantees

In this Section, we apply MCPT to three guaranteed products (roll-up, ratch-up and cliquet) and a product without guarantee. If not stated otherwise, we follow Ebert et al. (2012) in this section.

3.1 Financial Market

We assume a Black-Scholes financial market model (Black & Scholes (1973)). We consider a filtered probability space \((\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})\) with a finite time horizon \( T \in (0, \infty) \) and a real world measure \( \mathbb{P} \) satisfying the usual assumptions. \( \mathbb{F} = (\mathcal{F}_t)_{0 \leq t \leq T} \) and the \( \sigma \)-algebra \( \mathcal{F}_t \) contains the available information at time \( t \). The risky asset \( S \) is given by

\[
dS_t = S_t (\mu dt + \sigma dW_t), \quad S_0 = 1,
\]

where \( W \) is a standard Brownian motion with respect to \( \mathbb{P} \). The risk free asset \( B \) is given by \( dB_t = B_t r dt \). We assume \( \mu > r \geq 0 \) and \( \sigma > 0 \). Moreover, we define \( \theta \in [0,1] \) to be the fraction of wealth invested in the risky asset \( S \), and \( 1 - \theta \) the fraction invested in the risk free
We assume continuous rebalancing to keep these ratios stable. The portfolio value process \( V \) is then given by the \( \mathbb{P} \)-dynamic

\[
dV_t(\theta) = V_t(\theta) \left( \theta \frac{dS_t}{S_t} + (1 - \theta) \frac{dB_t}{B_t} \right), \quad V_0(\theta) = 1,
\]

which has the solution: \( V_t(\theta) = V_0(\theta) e^{\left( r + \theta (\mu - r) - \frac{1}{2} \theta^2 \sigma^2 \right) t + \theta \sigma W_t} \).

### 3.2 Contract Types

We study four investment (or insurance) contracts with different guarantee features and benefits at a fixed future (retirement) date \( 0 < T < \infty \) and inception \( t_0 = 0 \). The premium \( P \) paid at \( t_0 = 0 \) is assumed to be 1. The investment premium \( \alpha \) describes the fraction of the premium allocated to the investment account \( V \), while the remaining part \( 1 - \alpha \) is used to finance the guarantee. Moreover, we define lock-in dates \( t_1, \ldots, t_n = T \) as the endpoints of \( n \) equidistant subintervals of \( [0, T] \) with length \( \Delta t = \frac{T}{n} \).

The first considered guarantee feature is the **roll-up**. Its payoff at maturity \( T \) is given by

\[
A_T^{rol} := \max \left( e^{gT}, \alpha V_T \right) = \alpha V_T + \left[ e^{gT} - \alpha V_T \right]^+. 
\]

The roll-up essentially provides a guaranteed rate \( g \) on the original premium. It is a frequently offered guarantee feature, e.g., in the context of variable annuities (cf., e.g., Bauer et al. (2008)).

Estep & Kritzman (1988) argue that investors are not only interested in the protection of a pre-specified fixed level, but also in the protection of interim gains. Guarantees like ratchet or cliquet features are able to incorporate this effect. Therefore, as a second product, we consider the so-called **ratch-up**, which is a combination of a roll-up and a ratchet feature with the following payoff:

\[
A_T^{rat} := \max \left( e^{gT}, \alpha V_{t_1}, \ldots, \alpha V_T \right) = \alpha V_T + \left[ \max \left( e^{gT}, \alpha V_{t_1}, \ldots, \alpha V_{t_{n-1}} \right) - \alpha V_T \right]^+. 
\]
This product essentially pays the highest portfolio value at any lock-in date or a roll-up with rate $g$, whichever is higher.

The third and final guaranteed product will be referred to as cliquet product with payoff:

$$A_{cli}^T := \prod_{i=1}^{n} \max \left( e^{g \Delta t}, \frac{V_{t_i}}{V_{t_{i-1}}} \right) = \prod_{i=1}^{n} \left( \frac{V_{t_i}}{V_{t_{i-1}}} + \left[ e^{g \Delta t} - \frac{V_{t_i}}{V_{t_{i-1}}} \right]^+ \right)$$

In each period, this product locks in the higher of a guaranteed rate $g$ and the performance of the underlying portfolio $V$ (the latter only with respect to a portion $\frac{\alpha}{n}$ of the investment premium). This representation of a cliquet product from Ebert et al. (2012) is rather unusual. However, for $\tilde{g} = g - \log(\alpha)$, this representation coincides with the more common representation

$$A_{cli}^T := \alpha \prod_{i=1}^{n} \max \left( e^{\tilde{g} \Delta t}, \frac{V_{t_i}}{V_{t_{i-1}}} \right) = \alpha \prod_{i=1}^{n} \left( \frac{V_{t_i}}{V_{t_{i-1}}} + \left[ e^{\tilde{g} \Delta t} - \frac{V_{t_i}}{V_{t_{i-1}}} \right]^+ \right).$$

In the more common representation, the contract in each period simply earns the greater of the guaranteed rate and the performance of the underlying portfolio.

Besides the contracts with guarantee feature, we consider a contract without guarantee investing in the underlying $V$, which we refer to as constant mix (cm) contract. Note that here $\alpha = P = 1$ and therefore obviously $A_{cm}^T = V_T$.

We will only consider fair contracts with an identical initial arbitrage free price of 1, that is, we consider contracts $c \in \{rol, rat, cli\}$ with $A_0^c(g, \alpha, \theta) = 1$, where $(g, \alpha, \theta) \in [-\infty, r] \times (0, 1] \times [0, 1]$. Closed form solutions for the arbitrage free prices of the different products at $t = 0$ and at each lock-in date $t_1, \ldots, t_{n-1}$ are given in Appendix A. In the analysis, we use the approach of Ebert et al. (2012), that is, we fix $(\alpha, \theta)$ and determine for each contract the value of $g \in [-\infty, r]$, that makes the contract fair. This represents a more intuitive choice in a behavioral context than fixing $g$, since $1 - \alpha$ denotes the fair value of each guarantee and $\theta$

\footnote{If such a $g$ exists, then it is unique. Moreover, within this setting for the roll-up and the cliquet features such a $g$ exists for all combinations of $\alpha$ and $\theta$. But for large $\theta$ and small $\alpha$ no solution exists for the ratch-up feature.}
3.3 Results

In this Section, we present numerical results for the following financial market parameters: \( \mu = 0.06, \sigma = 0.3, r = 0.03 \) and \( T = n = 5 \), that is, the lock-in dates are at \( t = 1, \ldots, 5 \) and the periods are one year. We will perform sensitivity analyses in Section 3.4 including longer time horizons. For the Monte Carlo sample size we use \( l = 20,000 \) and the lower bound for \( \alpha \) is chosen to be 0.6.

3.3.1 Guarantee Levels and Terminal Distributions

Before analyzing the utility of the different contracts and the question, which guarantee type is preferred by which investor, we take a closer look at the features of the different contracts to point out the differences and similarities.

Figure 1 displays the different guarantee levels \( e^{gT} \) of the contracts for different fractions \( \theta \) invested in the risky asset. When compared to the roll-up, the ratch-up also locks in past peaks, which is more expensive. The cliquet guarantee is even more expensive since each period’s performance is maximized with the guaranteed return. Therefore for given \( (\alpha, \theta) \) we have

\[
\text{is a measure of the upside potential of the underlying.}
\]
(a) Constant Mix (Pure Stock)

(b) Roll-up with $g_{fair} = 0.0142$, guarantee level 1.0735 and guarantee level frequency 77.29%.

(c) Ratch-up with $g_{fair} = 0.0066$, guarantee level 1.0337 and guarantee level frequency 66.77%.

(d) Cliquet with $g_{fair} = -0.0938$, guarantee level 0.6257 and guarantee level frequency 2.78%.

Figure 2: Payoff distribution of the different contracts for $\alpha = 0.6$ and $\theta = 1$.

$g_{rol} \geq g_{rat} \geq g_{cli}$. Moreover, for very small values of $\theta$, the guarantee level increases slightly in $\theta$. This effect, which results from diversification, is stronger for longer time horizons. After reaching a peak, the guarantee level decreases for increasing $\theta$.

To get an impression of the risk-return-characteristics of the terminal payoffs of the different contracts, Figure 2 displays the payoff distributions for $\alpha = 0.6$ and $\theta = 1$. The upper left panel shows the payoff distribution of the pure stock investment, while the other three panels display the different guaranteed products.
The guarantee levels are reflected in the terminal payoff distributions. All distributions are right-skewed. The roll-up contract has the highest guarantee level (1.0735), and also the highest guarantee level frequency (77.29%), which is the probability that the terminal value coincides with the guarantee. It is followed by the ratch-up contract (1.0337 resp. 66.77%). The lower guarantee frequency for the ratch-up contract is a consequence of its design, which locks in past peaks, if they exceed the guarantee level. As a consequence of the lower guarantee level, the right tail of the ratch-up is heavier than the right tail of the roll-up, which indicates higher upside potential. The guarantee level of the cliquet contract amounts to 0.6257 and is by far the lowest. In turn, the guarantee level frequency is very low (2.78%). The cliquet contract protects only against rather high losses, but does so in each year, whereas the other two guaranteed contracts guarantee even a small gain compared to the initial premium. For smaller values of \( \theta \), the guarantee level of the cliquet is higher, inducing also a higher guarantee level frequency, e.g., for \( \theta = 0.5 \) the guarantee level of the cliquet contract is 1.0014 and the guarantee level frequency is 14.27%. Note that the guarantee level frequency is significantly lower for the cliquet contract, even for similar guarantee levels. This is due to the fact, that for the cliquet contract one period with a good performance is sufficient for a terminal value exceeding the guarantee.

### 3.3.2 Annual Price Changes

Since MCPT utility is driven by annual price changes, we first illustrate the differences between the contracts in this respect. Percentiles of the distribution of the annual price changes \( X_t = A_t^c - A_{t-1}^c \) with \( t \in \{1, \ldots, T\} \) and \( c \in \{cm, rol, rat, cli\} \) are displayed exemplarily for the year \( t = 3 \) in Figure 3 for different choices of \( \alpha \) and \( \theta \). In the case \( \alpha = 0.6 \) and \( \theta = 0.5 \) (upper left panel), the guarantee levels are rather similar: 1.1460 for the roll-up, 1.1431 for the ratch-up and 0.9673 for the cliquet contract. The Figure shows that the annual price changes for the guaranteed contracts are subject to less fluctuations than for the contract without guarantee (constant mix). While in the case \( \alpha = 0.6 \) and \( \theta = 0.5 \) negative price changes are very unlikely, the upper right panel shows that even for this value of \( \alpha \) the probability of negative price changes increases if \( \theta \) is increased.
The cliquet contract shows a different structure in the annual changes than the other two guaranteed contracts. It significantly reduces the probability for strong negative price changes, while the probability of medium negative price changes and the potential for large positive price changes is higher than with the other two guaranteed contracts, that is, the distribution of the annual price changes is more right-skewed. This is an important difference, since CPT investors tend to prefer right skewed distributions (cf. Barberis & Huang (2008) and Ebert & Strack (2015)). In the case $\alpha = 0.9$ (lower panels), there exists no solution for the ratch-up contract for both, $\theta = 0.5$ and $\theta = 1$. The distributions of the annual price changes of the roll-up and the cliquet contract spread more widely than for the previous cases due to the

Figure 3: Percentiles of the distribution of annual changes in the 3rd year ($X_3$) of the fair prices of the different contracts for different parameters $\alpha$ and $\theta$. The bars indicate the 1% – 5%, 5% – 10%, 10% – 25%, 25% – 75%, 75% – 90%, 90% – 95% and 95% – 99% percentiles and the black lines indicate the mean.

(a) $\alpha = 0.6, \theta = 0.5$
(b) $\alpha = 0.6, \theta = 1$
(c) $\alpha = 0.9, \theta = 0.5$
(d) $\alpha = 0.9, \theta = 1$
higher investment into the underlying investment account. Also, the distribution of the annual price changes of the cliquet contract is not as right-skewed as in the previous cases because of the significantly lower guarantee level.

### 3.3.3 Expected Utility Theory Analysis

Ebert *et al.* (2012) show that for a CRRA EUT investor with reasonable risk aversion parameter, all guaranteed contracts create disutility when compared to a constant mix contract. This is also consistent to the fact that for a CRRA EUT investor any deviation from the optimal Merton strategy leads to disutility (cf. also Merton (1971) or Tepla (2001)). These findings can be replicated in our model and hold for all pairs \((\alpha, \theta)\) and all reasonable financial market parameters.

### 3.3.4 Cumulative Prospect Theory Analysis

We were able to replicate the CPT findings from Ebert *et al.* (2012). Additionally, we performed analyses in a model with reference point adaptation. The results that we have replicated are: A CPT investor prefers either the constant mix or the roll-up contract, that is, if a guaranteed contract is preferred over the constant mix contract, then the roll-up always dominates the other guarantees even if each contract is specified with its optimal guarantee level. In particular for high values of \(\theta\) the roll-up outperforms the constant mix contract. Furthermore, the results show that in these cases either a guarantee level equal to the reference point or an insurance against large losses only is optimal for the CPT investor. The main results did not change under different static reference point choices, different CPT parameters or financial market parameters.

In addition, we have considered a model with reference point adaptation as proposed by Khuman *et al.* (2012), that is, \(\chi = s \max(A_c^0, \ldots, A_c^T) + (1 - s)A_c^0\) and \(s \in [0, 1]\) for \(c \in \{cm, rol, rat, cli\}\). The first part of the reference point includes the idea of retaining past peaks, whereas the second part is again the SQ. Our simulations under this model indicate, that this reference point adaptation is also not able to explain the demand for the more complex guar-
anteed products. Although the more complex guarantees do better than the roll-up contract for most of the combinations, for these combinations the constant mix contract outperforms all guaranteed contracts.

### 3.3.5 Multi Cumulative Prospect Theory Analysis

This Section presents the main results of our paper. We use MCPT as described in Section 2.2 to analyze the influence of the annual price changes on the subjective utility. We use the same CPT parameters as in the pure CPT case, that is, we fix $a = 0.88$, as suggested by Tversky & Kahneman (1992) and perform analyses for different values of $\lambda$ and $\gamma$. Moreover, we consider the case without discounting, that is, $\rho = 1$. As explained in Section 2.2, for each period we use the annual price change $X_t = A^c_t - A^c_{t-1}$ based on the SQ reference point for time $t$ and $c \in \{cm, rol, rat, cli\}$. We will derive certainty equivalent contracts ($CE^M$) with a fixed annual return $r^{CE}$, that is, $r^{CE}$ describes the fixed annual return that an investor would regard equally desirable as the considered contract $c$. Therefore $X_t^{CE} = A^{CE}_{t-1}(e^{r^{CE}} - 1)$ and $\sum_{t=1}^{T} \rho^t CPT(X_t^{CE}) = MCPT(A^c)$, which leads to:

$$MCPT(A^c) = \begin{cases} 
\sum_{t=1}^{T} \rho^t \left( X_t^{CE} e^{r^{CE}(t-1)} \right)^a, & MCPT(A^c) \geq 0 \\
-\lambda \sum_{t=1}^{T} \rho^t \left| X_t^{CE} e^{r^{CE}(t-1)} \right|^a, & MCPT(A^c) < 0 
\end{cases}$$

We solve the equation numerically for each contract to obtain the corresponding fixed annual return $r^{CE}$.

Figure 4 illustrates the influence of the different CPT features on the MCPT value for $\alpha = 0.9$. The upper left panel shows the MCPT without loss aversion and probability weighting, such that the only considered CPT feature is the S-shaped value function. Similar to the EUT and CPT case, for all contracts the certainty equivalent return increases with increasing $\theta$, the optimal value of $\theta$ is 1 and the constant mix strategy dominates the other contracts.

The upper right panel additionally includes loss aversion with $\lambda = 2.25$. In this setting, the
constant mix contract is dominated by the guaranteed contracts, because they reduce the probability of negative annual price changes (cf. Section 3.3.2). This is also the reason why the roll-up and the ratch-up dominate the cliquet contract in this case. Especially for higher values of $\theta$, the probability of annual losses is higher for the cliquet contract than for the other two guaranteed contracts, since (as explained in Section 3.3.2) the cliquet product has a significantly lower minimum guarantee level and hence a higher potential for a price drop. As for the CPT case, the certainty equivalent return increases slightly for low values of $\theta$ and decreases for high values of $\theta$, where the value of $\theta$ at the peak is higher, if the guarantee level is higher.

The lower left panel shows the influence of the probability weighting without loss aversion, that is, $\gamma = 0.65$ and $\lambda = 1$. Here, the cliquet contract dominates the other contracts for all values of
This can again be explained by the annual price change distributions (cf. Section 3.3.2) and the fact that CPT particularly overweights extreme events that occur with low probability. The annual price changes of the cliquet contract are extremely right-skewed, that is, they include some relatively high gains with low probabilities. These outcomes get overweighted and generate a higher certainty equivalent return. The constant mix contract also contains high gains that happen with low probabilities, but also high losses that happen with low probabilities, which reduce the certainty equivalent return. The certainty equivalent return of all contracts increases with increasing $\theta$ and the optimal value of $\theta$ is 1.

Finally, the lower right panel includes all CPT features. Here, the MCPT certainty equivalent return of the more complex guaranteed products (ratch-up and cliquet) exceeds the roll-up and the constant mix contract. Due to the loss aversion, the constant mix performs worst. The overall optimal certainty equivalent return is reached by the cliquet contract, which also performs best for most values of $\theta$. However, the optimal value of $\theta$, that is, the fraction invested in the risky asset, is lower than in the CPT case. This is due to the fact, that CPT investors with a longer evaluation period prefer more risky assets (cf. Benartzi & Thaler (1995) or Berkelaar et al. (2004)) than in our setting with an annual evaluation.

Figure 5 illustrates the same effects for $\alpha = 0.6$, that is, for higher guarantee levels. It is worth noting that especially for high values of $\theta$, the guaranteed contracts perform much better, than in the case $\alpha = 0.9$. The upper right plot shows that the higher guarantee levels reduce the losses in the annual price changes, that caused disutility in the $\alpha = 0.9$ case. Therefore, the MCPT certainty equivalent return difference between the constant mix and the guaranteed contracts is even larger in this case.

In a next step we look at different levels of $\alpha$ and $\theta$ simultaneously. Figure 6 displays the values of the MCPT certainty equivalent return for the four different contracts as a function of $\alpha$ and $\theta$. Note, that the lower bound for $\alpha$ is chosen to be 0.6.
For each fixed level of $\alpha$, the cliquet contract generates the highest certainty equivalent return. The overall maximum certainty equivalent return for the cliquet and therefore for all contracts is 4.79% for $\alpha = 0.6$ and $\theta \approx 0.5$. Moreover, the cliquet performs better than both, the roll-up and the constant mix contract for almost all pairs $(\alpha, \theta)$. The maximum for the roll-up contract is 3.01% for $\alpha \approx 0.75$ and $\theta \approx 0.325$ and as seen before the maximum for the constant mix contract is 3% at $\theta = 0$. This means that for an investor, who takes annual changes into account, the risk free asset is the most attractive underlying for a contract without guarantee. This finding might explain the demand for very safe assets, that can be observed in many markets.

$^8$If we allow for all levels of $\alpha$, this remains true and the optimal value of the cliquet contract is at $\alpha \approx 0.2$ and $\theta = 1$. 

Figure 5: $r^{CE}$ values for the constant mix and the three guaranteed products as a function of $\theta$ for $\alpha = 0.6$. 

(a) $\lambda = 1, \gamma = 1$

(b) $\lambda = 2.25, \gamma = 1$

(c) $\lambda = 1, \gamma = 0.65$

(d) $\lambda = 2.25, \gamma = 0.65$
3.3.6 A Combined CPT and MCPT Analysis

As described in Section 2.4, MCPT reflects the utility created by annual price changes. Investors might consider both, annual price changes and terminal value. Therefore, we now analyze combinations of the CPT and the MCPT utility, that is, we look at \( CPT^{com}(A^c) := sMCPT(A^c) + (1-s)CPT(X) \) with \( s \in [0, 1] \) and \( X = A_T^c - \chi \).

\(^{9}\) Note that in this setting \( \chi = \chi_1 = 1 \), where \( \chi_1 \) represents the reference point in MCPT at time 1.
Figure 7: $r_{CE}$ with $s = 0.3$ and $s = 0.5$ for the constant mix and the three guaranteed products as a function of $\theta$ and $\alpha = 0.6$ (upper panels) resp. $\alpha = 0.9$ (lower panels). CPT parameters: $\lambda = 2.25$, $\gamma = 0.65$.

equivalent contracts ($CE^{com}$) with a fixed annual return $r_{CE}$ that is determined by:

$$CPT^{com}(A^c) = \begin{cases} 
  s \sum_{t=1}^{T} \rho^t \left( X_1^{CE} e^{r_{CE}(t-1)} \right)^a + (1 - s) (X^{CE})^a, & CPT^{com}(A^c) \geq 0 \\
  -\lambda \left( s \sum_{t=1}^{T} \rho^t \left| X_1^{CE} e^{r_{CE}(t-1)} \right|^a + (1 - s) |X^{CE}|^a \right), & CPT^{com}(A^c) < 0.
\end{cases}$$

The upper panels in Figure 7 illustrate the certainty equivalents in the combined model for the case $\alpha = 0.6$ and lower panels for the case $\alpha = 0.9$, with $s = 0.3$ and $s = 0.5$. The lower panels in Figure 7 show for the case $\alpha = 0.9$, that both, the cliquet and the ratch-up contract outperform the constant mix contract for all values of $\theta$ in both considered combinations. Moreover, the ratch-up for $s = 0.3$ resp. the cliquet contract for $s = 0.5$ generate
the highest combined certainty equivalent return. Similar results can be seen in the upper panels for the case $\alpha = 0.6$. In this case the constant mix and even the roll-up are outperformed for all values of $\theta$ by at least one of the more complex guaranteed contracts. The highest combined certainty equivalent return in this case is given by the ratch-up with $\theta = 1$ in both considered combinations. Moreover, we have calculated the certainty equivalent return as a function of $\theta$ and $\alpha$ similar to the MCPT case for $s = 0.3$ and $s = 0.5$. The results show that the described findings also hold for other parameter combinations and the highest overall combined certainty equivalent return is generated by the ratch-up for $s = 0.3$ ($r^{CE} = 4.85\%$ for $\theta = 1$ and $\alpha = 0.6$) resp. the cliquet contract for $s = 0.5$ ($r^{CE} = 4.27\%$ for $\theta = 0.5$ and $\alpha = 0.6$).

We can conclude that in contrast to EUT and CPT (with and without reference point adaptation), MCPT can explain the demand for more complex guaranteed contracts (cliquet and ratch-up). This remains true even if value fluctuations only partly influence the investor’s subjective utility.

### 3.4 Sensitivity Analysis

We have performed sensitivity analyses with respect to different parameters. First, we used different financial market parameters, that is, $\mu$, $\sigma$ and $r$. Generally, under reasonable parameter settings our main findings for the MCPT are stable. For a fixed $\mu$, we find that an increasing volatility makes the simple products even less attractive, since more low probability events of large losses happen for the constant mix contract and the roll-up includes less low probability annual gains compared to the other two guaranteed contracts. Moreover, the better the market environment, that is, higher $\mu$ and lower $\sigma$, the higher the optimal fraction $\theta$ invested in the risky asset, while the differences between the products remain rather stable. Moreover, we have performed simulations with longer investment horizons $T$ (10 and 20 years). The main findings prevail, that is, the more complex guaranteed contracts still outperform the simple contracts. E.g., for $T = 10$ the overall maximum certainty equivalent annual return $r^{CE}$ is 4.15% generated by the cliquet contract for $\alpha = 0.6$ and $\theta \approx 0.33$. Contrary to the CPT, MCPT investors reduce (if possible) the fraction invested in the risky asset $\theta$ for longer
investment horizons to obtain the maximum MCPT certainty equivalent return for a fixed \( \alpha \).
However, the annualized guarantee rates \( e^{\theta} \) are rather similar. E.g., the annualized guarantee rate of the overall optimal cliquet contract in the case of \( T = 5 \) is 1.0014 (\( \alpha = 0.6, \theta \approx 0.5 \)) and 1.0025 in the case of \( T = 10 \) (\( \alpha = 0.6, \theta \approx 0.33 \)). Using different empirically reasonable value function parameters \( a \in [0.8, 1] \) (cf. Tversky & Kahneman (1992), Birnbaum & Chavez (1997) or Abdellaoui (2000)) does not change the findings significantly. We also repeated the analysis for different discounting factors \( \rho \). We have observed that for all reasonable values for the time discounting parameter \( \rho < 1 \) the influence is negligible.

Last, we have investigated the influence of the reference point adaptation, that is, the use of annual price changes as a basis for the annual CPT evaluation. For this, we ran the MCPT simulations also with reference points fixed to the initial fair price, that is, \( \chi_1 = \chi_2 = \cdots = \chi_T = 1 \). This means that investors still evaluate annually, but their reference point remains equal to the initial fair price of the contract. In this setting, we find that all guaranteed contracts outperform the constant mix contract in all cases. Without reference point adaptation, we also find that the roll-up dominates the more complex guaranteed contracts for all fixed levels of \( \alpha \). Therefore, the reference point adaptation is a necessary feature of MCPT to explain the demand for more complex guaranteed products.

4 Conclusion and Outlook

In this paper, we have proposed an extension of CPT that we call Multi Cumulative Prospect Theory (MCPT). It is based on the CPT utility generated by, e.g., annual changes of the contract value. This is motivated by the fact that investors tend to regularly re-evaluate their investment and adapt their reference point based on the evolution of the investment value. We propose that investors who are attracted by specific contract features, like annual lock-in guarantees, might be particularly inclined to take into account the subjective utility of potential future fluctuations of the contract value when making investment decisions. MCPT measures the subjective utility generated by potential interim changes of the contract value.
Nevertheless, the terminal value has an outstanding role when making an investment decision. Therefore, we also propose a combination of CPT and MCPT, which considers both, annual price changes and terminal value.

As an application we have analyzed three guaranteed products, which are common in many markets (roll-up, ratch-up and cliquet) and a contract without guarantee (constant mix). First, we could confirm previous results, that neither EUT nor CPT can explain the demand for the more complex types of guarantees. Moreover, we have performed an analysis with a CPT reference point adaptation and have found that this extension is (at least in our setting) also not able to explain the demand for the more complex guaranteed contracts.

When applying MCPT, the more complex guaranteed contracts, in particular the cliquet contract, outperform the other contracts (roll up and constant mix) in all considered cases. This is mainly caused by the more right-skewed distribution of the annual changes in the value of the cliquet contract compared to the other contracts. Hence, our approach is able to explain the demand for these contracts. Moreover, the contract without guarantee creates the more disutility, the higher the fraction invested in the risky asset $\theta$. If only products without guarantee are offered, then an MCPT investor prefers an investment in the risk free asset. Therefore, our approach can also explain the very large holdings of safe assets that can be observed in many countries.

Additionally, we have analyzed the contracts under the combined model, which considers both, the terminal value of the investment, and the annual value changes. Our results show that also in this combined model investors may have a preference for the more complex guaranteed products. This means that demand for more complex guarantees exists even if value fluctuations only partly influence the investor’s subjective utility.

The analyses in this paper provide a first indication that MCPT has some descriptive power in particular for long-term investments. Still, the influence of value fluctuations on the subjective
utility is not completely understood. Future research should therefore address the following questions: For which types of contract features do investors tend to take into account value fluctuations when making the investment decision? How strong is the influence of the value fluctuations on the subjective utility compared to the influence of the terminal value? Moreover, we have only analyzed the MCPT-utility at time $t = 0$. An analysis of the MCPT-utility during the contract duration could give additional insights on client’s surrender behavior. Also, we have only considered a restricted set of contracts and compared them through simulations, leaving several theoretical questions like: Under which general conditions does an optimum for a MCPT investor exist and what general properties hold for the MCPT? Furthermore, there are several other aspects that might affect the results and should therefore be considered in future research. Such aspects include the influence of a more dynamic reference point, that depends on the complete history of previous gains and losses (as opposed to the last value only), time dependent CPT parameters for loss aversion and probability weighting, etc. Also, experiments about suitable models for multiple reference points and reference point adaptation and experiments to verify that typical CPT parameters are suitable choices also for the evaluation of future value fluctuations would be desirable.

### A Pricing Formulae for the Considered Guaranteed Products

In this Appendix we give the arbitrage free prices for the three guaranteed products considered in Section 3 at $t_0 = 0$ and at each lock-in date $t_1, \ldots, t_{n-1}$. The proofs of the pricing formulas of Proposition A.1 use similar techniques as presented by Ebert et al. (2012), who only gives prices for $t = 0$. We therefore omit the details.

**Proposition A.1** (Arbitrage free Pricing)

Let $B_{t}^{Put}(S_t,t,\theta,K,T)$ denote the time $t$-price of a European put option with underlying $S,$
maturity $T$ and strike $K$, that is,
\[
\mathcal{B}^{\text{Put}}(S_t, t, \theta, K, T) = e^{-r(T-t)} K \mathcal{N}
\left(-h^{(2)} \left((T-t), \frac{S_t}{K}\right)\right) - S_t \mathcal{N}
\left(-h^{(1)} \left((T-t), \frac{S_t}{K}\right)\right)
\]

with
\[
h^{(1)}(t, z) := \frac{\ln(z) + (r + \frac{1}{2} \theta^2 \sigma^2) t}{\theta \sigma \sqrt{t}} \quad \text{and} \quad h^{(2)}(t, z) := h^{(1)}(t, z) - \theta \sigma \sqrt{t}.
\]

Here, $\mathcal{N}(\cdot)$ denotes the one-dimensional and $\mathcal{N}_d(\cdot)$ the $d$-dimensional cumulative standard normal distribution. Then the arbitrage free prices for $m \in \{0, \ldots, n-1\}$ are given by:

(i) Roll-up:
\[
A^{\text{rol}}_m(g, \alpha, \theta) = \alpha V_{tm} + \mathcal{B}^{\text{Put}}\left(\alpha V_{tm}, t_m, \theta, e^{gT}, T\right)
\]

(ii) Ratch-up:
\[
A^{\text{rat}}_m(g, \alpha, \theta) = \alpha V_{tm} + \mathbb{I}\left\{\left\{ V_j \leq e^{gT} \right\}_{1 \leq j \leq m}\right\}
\left( e^{-r(T-t_m)+gT} \mathcal{N}_{n-m}(v_1, \Sigma_m) - \alpha V_{tm} \mathcal{N}_{n-m}(v_2, \Sigma_m) \right)
+ \sum_{i=1}^{n-1} I_{m,i}
\]

where
\[
I_{m,i} := \mathbb{I}\{i \leq m\} \mathbb{I}\left\{\left\{ V_i \geq e^{gT} \right\}_{1 \leq j \leq m, j \neq i}\right\}
\alpha \mathcal{N}_{n-m}(v_3, \Sigma_m) \left( e^{-r(T-t_m)} V_i - V_{tm} \right)
+ \mathbb{I}_{\{i > m\}} \alpha V_{tm} \mathcal{N}_{n-m}(v_4, \Sigma_i) \left( e^{-r(T-t_i)} \mathcal{N}_{n-i}(v_5, \Sigma_i) - \mathcal{N}_{n-i}(v_6, \Sigma_i) \right)
\]

with
\[
v_1 := \left(-h^{(2)} \left(t_{m+1} - t_m, \frac{\alpha V_{tm}}{e^{gT}}\right), \ldots, -h^{(2)} \left(T - t_m, \frac{\alpha V_{tm}}{e^{gT}}\right)\right)
\]
\[
v_2 := \left(-h^{(1)} \left(t_{m+1} - t_m, \frac{\alpha V_{tm}}{e^{gT}}\right), \ldots, -h^{(1)} \left(T - t_m, \frac{\alpha V_{tm}}{e^{gT}}\right)\right)
\]
$$v_3 := \left( -h^{(1)} \left( t_{m+1} - t_m, \frac{V_m}{V_t} \right), \ldots, -h^{(1)} \left( T - t_m, \frac{V_m}{V_t} \right) \right)$$

$$v_4 := \left( h^{(1)} \left( t_i - t_m, \min \left( \frac{\alpha V_m}{e^{\theta g \Delta t}}, \frac{V_m}{V_t^i}, \ldots, \frac{V_m}{V_t^m} \right) \right), h^{(1)} \left( t_i - t_{m+1}, 1 \right), \ldots, h^{(1)} \left( t_i - t_{i-1}, 1 \right) \right)$$

$$v_5 := \left( -h^{(2)} \left( t_{i+1} - t_i, 1 \right), \ldots, -h^{(2)} \left( T - t_i, 1 \right) \right)$$

$$v_6 := \left( -h^{(1)} \left( t_{i+1} - t_i, 1 \right), \ldots, -h^{(1)} \left( T - t_i, 1 \right) \right)$$

and corresponding variance-covariance matrices: $\Sigma_i$, $\Sigma_m^i$, $\Sigma_m$.

(iii) Cliquet:

$$A_{tm}^{cli}(g, \alpha, \theta) = \left( \alpha^{\frac{1}{n}} + B^{Put} \left( \alpha^{\frac{1}{n}}, 0, \theta, e^{\theta g \Delta t}, \Delta t \right) \right)^{n-m} \prod_{i=1}^{m} \max \left( \alpha^{\frac{1}{n}} \frac{V_i}{V_{t_i-1}}, e^{\theta g \Delta t} \right)$$

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2 As You like It: Explaining the Popularity of Life-Cycle Funds with Multi Cumulative Prospect Theory

Source:

As You like It: Explaining the Popularity of Life-Cycle Funds with Multi Cumulative Prospect Theory

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Abstract

Life-cycle (or target-date) funds are funds which typically decrease their risk exposure over time. They have been very successful in many countries in particularly in the segment of old age provision. However, Expected Utility Theory (EUT) cannot explain their popularity. Moreover, recent results of Graf (2017) imply that not only EUT but also its behavioral counterpart Cumulative Prospect Theory (CPT) is often not able to explain the popularity of these products since for each life-cycle fund a corresponding balanced fund can be constructed which is preferable from the investor’s perspective in most circumstances. In a very recent paper, Ruß & Schelling (2018) have argued that potential future changes in an investment’s value already impact the decision of long-term investors at outset. Based on this, they have introduced Multi Cumulative Prospect Theory (MCPT) which is based on CPT and considers the subjective utility generated by annual value changes. This paper shows that for MCPT-investors, life-cycle funds are typically more attractive than their corresponding balanced funds since they reduce the potential losses towards the end of the investment horizon. Hence, our findings provide an explanation for inferior decisions in old age provision. This can serve as a basis to improve such decisions.

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1 Introduction

In a recent paper, Graf (2017) has analyzed life-cycle funds for single and regular contributions and under different asset models. His results imply that for any given life-cycle fund, a rational investor (maximizing expected utility) can find a superior balanced fund. Nevertheless, life-cycle funds have been and continue to be extremely successful. A recent study by Yang et al. (2015) shows that assets under management of life-cycle funds in the US have steadily increased from roughly $100bn in 2006 to $700bn in 2014. One of the reasons for this success is that a large number of pension plans allocate their contributions to these life-cycle funds, since – according to Charlson et al. (2010) – 96% of the large\(^1\) pension plans in the US selected life-cycle funds as their default asset allocation and most of the money allocated in these funds is left in the same fund until retirement. However, Mitchell et al. (2009) show that default options alone cannot explain the demand since life-cycle funds are also frequently chosen when no default option exists.

The results of Graf (2017)\(^2\) imply that in most cases, the sharp contrast between optimal utility maximizing behavior (in terms of terminal wealth) – that is, not invest in life-cycle funds – and observed actual behavior – that is, invest in life-cycle funds – can also not be explained by Cumulative Prospect Theory (CPT), the most popular behavioral counterpart to Expected Utility Theory that was introduced by Tversky & Kahneman (1992).

Several authors pointed out that investors tend to reevaluate their investment regularly, cf., e.g., Benartzi & Thaler (1995) or Gneezy & Potters (1997). In a very recent paper, Ruß & Schelling (2018) have argued that – particularly for rather long investment horizons – investors also get subjective utility and disutility from interim value gains and losses. Moreover, they argue that this already impacts the investment decision at outset. They develop a modification of CPT that considers potential future value fluctuations already in the decision making. They

\(^1\)Defined as including more than 5,000 employees.

\(^2\)Graf (2017) finds stochastic dominance for a single premium investment in a Black-Scholes economy and “practical” stochastic dominance for regular premium investments when (far) upper and lower tails of the distribution are ignored (cf. Figure 1 for a comparison of the resulting cumulated distribution functions).
show that their so-called Multi Cumulative Prospect Theory (MCPT) is able to explain the demand for certain complex guaranteed products that are very popular in many markets but should not be desired by either rational investors or CPT-investors. Since life-cycle funds and balanced funds – even if they were designed to have a similar probability distribution of terminal wealth – significantly differ with respect to the distributions of the potential value changes over time, they would create significantly different subjective utility for an MCPT-investor. The present paper therefore analyzes how such an investor’s choice differs from a rational investor or a CPT-investor.

Considering academic literature on life-cycle funds, three main streams of research can be identified: The first one is concerned with comparing possible returns of arbitrary life-cycle funds and arbitrary (unrelated) balanced funds. As a starting point, Blake et al. (2001) provide a Value-at-Risk based analysis of different investment strategies including a balanced and a life-cycle fund within a defined contribution plan setting. In addition, in the context of mutual funds, various authors analyze the results of some life-cycle strategies compared to some balanced funds using different methodologies such as expected utility or shortfall measures. Due to the rather arbitrary choice of the balanced and life-cycle funds under investigation in the different papers, their conclusions are ambiguous: some conclude that life-cycle funds should be preferred from an investor’s point of view (e.g., Pfau (2010)) some prefer balanced funds (e.g., Spitzer & Singh (2008) or Schleef & Eisinger (2007)) while others (e.g., Pang & Warshawsky (2008)) are indifferent.3

The second strain of literature is concerned with finding “optimal” life-cycle funds. Here, the focus is on deriving in some sense optimal path-dependent life-cycle strategies in contrast to purely deterministic life-cycle strategies (cf. Cairns et al. (2006) for an application of stochastic control or Basu et al. (2011) for a simulation study). In a recent work, Bernard & Kwak (2016) show that a generally decreasing risk exposure over time can be explained using SAHARA utility preferences as introduced by Chen et al. (2011) in a dynamic setting. However, they

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3These somewhat contradicting conclusions are in sharp contrast to the results by Graf (2017) (cf. Section 5.1 for more details) who constructs a “matching” balanced fund for any given life-cycle fund.
also show that typical life-cycle funds are not consistent with the dynamic setting.4

The third stream of literature analyses why life-cycle funds are popular from a behavioral perspective. In these descriptive models, the main goal is to understand how and why people make their decisions (in contrast to normative models that identify optimal decisions). Mitchell et al. (2009) show empirical evidence of default and framing effects in the context of life-cycle funds. As mentioned above, they also show that default alone cannot explain the demand. More generally, the role of behavioral decision making in the context of retirement saving has been analyzed by various authors such as Mitchell & Utkus (2004), Thaler & Benartzi (2004), Benartzi & Thaler (2007), Brown (2007), and many others.

However, a sound opinion or common understanding, whether a balanced fund or a life-cycle fund with some deterministic glide path5 is preferred by a certain type of investor does not exist. Based on the results from Graf (2017) and using MCPT, we will show that even if a life-cycle fund’s terminal value distribution is “practically” stochastically dominated6 by a suitably designed balanced fund it will still be preferred by a large group of investors. Hence our findings offer an explanation for inferior decisions in saving for retirement. Understanding such inferior decisions is an important requirement for improving such decisions.

The remainder of this paper is organized as follows: In Section 2, we briefly summarize the concept of MCPT. Section 3 describes the life-cycle funds and balanced funds that will be analyzed in this paper. In Section 4, we present the financial model used to derive the numerical results in Section 5 where we start with a short summary of the results from Graf (2017) followed by the results from our simulation study. In particular, we analyze how much weight has to be given to interim fluctuations (vs. terminal value) in the investment decision and also which level of loss aversion has to be present in order to prefer life-cycle funds over a balanced fund. Finally, Section 6 concludes.

4Chen & Vellekoop (2017) find similar results as Bernard & Kwak (2016) in a dynamic setting with consumption.
5The life-cycle fund’s glide path defines its asset allocation of risky and riskless assets over time.
6Cf. Section 5.1 for details.
2 Multi Cumulative Prospect Theory

Cumulative Prospect Theory (CPT), introduced by Tversky & Kahneman (1992), has become one of the most prominent behavioral counterparts to Expected Utility Theory (EUT). In contrast to EUT, CPT is not a normative theory that determines how people should rationally behave, but rather a descriptive theory that tries to predict how people actually make decisions that may be influenced by heuristics and biases. It considers gains and losses with respect to a certain reference point instead of total wealth. Moreover, it is based on an S-shaped value function $v$ which assumes that investors are typically loss averse and a probability distortion function $w$ which takes into account that investors tend to overweight events with small probabilities (particularly extreme events) and underweight events with high probabilities. Although CPT can explain behavior in real life that cannot be explained by EUT, for long investment horizons, even CPT frequently fails to explain typical behavior. In particular, there are many investment products that are very popular for long investment horizons which neither an EUT-investor nor a CPT-investor would buy.

One reason is that CPT (like EUT) is typically applied such that investment products only generate subjective utility in connection with actual cash flows, that is, in case of long-term saving products, only at maturity. However, even long-term investors tend to reevaluate their investment products regularly, for example, annually when they receive a financial statement, cf. Barberis et al. (2001). If a reported value is lower than previous year’s value, this will be perceived as a loss that will be valued higher than a gain of similar amount in a different year. Benartzi & Thaler (1995) propose the theory of myopic loss aversion, a combination of loss aversion and frequent investment evaluation, and provide an explanation for the equity premium puzzle. Moreover, mental accounting, introduced by Thaler (1985), implies that investors tend to take into account potential future fluctuations of the contract’s value already when making an investment decision. This motivates that the initial subjective utility of a long term investment not only depends on the distribution of the terminal outcome, but also on the distributions of the possible future interim changes. To capture this effect, Ruß & Schelling (2018) have introduced a modification of CPT, the so-called Multi Cumulative Prospect Theory.
(MCPT) which essentially uses CPT with multiple reference points and evaluation periods to measure the subjective utility of the potential interim value changes. Since the difference between CPT and MCPT typically becomes larger for an increasing investment horizon, MCPT is therefore particularly useful to explain and predict actual behavior for long-term investment decisions. The remainder of this section introduces MCPT closely following Ruß & Schelling (2018).

MCPT considers an investor and an investment contract $A$ with time horizon $[0, T]$, $T \in \mathbb{N}$, at time 0, that is, at the time of decision making. For the sake of simplicity, in this paper we only consider annual future interim evaluations. Since in many countries for funds and fund-linked products, the current market value of the product has to be communicated to the client on a regular basis, we consider for all $t \in \{1, \ldots, T\}$ the annual gain or loss $X_t := A_t - \chi_t$, where $A_t$ is the market value of the investment $A$ at time $t$ and $\chi_t$ is the reference point for time $t$.

The natural reference point choice (status quo reference point) for each period is given by the market value of the contract at the end of the previous year, that is, $\chi_t = A_{t-1}$. Note that in the case of regular premiums the reference point equals the market value of the contract at $t - 1$ plus the premium invested at $t - 1$.

We can now evaluate the CPT value of each annual value change $X_t$ by (cf. Tversky & Kahneman (1992) and Hens & Rieger (2010))

$$CPT(X_t) := \int_{-\infty}^{0} v(x) \, d(w(F_t(x))) + \int_{0}^{\infty} v(x) \, d(-w(1 - F_t(x)))$$

where $F_t(x) = \int_{-\infty}^{x} d\mu_{X_t}$ with $\mu_{X_t}$ the probability measure given by the random variable $X_t$ and $v$ denotes the typical S-shaped CPT value function $v(x) := x^a \mathbb{I}_{\{x \geq 0\}} - \lambda|x|^b \mathbb{I}_{\{x < 0\}}$ with parameters $0 < a, b \leq 1$. We use the common assumption\(^7\) and set $a = b$. Moreover, the probability distortion function $w$ is given by $w(x) := \frac{p^\gamma}{(p^\gamma + (1-p)\gamma)^\frac{1}{\gamma}}$ with $\gamma \in (0.28, 1]$, where the lower boundary for $\gamma$ is chosen, such that $w(p)$ is strictly monotonically increasing for $p \in [0, 1]$.

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The MCPT utility at time $t = 0$ is then given by

$$MCPT(A) := \sum_{t=1}^{T} \eta^t \cdot CPT(X_t)$$

with a subjective discounting parameter $\eta \in \mathbb{R}_+$. The MCPT utility reflects the subjective utility of the potential annual changes. Ruß & Schelling (2018) also suggest a combined model given by the weighted sum of the CPT and the MCPT utility. In doing so, the combined model captures the subjective utility of the terminal value change $X = A_T - \chi$ (CPT) with respect to a reference point $\chi$ and the annual value changes $X_t = A_t - \chi_t$ (MCPT). The combined model is given by

$$CPT^{\text{com}}(A) := s \cdot MCPT(A) + (1 - s) \cdot CPT(X)$$

where $s \in [0, 1]$ controls the impact of the annual value changes on the combined subjective utility. Note that the CPT reference point $\chi$ for the terminal value change in the single premium case is given by the single premium $P_0$. In case of regular premiums, we set this reference point $\chi$ to the sum of all premiums, that is, $\chi = \sum_{t=0}^{T-1} P_t$. Also note that $s = 0$ represents the pure CPT utility and $s = 1$ the pure MCPT utility.

## 3 Life-Cycle and Balanced Funds

Life-cycle and balanced funds generally invest in a mix of stocks and bonds. A balanced fund has a constant portion $x_S \in [0, 1]$ of its capital invested in stocks and the remaining part in bonds. We further assume a fixed duration $d$ for the bond portion and therefore model the bond portfolio by a “rolling” investment in zero-coupon bonds with time to maturity $d$. In contrast, a life-cycle fund applies a time-dependent (however not path-dependent) asset allocation strategy, where $(x_{S,t})_t \in [0, 1]$ denotes the stock portion at time $t$.

In our numerical analyses in Section 5, we primarily focus on the “classical” deterministic glide
path where for $T > 1$ the life-cycle fund starts with a complete investment in stocks, that is, $x_{S,0} = 1$ and each year linearly decreases its stock exposure to $x_{S,T-1} = 0$. We assume the asset allocation to be constant throughout each year. The potential impact of other glide paths will be considered in the sensitivity analyses (cf. Section 5.3).

4 Financial Model

We use a slightly modified version of the Heston model (cf. Heston (1993)) for stock markets and the Cox-Ingersoll-Ross model (cf. Cox et al. (1985)) for interest rate markets.

Let $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$ be a filtered probability space equipped with the natural filtration $\mathbb{F} = (\mathcal{F}_t)_{t \in [0,T]}$ for $t \in [0,T]$ generated by $\mathbb{P}$-Brownian motions $W_1(t)$, $W_2(t)$ and $W_3(t)$. We assume $dW_2(t)dW_3(t) = \rho dt$ and $dW_1(t)dW_2(t) = dW_1(t)dW_3(t) = 0$. Further let $r(t)$ denote the short-rate and $S(t)$ denote the stock's spot price at time $t$, respectively. The (real-world) asset model is then summarized with the $\mathbb{P}$-dynamics

\[
\begin{align*}
    dr(t) &= \kappa_r (\theta_r - r(t)) dt + \sigma_r \sqrt{r(t)} dW_1(t), \quad r(0) = r_0, \\
    dS(t) &= S(t) \left( (r(t) + \lambda_S) dt + \sqrt{V(t)} dW_2(t) \right), \quad S(0) = S_0, \\
    dV(t) &= \kappa_V (\theta_V - V(t)) dt + \sigma_V \sqrt{V(t)} dW_3(t), \quad V(0) = V_0,
\end{align*}
\]

where $\lambda_S$ denotes the equity risk premium and $\sqrt{V(t)}$ is the stochastic evolution of stock-volatility over time. In this model, zero-bond prices $P(t,d)$ at time $t$ with time-to-maturity $d$ can be derived using standard no-arbitrage arguments\(^8\): $P(t,d) = A(d) e^{-B(d)r(t)}$ with $A(d) = \left[ \frac{2h - \exp((\kappa_r + h) - d/2)}{(\kappa_r + h)(\exp(hd) - 1) + 2h} \right]^{\frac{2\kappa_r \theta_r}{\sigma_r^2}}$ and $B(d) = \frac{2\exp(hd) - 1}{(\kappa_r + h)(\exp(hd) - 1) + 2h}$ where $h = \sqrt{\kappa_r^2 + 2 \cdot \sigma_r^2}$, $\tilde{\kappa}_r = \kappa_r + \lambda_r \sigma_r$, $\tilde{\theta}_r = \frac{\kappa_r \theta_r}{\kappa_r + \lambda_r \sigma_r}$ and $\lambda_r$ denotes the market price of interest rate risk.

To be able to compare our results with those from Graf (2017), we use the same capital market parameters that are summarized in Table 1. Sensitivity analyses with respect to different capital

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\(^8\)Cf., e.g., Bingham & Kiesel (2004).
market assumptions can be found in Section 5.3.

5 Results

In this section we will present our results: First, Section 5.1 summarizes existing results for a rational EUT-investor. Then, Section 5.2 gives the corresponding results for an MCPT-investor. Finally, Section 5.3 closes with a variety of sensitivity analyses.

All numerical results in this Section are based on Monte Carlo simulations with 20,000 trajectories. Within this simulation, we apply daily rebalancing between stocks and bonds in all funds and further assume 252 trading days per year.

5.1 The Rational Investor: Existing Results under EUT

Graf (2017) has shown that in a simple Black-Scholes model\(^9\) and for a single premium investment, for every life-cycle fund with given time dependent asset allocation \((x_{S,t})_t \in [0, 1]\) and some management fee, there exists a uniquely defined balanced fund with constant stock ratio \(x^*_S = x_S\) and a management fee that exceeds the management fee of the life-cycle fund such that both funds have the same probability distribution of wealth at maturity. This implies that a balanced fund with constant stock ratio \(x^*_S\) that comes with the same management fee as the life-cycle fund, dominates the life-cycle fund.

Since there is no reason why a (simpler) balanced fund should have a higher management fee than a (more complex) life-cycle fund, and since under EUT (as well as CPT), utility is only derived from the terminal value, an EUT (or CPT)-investor would (in this setting) always pre-

\(^9\)That is, stocks are modeled via a geometric Brownian motion and the interest rate is assumed to be constant.
fer a balanced fund with suitable stock ratio over any given life-cycle fund.

Graf (2017) also shows that in the case of regular premiums dominance is lost even in the simple Black-Scholes economy. However, for any life-cycle fund with asset allocation \((x_{S,t})_t \in [0,1]\) and some management fee, there exists a uniquely defined balanced fund with constant stock ratio \(x_{S}^* = x_S\) and a management fee that exceeds the management fee of the life-cycle fund such that the distribution of the two funds’ maturity values have the same expected value and variance. Again, if the balanced fund comes with the same management fee, it has a higher expected value and the same variance and the distribution would be “practically” dominating, crossing the distribution of the life-cycle fund only at the very far tail.

Finally, Graf (2017) analyzes funds under more complex asset models (e.g., the one used in this paper). Here, for any given life-cycle fund, first a “matching” balanced fund is constructed by using the stock ratio that would result in the same expected value and variance in a Black-Scholes economy. In a second step, these two funds are compared under the respective more complex asset model. Although under such asset models stochastic dominance is lost even in the single premium case, the results still show “practical” stochastic dominance for the single and regular premium case (cf. Figure 1). For the classical life-cycle fund’s glide path as defined
in Section 5.2 and an investment horizon of 35 years, Figure 1 shows the estimated cumulative
distribution function of the life-cycle fund and an appropriately calibrated balanced funds con-
sidering a single premium (left panel) and regular premium investment (right panel).

If the life-cycle fund has a management fee of 0%, the matching balanced fund has a stock
portion of 0.5816 and a management fee of 0.244% p.a. for the single premium case and a stock
portion of 0.4173 and a management fee of 0.094% p.a. for the regular premium case.

As a consequence, the high popularity of life-cycle funds that can be observed in many countries
cannot be explained by EUT or CPT. However, it seems natural that potential interim value
changes (that are not captured by EUT or CPT) also have an impact on investors’ preferences.
We will use MCPT to analyze this aspect in the next section.

5.2 CPT- and MCPT-Investors: Numerical Analyses

In the remainder of this section, we will analyze four different products and assume an invest-
ment horizon of 35 years. Note that we do not aim to derive the “optimal” investment strategy
in a general sense but rather focus on the typical decision problem between a small number of
alternative choices which long-term investors are often confronted with (e.g., when consulting
a financial advisor for retirement saving). This is in line with studies which show that investors
tend to consider rather small samples of investment choices isolated from other choices or fu-
ture opportunities, cf. Kahneman & Lovallo (1993). In addition to a life-cycle fund (denoted by
LC in the following figures) and the corresponding balanced fund (BF) that approximates the
terminal distribution of wealth of the LC as described in Section 5.1, we also consider direct
investments in the stock and the bond, respectively. Further, we consider the case of a single
premium investment of 1 at $t = 0$ as well as regular premium payments of 1 at $t = 0, 1, \ldots, 34$.

Note that we do not consider any management fees for any product. In particular, we do not

\footnote{For the sake of clarity: The moments are matched in a Black-Scholes economy. The resulting balanced fund is then analysed under the asset model described in Section 4.}
consider the additional management fee that would be required to match the moments of the balanced fund and the life-cycle fund, since in practice, a balanced fund would not be more expensive than a life-cycle fund. This will prevent a demand for the life-cycle fund only because the “competing” product is more expensive.

We use MCPT to analyze the influence of the annual price changes on the subjective utility. We fix $a = b = 0.88$ as suggested by Tversky & Kahneman (1992) and perform analyses for different values of $\lambda$ and $\gamma$. Moreover, we consider the case without discounting, that is, $\eta = 1$.

As in Ruß & Schelling (2018), we derive certainty equivalent contracts with a fixed annual return $r^{CE}$. We solve the following equation numerically for each contract to obtain the corresponding fixed annual return $r^{CE}$ that an MCPT-investor would regard equally desirable as the considered contract $A$.

$$CPT^{com}(A) = \begin{cases} 
  s \cdot \sum_{t=0}^{T-1} \left( \sum_{\tau=0}^{t} P_{\tau} \left( e^{r^{CE}(t-\tau)} - 1 \right) \right)^{a} + \\
  (1 - s) \cdot \left( \sum_{t=0}^{T-1} P_{t} \left( e^{r^{CE}(T-t)} - 1 \right) \right)^{a}, & CPT^{com}(A) \geq 0 \\
  -\lambda \cdot s \cdot \sum_{t=0}^{T-1} \left( \sum_{\tau=0}^{t} P_{\tau} \left( e^{r^{CE}(t-\tau)} - 1 \right) \right)^{a} + \\
  -\lambda \cdot (1 - s) \cdot \left( \sum_{t=0}^{T-1} P_{t} \left( e^{r^{CE}(T-t)} - 1 \right) \right)^{a}, & CPT^{com}(A) < 0.
\end{cases}$$

Figure 2 shows the certainty equivalent returns as a function of loss aversion for a CPT-investor, that is, an investor who does not value interim fluctuations ($s = 0$). Such an investor would always prefer a pure stock investment. This is typical under CPT for a long time horizon where losses are rather unlikely and hence loss aversion plays only a minor role. These results are very similar for single premium (upper panels) and regular premiums (lower panels). Furthermore, the probability distortion (that is switched off in the left panels and switched on in the right panels) leads to an overestimation of very high gains that incur with low probability which makes the stock investment even more attractive.
If we compare the life-cycle to the balanced fund, we find that in the single premium case and in the regular premium case without probability distortion, the balanced fund is preferred by a CPT-investor. In the regular premium case with probability distortion, however, a CPT-investor would (slightly) prefer the life-cycle fund. This is due to the fact that the left tail of the cumulative distribution function of the terminal value of the balance fund is slightly heavier, that is, the probabilities for high losses are slightly higher for the balanced fund than for the life-cycle fund. Due to the overestimation of low probability events, these outcomes affect the CPT-investor disproportionally.

In summary, however, a CPT-investor would always prefer a pure stock investment over both,
Figure 3: MCPT ($s = 1$) Certainty Equivalents as a function of $\lambda$. Upper panels single premium – lower panels regular premiums. Left panels no probability distortion ($\gamma = 1$) – right panels with probability distortion ($\gamma = 0.65$).

(a) $a = 0.88, \gamma = 1$

(b) $a = 0.88, \gamma = 0.65$

Figure 3 shows the same results for a pure MCPT-investor, that is, an investor that only values interim fluctuations and does not assign any weight to the terminal value ($s = 1$). The pattern is structurally similar in all four panels: For very low loss aversion, the investor prefers a pure stock investment, for rather high loss aversion, a pure bond investment is preferred. In between, the life-cycle fund is the most attractive choice for an MCPT-investor. Interestingly, for typical loss aversions between 1.6 and 2.2\(^{11}\), the life-cycle fund is either the most attractive choice for

\(^{11}\)These are loss aversion levels that seem to prevail for most people according to empirical studies, cf. Tversky
an MCPT-investor or a very close second after the pure bond investment. This is true in all four cases (single and regular premium, with and without probability distortion).

The reason why typical MCPT-investors prefer the life-cycle fund over the balanced fund can be seen in Figure 4. In the first years, the life-cycle fund generates higher possible annual losses than the balanced fund, because of the higher stock ratio. However, due to its decreasing stock ratio, potential fluctuations in later years are much smaller for the life-cycle fund than for the balanced fund. This difference in later years is particularly large in the regular premium case where the portfolio value tends to be strongly increasing over time. Due to loss aversion, the & Kahneman (1992) or Rieger et al. (2017).
(much) higher possible annual losses in later years outweigh the higher possible annual gains and therefore make the balanced fund less attractive for an MCPT-investor.

Futher, we analyze which weight \( s \) has to be assigned to interim changes in order to prefer a life-cycle fund. In Figure 5 we therefore fix the loss aversion parameter \( \lambda \) at a value of 2.2 (cf. Tversky & Kahneman (1992)) and display certainty equivalent returns as a function of \( s \).

Again, we observe a rather similar pattern in all four panels: For low values of \( s \), the pure stock investment is preferable (consistent with Figure 2). For values above roughly 0.5, however, the life-cycle fund is preferred. In the case with probability distortion, for values very close to 1 the certainty equivalent return of the pure bond investment exceeds the life-cycle fund’s. It is...
noteworthy that if only the balanced fund and the life-cycle fund are compared, the life-cycle fund is preferred for all (regular premiums) respectively almost all (single premium) values of $s \in [0, 1]$.

Finally, Figure 6 gives the certainty equivalent returns in the combined model as a function of the loss aversion parameter $\lambda$. Note that here, in all four panels probability distortion is considered. The left panels are for $s = 0.5$ (that is, an investor that places only moderate weight on interim fluctuations) whereas the right panels are for $s = 0.8$ (that is, a higher weight is placed on interim fluctuations). Again, we find a similar pattern in all four panels: For low loss aversion, the pure stock investment is preferred, for very high loss aversion, the pure bond...
investment is preferred (although for \( s = 0.5 \) this situation would only be observed for values that are not displayed anymore) and in between, the life-cycle fund is preferred, in particular for loss aversion parameters in the typical range of around two. Here, as in all previous figures, it is striking that for all analyzed parameter combinations where the balanced fund would be preferred over the life-cycle funds, a pure stock investment would be preferred over both.

We can conclude that MCPT can explain the popularity of life-cycle funds from a behavioral perspective and that this remains true even if the annual value fluctuations only partly influence the investor’s subjective utility.

5.3 Sensitivity Analysis

We will now present sensitivity analyses with respect to different parameters. Note that in some of the sensitivity analyses, the stock ratio of the “matching” balanced fund might differ from the balanced fund in Section 5.2 since, e.g., for a different investment horizon, a different balanced fund would match the considered life-cycle fund.

First – given the current low interest rate environment – we have analyzed a variety of scenarios with lower interest rate levels. Other parameters unchanged, we find that by reducing the initial interest rate \( r(0) \) and the long-term interest rate level \( \theta_r \) all products become less attractive for an MCPT-investor, because their expected return is reduced. Moreover, the range of loss aversions for which the life-cycle fund is the most attractive investment becomes smaller. Nevertheless, for reasonable interest rate parameters, the life-cycle fund is more attractive than the corresponding balanced fund for all MCPT-investors with a loss aversion \( \lambda \geq 1 \). Figure 7 shows by way of example the results for \( r(0) = 0.5\% \) and \( \theta_r = 3\% \).

If the equity risk premium \( \lambda_S \) increases and/or the long-term stock volatility \( \theta_V \) decreases, then a pure stock investment becomes more and a pure bond investment less attractive. The relation between the products remains rather stable. Note that in this case the range of loss aversions for which the life-cycle fund is the most attractive product is slightly shifted to higher
Figure 7: MCPT Certainty Equivalents as a function of $\lambda$ for $\gamma = 0.65$ for lower interest rate level. Upper panels single premium – lower panels regular premiums. Left panels $r(0) = 4.5\%$, $\theta_r = 4.5\%$ – right panels $r(0) = 0.5\%$ and $\theta_r = 3\%$.

values.

Figure 8 displays the impact of a shorter investment horizon $T$ (5, 10 and 20 years) compared to $T = 35$ in the single premium case. Here, the range of loss aversions for which the life-cycle fund is the most attractive product becomes smaller. However, for investors with a loss aversion $\lambda \geq 1$, the life-cycle fund is more attractive than the corresponding balanced fund also for shorter investment horizons. We find similar results also in the regular premium case.

We have also performed sensitivity analyses for the discounting parameter $\eta$ and have observed that for all reasonable values this parameter’s impact is negligible.
Finally, we have analyzed a so-called “deferred” strategy which is another popular life-cycle strategy (again with maturity $T = 35$ years). It is completely invested in stocks for the first 30 years. Over the last five years, the stock exposure is decreased linearly to $x_{S,T-1} = 0$. We again find that the life-cycle fund is preferred over the corresponding balanced fund for all loss aversion levels $\lambda > 1$ (cf. Figure 9). However, since this life-cycle fund and hence also the matching balanced fund are both rather risky, the bond investment is already preferred by MCPT-investors with a loss aversion level $\lambda$ above roughly 1.85 (single premium), respectively 1.75 (regular premiums). Nevertheless, further analyses show that the loss aversion level above which the pure bond investment is preferred is higher for deferred strategies with a lower initial
stock exposure.\textsuperscript{12} This shows that MCPT is also able to explain the popularity of other typical life-cycle strategies which reduce the stock ratio only in the last years of the investment horizon.

6 Conclusion

In this paper, we have analyzed the demand for life-cycle and balanced funds. We have particularly considered “matching” pairs of life-cycle and balanced funds that come with a comparable probability distribution of terminal wealth.

We have shown that the observed high demand for life-cycle funds cannot be explained by either EUT or CPT, since – based on a result in Graf (2017) – for any given life-cycle fund a corresponding balanced fund which (“practically”) stochastically dominates the considered life-cycle fund can be designed.

For an investor who evaluates investment options according to MCPT – as introduced by Ruß & Schelling (2018) – life-cycle funds are typically more attractive than “matching” balanced

\textsuperscript{12}We have also analyzed deferred strategies with initial stock exposure of 50% and 80%. The corresponding loss aversion parameters for an initial stock exposure of 80% were roughly 1.9 (single premium), respectively 1.8 (regular premiums) and for an initial stock exposure of 50% roughly 1.95 (single premium), respectively 1.85 (regular premiums).
funds. In contrast to CPT-investors, MCPT-investors also gain subjective utility and disutility from interim value changes. Therefore, life-cycle funds which reduce the potential losses throughout the investment horizon are generally preferred by MCPT-investors over balanced funds. Our findings hold for different degrees of risk appetite, different life-cycle glide paths and under various capital market assumptions. Hence, in contrast to EUT and CPT, MCPT is able to explain the observed popularity of life-cycle funds. Moreover, combined with the results from Ruß & Schelling (2018), this provides strong evidence that particularly for long investment horizons MCPT can better explain observed demand than CPT. The major assumption of MCPT is that an investor (consciously or subconsciously) already considers future utility or disutility stemming from interim value changes when making the investment decision. If one is willing to accept this assumption, then MCPT provides a convincing explanation for the popularity of life-cycle funds and various other (suboptimal) investment/old age provision products. Hence, product providers as well as public pension schemes may take MCPT into account to analyze if their currently offered or newly developed products will be attractive to customers.

Understanding the reasons for the demand for suboptimal products is an important prerequisite to changing consumer behavior. Potential ways of helping consumers make better decisions are beyond the scope of this paper. Future research should therefore analyze if and how investment products should be designed such that they are still desirable from an MCPT-investor’s point of view on the one hand, but at the same time provide the investor with a better (that is, closer to a rationally optimal) investment decision under EUT. Since more and more countries require that investors receive regular information about the value of their product, our results strongly indicate that it might be preferable to communicate interim values as a performance since the start of the product rather than a performance during the last period since the latter will typically more frequently change between gains and losses.

However, future experimental and empirical studies are necessary to improve understanding of the impact of potential future value fluctuations on the subjective utility when making a long-term investment decision.
References


2 As You like It: Explaining the Popularity of Life-Cycle Funds with MCPT Research Papers


3 Return Smoothing and Risk Sharing Elements in Life Insurance from a Client Perspective

Source:
Return Smoothing and Risk Sharing Elements in Life Insurance from a Client Perspective

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Abstract

In many countries, traditional participating life insurance (TPLI) contracts are typically equipped with a cliquet-style (year-to-year) guarantee. Life insurers pool the assets and liabilities of a heterogeneous portfolio of TPLI contracts. This allows for intergenerational risk sharing. Together with certain smoothing elements in the collective investment, it also results in rather stable returns for the policyholders. Despite the current low interest rate environment, TPLI contracts are still popular in the segment of retirement savings. Standard approaches which focus solely on the cash-flow at maturity cannot explain their popularity. In a recent paper, Ruß & Schelling (2018) have introduced a descriptive model of decision making which takes into account that potential future changes in the account value impact the decision of long-term investors at outset. Based on this, we illustrate how smoothing and risk sharing elements provided by a life insurer can significantly increase the subjective utility for such investors. Furthermore, we show that for these investors TPLI contracts are more attractive than common unit-linked (guaranteed) products. Hence, our findings explain the popularity of TPLI contracts and provide helpful insights into decision making in the context of retirement savings.

Keywords: Cumulative Prospect Theory, Myopic Loss Aversion, Mental Accounting, Smoothing, Risk Sharing, Retirement Savings, Traditional Participating Life Insurance

JEL: D14, D81, G11, G22, G41, J26, J32

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1 Introduction

Traditional participating life insurance (TPLI) contracts (also referred to as with-profit life insurance contracts) have been the core business of life insurers for many years. In contrast to individual retirement savings products, life insurers pool the assets and liabilities of a heterogeneous portfolio of TPLI contracts which allows for intergenerational risk sharing. In many countries, TPLI contracts are typically equipped with an cliquet-style (year-to-year) guarantee where a guaranteed return must be credited to the policyholder’s individual account at the end of each year. Additionally, TPLI contracts receive a surplus participation which is based on the return of a collective investment which is subject to various smoothing elements. In particular, rather stable returns are achieved by building up collective reserves on both sides of the balance sheet in good years and dissolving these reserves to compensate for years with poor (or even negative) returns.¹ Goecke (2013) shows that such smoothing and risk sharing elements can (in absence of a guarantee) heavily reduce the short-term risk without significantly affecting the long-term risk-return-profile. In that sense, life insurers operate like a buffer between the capital market and the policyholders.

However, the current low interest rate environment has forced life insures to reduce guaranteed rates for new contracts. While smoothing elements can reduce the volatility of returns, they cannot compensate for a long-term decline in the capital market returns. Hence, also realized returns for TPLI contracts have decreased over the past years. Furthermore, due to insurance portfolios with long-term contracts and rather high guaranteed rates (especially in old contracts) in combination with rather restrictive solvency requirements, the insurer’s asset allocation allows only for low risk taking. For long-term investors this very likely results in a suboptimal distribution of the terminal value. In addition, smoothing and intergenerational risk sharing mechanisms are opaque by nature. For all these reasons, TPLI contracts have been heavily criticized by consumer protection organizations.² Consequently, life insurers are cur-

¹There is a broad literature on different aspects of TPLI contracts. Most of the literature focuses on the valuation and product design in the context of capital requirement issues. For an overview we refer to Goecke (2013) or Reuß et al. (2015).
²In this regard, a frequently cited criticism is that new contracts subsidize old contracts (with much higher guaranteed rates) and hence suffer from an ex-ante “collective malus”. On the other hand, the new contracts
rently reinventing their business. In particular, they tend to develop capital efficient versions of TPLI contracts with different types of guarantees or offer more (individualized) unit-linked contracts. Despite these tendencies, versions of TPLI contracts are still very popular in the segment of retirement savings (this is also true for slightly modified products which also make use of the same collective smoothing and risk sharing elements). Moreover, a complete shift to individualized contracts leads to a loss of intergenerational risk sharing and questions the role of the life insurer in this context.

The aim of this paper is to explain the popularity of TPLI contracts and to shed light on how smoothing and risk sharing elements are perceived by long-term investors. Studies show that Expected Utility Theory (EUT) and even Cumulative Prospect Theory (CPT) investors who focus solely on the terminal value would not buy products with cliquet-style guarantees, cf., e.g., Ebert et al. (2012). Gollier (2008) shows that an intergenerational risk transfer can be social welfare increasing and Goecke (2013) demonstrates advantages of collective over individual investments. In particular, Goecke (2013) suggests that investors reevaluate their investment regularly and that a volatile performance causes stress. Also, several other authors pointed out that investors show such a tendency, cf., e.g., Benartzi & Thaler (1995) and Gneezy & Potters (1997) as well as Koranda & Post (2014) with the focus on an index linked product. In a recent paper, Ruß & Schelling (2018) have argued that in particular long-term investors also get subjective utility and disutility from interim gains and losses in the account value. They argue that this already impacts the investment decision at outset and propose a modification of CPT that takes this into account. The so-called Multi Cumulative Prospect Theory (MCPT) is able to explain the demand for cliquet-style guarantees in a simple model framework (Black-Scholes benefit from assets (particularly bonds with rather high coupons) that have been bought in the past, resulting in an ex ante “collective bonus”. It is not intuitively clear which effect is larger. Recent research tries to shed light on these effects. Hieber et al. (2016) introduce conditions for a fair valuation of insurance contracts in the case of a heterogeneous insurance portfolio that ensure that new contracts are not exposed to an ex ante “collective malus” (and vice versa do not receive a ex ante “collective bonus”). Further, in a similar framework Eckert et al. (2018) propose a measure to quantify the “collective malus/bonus” of certain contracts.

3Capital efficiency can be interpreted as profitability in relation to capital requirement, cf. Reuß et al. (2015) for more details.

4E.g., certain index linked products, cf. Alexandrova et al. (2017) for more details.

5CPT introduced by Tversky & Kahneman (1992) is one of the most popular behavioral counterparts to EUT. Most importantly, it takes into account that actual decision making is often based on heuristics which can lead to systematic biases. Cf. Section 2 for more details.
market with constant risk-free rate, single premium, etc.). Further, Graf et al. (2018) show that MCPT is also able to explain the demand for life-cycle funds which decrease the risk exposure when approaching retirement. Both results suggest that MCPT is more accurate in predicting decision making of long-term investors than standard approaches like EUT and CPT. Based on these insights it seems natural that return smoothing and risk sharing elements provided by life insures are essential aspects for long-term investors when making the investment decision.

As we are particularly interested in analyzing the impact of smoothing and risk sharing elements we model these elements in detail by means of a stylized life insurance company based on the situation in Germany. We also consider a rather realistic model framework with respect to other aspects like stochastic interest rates, different types of charges, regular premium payments, etc. We will confirm that CPT in its standard form is not able to explain the popularity of TPLI contracts. Subsequently, we will show that MCPT-investors strongly prefer smoothed returns as well as TPLI contracts (compared to common unit-linked products). We will show that this is also true in the case of a (moderate) ex-ante collective malus and even if the subjective utility is only partly influenced by potential annual changes. Hence our findings offer a convincing explanation for observed decisions in retirement savings. Understanding the decision making is an important requirement to improve product design and ultimately help long-term investors to make the right choice to ensure a desired standard of living in old age.

The remainder of the paper is organized as follows: In Section 2, we briefly introduce the concept of MCPT. Section 3 describes the TPLI based on a stylized life insurance company. In particular, we model assets and liabilities and describe in detail the implemented smoothing and risk sharing elements. In Section 4 we specify the model parametrization and present the results of our analyses focusing on the impact of smoothing and risk sharing elements from a long-term investor’s perspective. Subsequently, in Section 5 we compare TPLI contracts with various unit-linked products to analyze the popularity of TPLI contracts compared to other common investment choices in retirement savings. Finally, Section 6 concludes and provides an outlook for future research.
2 Modeling Decision Making of Long-term Investors

Cumulative Prospect Theory (CPT) introduced by Tversky & Kahneman (1992) has been developed as a descriptive theory to model and predict how humans actually make decisions. It considers gains and losses with respect to a reference point and is based on an S-shaped value function \( v \) which assumes that investors are typically loss averse and a probability distortion function \( w \) which takes into account that investors tend to overweight tail events with small probabilities and underweight events with high probabilities. Although CPT explains actual human behavior that cannot be explained by Expected Utility Theory (EUT), even CPT frequently fails to explain typical behavior of long-term investors. In particular, there are many long-term investment products that are very popular which neither an EUT-investor nor a CPT-investor would buy (cf. Ebert et al. (2012), Ruß & Schelling (2018) or Graf et al. (2018)).

One reason is that CPT (like EUT) is typically applied such that investment products only generate subjective utility in connection with actual cash flows - thus, in case of long-term investments only at maturity. However, even for long-term investments, investors regularly evaluate their investment. If a reported value is lower than a previous value, this will be perceived as a loss which typically looms larger than a gain of similar amount, cf., e.g., Barberis et al. (2001). This motivates that for long-term investors, the initial subjective utility of an investment is not only dependent on the distribution of the terminal wealth (relative to some reference point), but also on the possible future interim value changes. To capture this effect, Ruß & Schelling (2018) have introduced a modification of CPT, the so-called Multi Cumulative Prospect Theory (MCPT) which essentially uses CPT with multiple reference points and evaluation periods to measure the subjective utility of the potential interim value changes. Since the difference between CPT and MCPT typically becomes larger for an increasing investment horizon, MCPT is particularly useful to explain and predict actual behavior for long-term in-

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6 This Section is closely following Ruß & Schelling (2018) and Graf et al. (2018).
7 Benartzi & Thaler (1995) propose the theory of myopic loss aversion, a combination of loss aversion and frequent investment evaluation, and provide an explanation for the equity premium puzzle and the preference of long-term investors for low-risk investments. Moreover, mental accounting, introduced by Thaler (1985), implies that investors tend to take into account potential future fluctuations of the contract’s value already when making an investment decision.
MCPT considers an investor and an investment $\Xi$ with time horizon $[0, T]$, $T \in \mathbb{N}$, at time $t = 0$. Throughout this paper we assume that premiums are paid annually in advance at time $t+$ for $t \in \{0, \ldots, T-1\}$. Moreover, we assume that future interim evaluations take place annually. We consider for all $t \in \{1, \ldots, T\}$ the annual gain or loss $X_t := A_t - \chi_t$, where $A_t$ is the account value of the investment $\Xi$ at time $t$ (before premium payment) and $\chi_t$ is the reference point for time $t$. The natural reference point choice for each period is given by $\chi_t = A_{t-1} + P_t$, that is, the (reported) account value of the contract at time $t-1$ plus the premium $P$ paid at time $(t-1)+$.\(^8\)

We can evaluate the CPT value of each annual value change $X_t$ by

$$CPT(X_t) = \int_{-\infty}^0 v(x) d(w(F_t(x))) + \int_{0}^{\infty} v(x) d(-w(1 - F_t(x))),$$

where $F_t(x) = \mathbb{P}(X_t \leq x)$ and $v$ is the investor’s value-function which is defined as $v(x) := x^a \mathbb{1}\{x \geq 0\} - \lambda |x|^a \mathbb{1}\{x < 0\}$ where $\lambda > 0$ is the loss aversion parameter and $a \in \mathbb{R}_+$ controls the risk appetite. The probability distortion function is given by $w(p) := \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{\frac{1}{\gamma}}}$ with $\gamma \in (0.28, 1]$, where the lower boundary for $\gamma$ is chosen such that $w$ is strictly monotonically increasing for $p \in [0, 1]$. The MCPT utility at time $t = 0$ is then defined by

$$MCPT(\Xi) := \sum_{t=1}^{T} \eta^t CPT(X_t)$$

with a discounting parameter $\eta \in \mathbb{R}_+$.\(^9\)

The MCPT utility reflects the subjective utility of the potential annual changes. Ruß & Schelling (2018) also suggest a combined model given by the weighted sum of the CPT and the MCPT utility. In doing so, the combined model captures also the subjective utility of the terminal value relative to a reference point $\chi$, that is, $X = A_T - \chi$. The natural CPT reference

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8 Note that the premium $P$ typically differs from the savings premium which is reduced by premium proportional charges, cf. Section 3.
point $\chi$ is given by the sum of all premiums ($\chi = T \cdot P$). The combined model is given by

$$CPT^{\text{com}}(\Xi) := s \cdot MCPT(\Xi) + (1 - s) \cdot CPT(X)$$

(2)

where $s \in [0, 1]$ controls the influence of the annual value changes on the subjective utility.

### 3 Traditional Participating Life Insurance

In this Section we model a traditional participating life insurance (TPLI) contract within a stylized insurance company. As insurance portfolios are heterogeneous, for example with respect to the guaranteed rates, TPLI contracts are influenced by intergenerational effects between different cohorts, cf. Hieber et al. (2016). In particular, policyholders of all cohorts participate in the returns of the same assets and these returns are subject to smoothing elements on both sides of the insurer’s balance sheet (see below). Since a key question to be answered in this paper is the attractiveness these elements for a long-term investor, we model these aspects very detailed based on the situation in Germany.

We consider a TPLI contract with a duration of $T$ years and a policyholder with initial age of $x$ years. We assume an annual premium $P$ paid in advance at time $t+$ for $t \in \{0, \ldots, T - 1\}$. The contract provides a cliquet-style annual guaranteed rate $i^0_g$ on the account value. Additionally, the TPLI contract receives a surplus participation which is subject to regulation, but allows for some discretion by the insurance company. In particular, we assume that at the end of each year the insurance company specifies a total interest rate which is credited in the subsequent year to the account value of the policyholder (details below).\footnote{Note that this common practice for German life insurers.} The premium can be derived by the actuarial principle of equivalence based on the guaranteed benefit $G$ and annual charges $c_t^p$ as a percentage of the premium, that is, $P = \sum_{t=0}^{T-1} \frac{G}{(1-c_t^p)(1+i_0^g)^{T-t}}$. In case of death the current account value is paid out.\footnote{Hence, the death benefit does not impact the premium.} Throughout the paper, we call $P \cdot (1 - c_t^p)$ the savings premium. Further, we assume that first- and second-order mortality rates and charges coincide and no lapses,
tax payments etc. are considered, such that the investment surplus is the only source of surplus.

The TPLI contract is based on a life insurance company described by a balance sheet and specific management rules.\textsuperscript{11} The insurance portfolio at the initial date \( t = 0 \), has been built up over the previous \( T \) years and consists of \( T - 1 \) cohorts of contracts with remaining time to maturity 1 to \( T - 1 \) years. At the beginning of each year \( t \) a new cohort of \( l^{(t)}(x) \) policyholders joins. The number of policyholders of this cohort remaining in the portfolio at time \( t+k \) is given by \( l^{(t)}(x+k) = l^{(t)}(x+k-1) \cdot (1 - q_{x+k-1}) \) for \( k \in \{1, \ldots, T\} \) with \( q_x \) denoting the mortality rate of an \( x \)-year old person. While premium, duration, initial age of the policyholder and charges (as percentage of the premium) are assumed to be equal for all cohorts, the guaranteed rates and hence the guaranteed benefits are modeled cohort specific.\textsuperscript{12} We denote the guaranteed rate of a cohort with initial date \( t \) as \( i_g^t \). For the new cohorts joining the company after \( t = 0 \) the guaranteed rate is calculated as 60\% of the average return of zero bonds with maturity of 10 years over the last 5 years, where the result is rounded down to a tenth of a percentage point and zero representing the minimum.\textsuperscript{13}

The balance sheet of the insurance company at the beginning of year \( t \) with balance sheet total \( BS_t \) is displayed in table 1. The assets are given by the book values of a bond portfolio \( BV^B_t \) and of a stock portfolio \( BV^S_t \). The bond portfolio consists of coupon bonds (yielding at par) with initial maturity \( T_B = 10 \). We follow German local GAAP (HGB) accounting rules\textsuperscript{14} and assume that bonds are recognized at acquisition costs and stocks at strict lower-of-cost-

\begin{table}[h]
\centering
\begin{tabular}{c|c|c|c|c}
\hline
\textbf{Assets} & \textbf{Liabilities} \\
\hline
\( BV^B_t \) & \( BV^S_t \) & IR_t & AR_t & \( RfB_t = RfB^D_t + RfB^S_t \) \\
\hline
\( BS_t \) & \( BS_t \) & \\
\hline
\end{tabular}
\caption{Structure of the balance sheet at time \( t \).}
\end{table}

\textsuperscript{11}Similar models have been used by Reuß et al. (2016), Reuß et al. (2015), Burkhart et al. (2015) and Seyboth (2011).

\textsuperscript{12}The guaranteed interest rate for the initial cohorts and for the cohort joining at time \( t = 0 \) are assumed to be given (see Section 4.1 for details).

\textsuperscript{13}This is in line with EU-regulation on maximum allowed guaranteed interest rates, cf. EU (2002).

\textsuperscript{14}Cf. Reuß et al. (2016) for details.
or-market principle. Differences in market and book values may result in unrealized gains and losses (UGL). According to local GAAP, unrealized losses on stocks have to be realized at the end of the year, that is, the ratio $d_{neg} = 100\%$ of the unrealized losses on stocks is realized annually. Furthermore, we assume that in case of unrealized gains on stocks the ratio $d_{pos}$ is realized annually in order to stabilize the investment return.

The insurance company follows a strategic asset allocation by annually rebalancing the assets based on a stock ratio $q_t \in [q_{\min}, q_{\max}]$ (in terms of market values) at the end of the year. If necessary, bonds are sold proportionally to their market values. Further, the insurer increases the stock ratio if the weighted average of the coupon rates (at the end of the year and before rebalancing) $\overline{cp}_t$ is rather low compared to the weighted average guaranteed rate of all contracts in the portfolio $\overline{ig}$. More precisely, we define the stock ratio by $q_t = \min \left\{ \max \left\{ q_{\min} \cdot \left( 1 + \left( \frac{1 + \overline{cp}_t}{1 + \pi\cdot\overline{cp}_t} - 1 \right) \cdot 100 \right), q_{\min} \right\}, q_{\max} \right\}$ with adjustment factor $\pi\cdot\overline{cp} \geq 0$.

The rebalancing of the assets takes place at the end of each year and takes into account the cash flow $CF_{t+}$ at the beginning of the year $^{16}$ which is invested in a riskless bank account earning the interest rate $r(1)$, and the cash flow at the end of the year. $^{17}$ The total (book value) investment return rate is then given by

$$i^*_t = \frac{CF_{t+} \cdot r(1) + CP_{t+1} + UGL_{t+1}}{BV^S_t + BV^B_t + CF_t}$$

with $UGL_{t+1}$ denoting the realized portion of the UGL.

The liabilities consist of the insurer’s profit (loss) $IR_t$ at the end of year $t - 1$, the sum of the actuarial reserves of all contracts $^{18}$ $AR_t$, and the reserves for premium refunds $RfB_t$, sometimes also referred to as uncommitted provision for premium refunds which are instrumental in

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$^{15}$Cf. Burkhart et al. (2015) or Seyboth (2011) for further details.

$^{16}$Given by the premium payments less expenses and the insurer’s profits. Cf. appendix B for details on the insurer’s future profits in the stochastic simulation.

$^{17}$Given by coupon payments $CP_{t+1}$ plus nominal repayments of bonds at maturity minus benefit payments to the policyholders, cf. Burkhart et al. (2015) for details.

$^{18}$The actuarial reserve $kAR_t$ of one contract at the end of the $k$-th year of its duration at time $t$ can be calculated recursively by $kAR_t = (k-1AR_{t-1} + P \cdot (1 - \overline{c}_t) \cdot (1 + i^*_t \cdot k))$ with $0AR_t = 0$. 

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smoothing investment returns within TPLI’s, cf. Alexandrova et al. (2017). The RfB is modeled by two parts: credited non revisable bonus reserve \( RfB^D_t \) and a terminal bonus fund \( RfB^T_t \) which can be used by the company for smoothing returns and as a buffer to cover losses. It follows that the account value of one contract with remaining duration \( T - k \) at time \( t+1 \) is given by \( kA_{t+1} = kAR_t + kRfB^D_t + P \cdot (1 - \delta^g) \). In subsequent years, the guaranteed rate applies to the account value and hence also to the credited non revisable bonus reserve \( kRfB^D_t \).

Next, we describe the mechanisms of the surplus distribution. Based on the investment return rate \( i^*_{t+1} \), we can determine the total investment return of the insurance company by 
\[
R^*_t = (RfB^S_t + A^S_{t+1}) \cdot i^*_{t+1}
\]
where \( A^S_{t+1} \) denotes the sum of all account values in the portfolio. The total investment surplus at the end of the year is given by 
\[
Sp_{t+1} = R^*_t - R^g_t
\]
where 
\[
R^g_t = \sum_{k=0}^{T-1} j^{(t-k)} \cdot kA_{t+k} \cdot i^g_{t-k}
\]
denotes the sum of the guaranteed interest credited to the policyholders. The part of the investment surplus that is distributed to the policyholders is given by
\[
PS_{t+1} = \max \left\{ 0; \alpha^{Sp} R^*_t - R^g_t \right\}
\]
 Ideally, this part of the investment surplus is taken to finance the part of the (cohort specific) total interest rates \( R^*_t \) that exceeds the guaranteed rate, that is, 
\[
\Delta R^g_t := \sum_{k=0}^{T-1} j^{(t-k)} \cdot kA_{t+k} \cdot (i^*_{t+k} - i^g_{t-k})
\]
However, it is not always the case that the investment surplus is sufficient to cover all total interest payments. In this case the insurer is allowed to dissolve reserves in the terminal bonus fund (and possibly also other unrealized gains) and, if necessary, the insurer covers the residual.

The remaining part of the investment surplus represents the insurer’s profit or loss \( IR_{t+1} \).

Finally, we describe how the insurance company decides on the total interest rate which defines the annual return for the policyholder on the account value. We assume that the total interest rate is based on an adjusted investment return \( i_t \) which is subject to various smoothing

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\(^{19}\) \( kRfB^D_t \) denotes the part that has been credited to one contract with time to maturity \( T - k \) at time \( t \).

\(^{20}\) According to the German MindZV (Mindestzuführungsverordnung) \( \alpha^{Sp} \geq 0.9 \).

\(^{21}\) Note that the part exceeding the guaranteed rate is credited to the non revisable bonus reserve \( kRfB^D_t \).

\(^{22}\) In detail, if \( Sp_{t+1} \geq 0 \) and \( \Delta R^g_{t+1} \leq PS_{t+1} \), then the investment surplus suffices to cover all total interest payments and the remaining part of \( PS_{t+1} \) is credited to the terminal bonus fund. If \( Sp_{t+1} \geq 0 \) and \( \Delta R^g_{t+1} > PS_{t+1} \) or if even \( Sp_{t+1} < 0 \) then the investment surplus is not sufficient to cover all interest payments to the policyholders. In this case, the residual is covered by the terminal bonus fund. If the terminal bonus fund is not sufficient to cover the residual, first remaining unrealized gains are realized before the insurer is liable.

\(^{23}\) Appendix B provides details under the considered settings.
elements. Firstly, it is based on the average (book value) investment returns\(^{24}\) of the last 3 years\(^{25}\), that is, \(\bar{i}_t = \frac{\sum_{j=0}^{2} i^*_t}{3}\). Secondly, we assume that the insurer reduces (increases) \(i_t\) in case of rather low (high) reserves. Additionally, for expiring contracts \(i_t\) is increased by a terminal bonus rate \(i^\text{term}_t\) depending on the current reserves. Based on this, the total interest rate of each cohort \(k_t\) is defined by the maximum of the corresponding guaranteed rate \(i^g_{t-k}\) and the adjusted investment return \(i_t\).

More precisely, the adjusted investment return is defined by

\[
i_t = \pi^* \bar{i}_t + \pi^\rho (\rho_t - \rho_{\text{target}}) + \pi^\hat{\rho} (\hat{\rho}_t - \hat{\rho}_{\text{target}}) \quad \text{with} \quad \rho_t = \frac{R_f B^S_t + U G_t}{B S_t} \quad \text{and} \quad \hat{\rho}_t = \frac{R_f B^S_t}{B S_t},
\]

where \(\rho_t\) defines the current reserve ratio\(^{26}\) and \(\hat{\rho}_t\) the current terminal bonus reserve ratio. Further, \(\rho_{\text{target}}\) and \(\hat{\rho}_{\text{target}}\) denote the target reserve ratios and \(\rho_{\min}\) and \(\hat{\rho}_{\min}\) the corresponding minimal values. Additionally, \(\pi^* \geq 0, \pi^\rho \geq 0\) and \(\pi^\hat{\rho} \geq 0\) denote adjustment factors to control the impacts of the different aspects. The total interest rate at time \(t\) for the cohort with initial date \(t - k\) applied in the period \([t, t+1]\) is then defined as

\[
k_t = i^g_{t-k} + \max \{i_t - i^g_{t-k}, 0\} \cdot 1_{\{\hat{\rho}_t \geq \hat{\rho}_{\min} \land \rho_t \geq \rho_{\min}\}}
\]

and at maturity (or in case of death) as \(k_t = \max \{i_t + i^\text{term}_t, i^g_{t-k}\}\). We define \(i^\text{term}_t = \tau_t \cdot \frac{R_f B^S_t}{A_t^+}\) with adjustment factor \(\tau_t \in [\tau_{\min}, \tau_{\max}]\) which controls that the terminal bonus rate is higher (lower) in case of higher (lower) terminal reserves\(^{27}\).

### 4 Analyzing Smoothing and Risk Sharing Elements

In this Section we will analyze the effect of smoothing and risk sharing elements from a long term investor’s perspective. First, in Section 4.1 we specify the parameter setting and the

\(^{24}\)Note that the (book value) investment return depend on realized gains and losses.

\(^{25}\)This is in accordance to the key figure C10 published by GDV (2016).

\(^{26}\)This definition is in line with the key figure D10 catalog for German life insurers given by GDV (2016).

\(^{27}\)\(\tau_t = \tau_{\min} + (\tau_{\max} - \tau_{\min}) \cdot \frac{\hat{\rho}_t - \hat{\rho}_{\min}}{\hat{\rho}_{\text{target}} - \hat{\rho}_{\min}}\) for \(\hat{\rho}_{\min} \leq \hat{\rho}_t \leq \hat{\rho}_{\text{target}}\) and \(\tau_t = \tau_{\min} \cdot 1_{\{\hat{\rho}_t < \hat{\rho}_{\min}\}} + \tau_{\max} \cdot 1_{\{\hat{\rho}_{\text{target}} < \hat{\rho}_t\}}\), else.
considered TPLI contract types. Then, Section 4.2 presents the results.

### 4.1 Specification and Starting Conditions

The assets are based on a financial market model which is given by a stock process $S$ following a geometric Brownian motion and a short rate process $r$ described by a Vasicek model, cf. Vasicek (1977). The parameters have been chosen in accordance with the European money market and recent literature. A detailed description is given in appendix A.

We analyze the performance of one contract starting in $t = 0$ with guaranteed rate $i_0^g = 1.25\%$ (chosen to be in line with the maximum rate allowed by the German regulation in 2016, cf. DAV (2017)) and annual premium $P = 1\,\text{€}$. We denote the account value at time $t$ of this contract as $A_t$. Further, for the insurance portfolio we assume that all policyholders are 40 years old at inception of their contract. All contracts have an initial duration of $T = 20$ years. Annual charges $c_t^p$ consist of annual administration charges $\beta = 5\%$ (as percentage of the premium) and initial acquisition charges $\alpha = 2.5\%$ (as percentage of the premium sum), which are equally deducted over the first five years.\(^{28}\) Hence, $c_t^p = \beta + \frac{\alpha T}{5} 1_{t \in \{0,\ldots,4\}}$. Mortality is based on the German standard mortality table (DAV 2008 T) and we do not consider surrender.\(^{29}\) Moreover, at the beginning of each year $t$, a new cohort of $l_{x}^{(t)} = 1000$ policyholders joins the insurance portfolio. The initial portfolio\(^{30}\) at time $t = 0$ is derived by a projection based on a deterministic (past) scenario with the first cohort joining in 1988 ($t = -28$). The guaranteed rates for the initial cohorts are assumed to coincide with the maximum rate allowed by the German regulation between 1988 and 2015, cf. DAV (2017). All values are given in appendix B in table 7.

At time $t = 0$, the book value of the assets coincides with the book value of the liabilities. As a management rule, we assume that the stock ratio\(^{31}\) is between $7.5\%$ and $15\%$ in the deter-

\(^{28}\)The value has been chosen according to the German Life Insurance Reform Act (LVRG) from 2015.

\(^{29}\)Note that we consider mortality only for the purpose of risk sharing and smoothing effects in the insurance portfolio. We assume that the investor focuses solely on the case of survival until maturity.

\(^{30}\)That is, the initial cohort sizes, the corresponding actuarial reserves and the reserves for premium refunds.

\(^{31}\)The average ratio of German life insurance companies invested in stocks and comparable assets in 2015 was $10.4\%$, cf. GDV (2016).
ministic (past) scenario and between 10% and 17.5% in the stochastic (future) projection.\textsuperscript{32}

The coupon bond portfolio is split in bonds with time to maturities between 1 and $T_B = 10$ years, whereby the proportions result from the deterministic scenario. For the deterministic scenario we use coupon and spot rates based on the historical annual average yields of German government coupon bonds with maturity between 1 and 10 years.\textsuperscript{33} The annual stock returns are based on the historical returns of the German stock index DAX between 1988 and 2015 provided by the Deutsche Bundesbank (2016), cf. appendix B table 7. Further, we set $\rho_{\text{target}} = 12\%$, $\tilde{\rho}_{\text{target}} = 6\%$, $\rho_{\text{min}} = 4\%$ and $\tilde{\rho}_{\text{min}} = 2\%$.\textsuperscript{34} The adjustment factors are set to $\pi^* = 0.9$, $\pi^o = 0.1$, $\pi^\varphi = 0.1$ and $\tau_{\text{min}} = 0$, $\tau_{\text{max}} = 0.3$, $\pi^\varphi = 0.75$. This ensures that the total interest rate is primarily affected by the average investment return rate of the last three years. Nevertheless, the higher the gap between the target (terminal) reserve ratio and the current (terminal) reserve ratio, the larger the adjustment of the total interest rate. The parameter for the management rules are summarized in table 2.

The deterministic scenario results in the initial balance sheet ($t = 0$) displayed in table 3. Further initial key values resulting from the deterministic scenario are summarized in table 4.

\begin{table}[h]
\centering
\begin{tabular}{ccccccccccc}
\hline
$q_{\min}$ (%) & $q_{\max}$ (%) & $T_B$ (years) & $d_{\text{pos}}$ (%) & $d_{\text{neg}}$ (%) & $\alpha_{S^p}$ (%) & $\pi^*$ & $\tau_{\text{min}}$ & $\tau_{\text{max}}$ \\
\hline
7.5 (10) & 15 (17.5) & 10 & 20 & 100 & 90 & 0.9 & 0 & 0.3 \\
\hline
$\tilde{\rho}_{\text{target}}$ (%) & $\rho_{\text{min}}$ (%) & $\rho_{\text{target}}$ (%) & $\rho_{\text{min}}$ (%) & $\pi^o$ & $\pi^\varphi$ & $\pi^\varphi$ & \\
6 & 2 & 12 & 4 & 0.1 & 0.1 & 0.75 \\
\hline
\end{tabular}
\caption{Parameter setting for the management rules in the base case.}
\end{table}

\textsuperscript{32}The higher corridor for the stock ratio in the future projection is motivated by the sustained trend of insurers to reset their risk limits and to increase their appetite for higher risk investments (including a shift from public to private assets). E.g., an annual international survey conducted by BlackRock in 2018 finds that almost half (47\%) of insurers surveyed plan to increase portfolio risk exposure over the next 1-2 years, while only 4\% plan to reduce risk exposure, cf. BlackRock (2018). Moreover, in the last four surveys (since 2015) at most 12\% of the surveyed insurers planned to reduce risk exposure, while in 2015 and 2016, 57\% and 47\% planned to increase risk exposure, respectively. However, we also provide a sensitivity analysis with respect to the stock ratio in Section 5.

\textsuperscript{33}Data from Deutsche Bundesbank (2016). Note that for the sake of a smooth shift from historical to model based yield curves, we calculate the yield curves for $t \in \{-3, -2, -1\}$ based on zero bond prices in the stochastic financial market and the average three-month EURIBOR rates of the last six months of the respective year.

\textsuperscript{34}The target reserve ratio is set approximately to the average of the corresponding ratio D10 of the key figure catalog for German life insurance companies given by GDV (2016) between 2007 and 2015 (data available since 2007) reduced by roughly 3\% because we do not consider any further equity in our model.
The initial (terminal) reserve ratio is given by \( \rho_0 = 9.5\% \) (\( \tilde{\rho}_0 = 3.65\% \)) and is therefore below the target. The stock ratio is \( q_0 = 11.58\% \) and the total interest rate for the first year is given by \( \max(3.2\%, i^0_{-k}) \), that is, all policyholders earn at least 3.2\% on their account value. The additional terminal bonus rate in the first year amounts to 0.54\%. Hence, the total interest rate for expiring contracts in the first year is given by 3.74\%.\(^{35}\)

The contract which is based on these assumptions is considered as the base case and is denoted as contract A. In this case the initial (terminal) reserve ratio equals 79\% (60\%) of the target. Further, the guaranteed rates of most contracts in the initial insurance portfolio are significantly higher than the guaranteed rate of contract A, cf. table 7 in appendix B. This causes on average a disadvantage for contract A, that is, contract A is expected to suffer more than profit from intergenerational effects.\(^{36}\) Eckert \textit{et al.} (2018) try to formalize this and define that a contract receives an “ex ante collective bonus”\(^{37}\) if on average it will earn more than an investment in a reference portfolio that replicates the market values of the assets of the insurance company, that is, \( CB = \mathbb{E}_{\mathbb{Q}} \left[ e^{-\int_0^T r_u du} \left( A_T - A_T^{ref} \right) \right] > 0 \) with \( A_T^{ref} = \sum_{t=0}^{T-1} P(1-c^p_t) \prod_{k=t}^{T-1} \text{Perf}^{MV_A}_t \) denoting the terminal value of an investment in a reference portfolio with annual return \( \text{Perf}^{MV_A}_t \) and \( \mathbb{Q} \) the risk-neutral measure. For the sake of better comparability, we consider the ex ante collective bonus in relation to the fair value of the alternative investment, that is, \( CB^\% = \frac{CB}{FV} - 1 \) with \( FV = \mathbb{E}_{\mathbb{Q}} \left[ e^{-\int_0^T r_u du} A_T^{ref} \right] \). If \( CB < 0 \) and hence \( CB^\% < 0 \), we say that the contract is exposed to an “ex ante collective malus”. This is the case for contract A where

\(^{35}\)These values are similar to the values of most life insurers in Germany in 2015, cf. ASSEKURATA (2015).

\(^{36}\)Cf. also Hieber \textit{et al.} (2016) for more details on these effects.

\(^{37}\)Note that this also includes payments to or from the insurer (insurer’s profit), cf. appendix B.
$CB\% = -6.12\%$. To separate the impact of smoothing and risk sharing from the impact of systematic intergenerational effects at some point in time, we consider the following three additional contract settings:

**B:** We assume the same initial setting as in case A, but with adjusted initial (terminal) reserve ratio of 100%.\(^{38}\) $CB\%$ is in this case $-5.08\%$.

**C:** We additionally assume that all contracts in the insurance portfolio have the same guaranteed rate of 1.25\%. We generate the initial portfolio based on this assumption and adjust the initial (terminal) reserve ratio to 100\%. This results in $CB\% = -2.31\%$.

**D:** We consider the same setting as in case C. Additionally, we increase the surplus participation rate to $\alpha^{Sp} = 97\%$ for all policyholders in order to obtain $CB\% \approx 0\%$.

It is worthwhile noting that in all considered cases the present value of the insurer’s future profit (PVFP) is positive. Details and further key figures are described in appendix B.

Furthermore, to analyze the asset smoothing elements which are based on a collective investment, we consider also two fictitious contracts:

**E:** Contract E invests the savings premium $P(1 - c^p_t)$ in the reference portfolio replicating the market value of the assets of the insurance company under the setting of case D.\(^{39}\) This contract represents the case of an investment without smoothing and risk sharing. Obviously, $CB\% = 0\%$ in this case.

**F:** Contract F is assumed to invest the savings premium in an investment that earns the average investment return $\bar{i}_t$ of the insurance company under the setting of case D. This contract represents the case with asset smoothing but without further risk sharing effects. The asset smoothing results in an ex ante collective malus $CB\% = -1.38\%$.

For the sake of comparability we assume that contracts E and F come with the same premium and annual charges $c^p_t$ as the other contracts.

\(^{38}\)In order to meet the balance equation assets are increased proportionally.

\(^{39}\)Note that the returns of the reference portfolio depend on the insurance portfolio structure. However, the differences between the considered cases are negligible for our analysis.
4.2 Results

Due to the complexity of the model, all results are based on Monte Carlo simulations with 20,000 trajectories. The numerical analysis is based on a stochastic simulation of the financial market under the real-world measure $\mathbb{P}$ (as well as under the risk-neutral measure $\mathbb{Q}$ for the purpose of fair valuation) which is done on a daily basis assuming 252 trading days per year.

4.2.1 Key Figures

First, we investigate the distribution of the terminal value and the annual changes in the account value since these distributions are main drivers of the further results.

Figure 1 displays the percentiles of the terminal value of the different TPLI contracts A–D, as well as of the fictitious contracts E and F. We find that the distributions of the terminal value are very similar. All distributions are slightly right skewed and have a median between approximately 24.8 € (A) and 26.2 € (D). The displayed percentiles are all in the range of 21.2 € and 32.3 € and hence always above the accumulated premiums. The terminal value of the TPLI contracts increases slightly for a lower ex ante collective malus. The percentiles of product E and F show that the asset smoothing elements implemented by the insurance company reduce the variability of the terminal value without significantly reducing its expected value.
Figure 2: Percentiles of the annual changes of TPLI contracts A and D as well as of the contracts E and F.

Figure 2 shows the percentiles of the annual changes in the account value of the considered TPLI contracts A and D, as well as of the fictitious contracts E and F. The changes in the account value are defined as $X_t = A_t - \overline{A}_{(t-1)+}$ for $t \in \{1, \ldots, T\}$ with $\overline{A}_{(t-1)+}$ denoting the account value at time $t - 1$ plus the premium $P$ paid at time $(t - 1)+$. The upper panels show that the patterns of the annual changes of the TPLI contracts do not significantly differ (thus, we refrain from displaying the annual changes for contract B and C). In the first five years they are slightly negative due to the acquisition charges which are deducted over the first five years. Subsequently, the annual changes are in almost all cases positive and (on average) increasing from year to year due to the higher account value. The annual change in the last year is (on average) significantly higher due to the additional terminal bonus. The lower left panel displays the annual changes of the unsmoothed fictitious contract E. The percentiles show that the dis-
Figure 3: \( r_{CE} \) in the CPT case depended on \( \lambda \) with (left panel) and without (right panel) probability distortion for the TPLI contracts A–D, as well as for the fictitious contracts E and F.

The distribution of the annual changes of contract E are much wider compared to the other contracts and include in particular a significant risk of annual losses. The annual changes of contract F illustrate that the implemented asset smoothing elements result in much tighter distributions of the annual changes (lower right panel). While the median values are very similar as for contract E, the asset smoothing elements heavily reduce the risk of annual losses (and also the potential for high annual gains). Moreover, the results show that asset smoothing elements based on a collective investment alone (without an embedded guarantee) already dramatically reduces the probability for annual losses.

In combination with the results displayed in Figure 1 this shows that the smoothing elements based on the collective investment of a life insurer can heavily reduce the variability of annual returns without significantly changing the risk-return characteristics of the terminal value.

4.2.2 CPT and MCPT Analysis

As described in Section 2 we consider CPT, MCPT and a combined model to analyze investor preferences. We use MCPT to analyze the influence of the annual changes in the account value on the subjective utility. If not stated otherwise, we fix \( a = 0.88 \) and \( \gamma = 0.65 \) as suggested by Tversky & Kahneman (1992) and perform analyses for different values of \( \lambda \). Moreover, we
focus on the case without discounting, that is, \( \eta = 1 \). As Ruß & Schelling (2018), we derive certainty equivalent contracts. We solve the following equation numerically for each contract to obtain the corresponding fixed annual return \( r^{CE} \) that an investor (CPT-investor for \( s = 0 \) and MCPT-investor for \( s = 1 \)) would regard equally desirable as the considered contract \( \Xi \).

\[
CPT^{com}(\Xi) = \begin{cases} 
    s \cdot \sum_{t=0}^{T-1} \left( \sum_{k=0}^{t} \left( P(1 - c^p_t)e^{r^{CE}(t-k)} - P \right) \right) + \\
    (1 - s) \cdot \left( \sum_{t=0}^{T-1} \left( P(1 - c^p_t)e^{r^{CE}(T-t)} - P \right) \right), & CPT^{com}(\Xi) \geq 0 \\
    -\lambda \cdot s \cdot \sum_{t=0}^{T-1} \left( \sum_{k=0}^{t} \left( P(1 - c^p_t)e^{r^{CE}(t-k)} - P \right) \right), & CPT^{com}(\Xi) < 0 \\
    -\lambda \cdot (1 - s) \cdot \left( \sum_{t=0}^{T-1} \left( P(1 - c^p_t)e^{r^{CE}(T-t)} - P \right) \right), & CPT^{com}(\Xi) < 0 
\end{cases}
\]

Figure 3 shows the certainty equivalent returns as a function of loss aversion (\( \lambda \)) for a CPT-investor who does not value annual changes (\( s = 0 \)) with (left panel) and without (right panel) probability distortion. We find that probability distortion only slightly reduces the certainty equivalents without changing the pattern of the result. For a CPT-investor the results for E and F show that asset smoothing elements slightly reduce the subjective utility. Hence, a pure CPT-investor would prefer the unsmoothed contract E since smoothing mainly reduces interim fluctuations which are not considered under CPT. While the TPLI contracts A, B, and C are less attractive than the fictitious contracts, contract D is the most appealing contract. Not surprisingly, the results illustrate that an ex ante collective malus makes the TPLI contracts A, B, and C less appealing. However, the results for contract D show that the embedded guarantee can also increase the subjective utility if the smoothing and risk sharing elements do not result in an ex ante collective malus for the contract.

Further, the results show that under CPT loss aversion plays no role for these types of products. The TPLI contracts come with an embedded guarantee which prevents losses and the fictitious products are based on a rather conservative investment (insurers asset stock ratio is between 10% and 20%) which makes losses in case of a long-term investment very unlikely. Hence,
applying CPT to describe actual human preferences in such cases assumes that the investor’s
degree of loss aversion does not impact the decision at all (at least in the common status quo
case where the reference point is given by the accumulated premiums). This (obvious) result
casts further doubts that CPT in its standard form is appropriate to describe actual decision
making in the context of long-term investments.

Figure 4 shows the results for an MCPT-investor who only values annual changes and does
not assign any weight to the terminal value \((s = 1)\). We find that the patterns for the TPLI
contracts A–D differ only slightly in the case with (left panel) and without (right panel) prob-
ability distortion. In contrast to the CPT case, the \(r^{CE}\) decreases in \(\lambda\), that is, loss aversion
with respect to annual changes reduces the attractiveness of the considered contracts. This is
mainly caused by the acquisition charges which generate losses in the first years. Again, we find
that an ex ante collective malus makes the TPLI contract less appealing. More interestingly,
the results for contracts E and F show the huge impact of the asset smoothing elements on
the attractiveness of the TPLI contracts. Without asset smoothing elements the \(r^{CE}\) declines
heavily with increasing loss aversion. We find that loss averse MCPT-investors \((\lambda > 1)\) prefer
in all cases contract F over contract E. The left panel shows that probability distortion, particu-
larly the overweighting of the small probabilities of rather high annual losses, makes contract E
even less appealing for loss averse investors. Conversely, probability distortion makes contract F more appealing due to the overweighting of rather high gains and the absence of high losses. Comparing the TPLI contracts A–D with contract F shows that the asset smoothing elements based on a collective investment are the main reason why TPLI contracts are attractive for loss averse MCPT-investors. This explains why TPLI contracts are even appealing in the case of low guaranteed rates.

Finally, we analyze investors who consider both, annual changes and the terminal value. Figure 5 shows the $r^{CE}$ in the CPT case depended on $s \in [0,1]$ without (left panel) and with typical loss aversion $\lambda = 2$ (right panel) for the TPLI contracts A–D, as well as of the fictitious contracts E and F.

Summarizing, this Section shows that for (loss averse) investors who gain subjective utility and disutility from potential annual changes, return smoothing elements based on a collective investment heavily increase the attractiveness of TPLI contracts. This is even true in case of a
significant ex ante collective malus and without guarantee. So far, however, we have analyzed the TPLI contracts in isolation from other common investments choices for retirement savings. Thus, to understand the popularity of TPLI contracts, we additionally need to analyze the preferences of long-term investors under the consideration of other common investment choices. This will be done in the next Section.

5 Explaining the Popularity of TPLI Contracts

In this Section we compare TPLI contracts with common unit-linked products. Note that we do not aim to find the “optimal” investment choice but rather analyze the typical decision problem between a small number of choices which long-term investors are often confronted with (e.g., when consulting a financial advisor for retirement savings).\textsuperscript{40} In Section 5.1 we define the unit-linked products. Then, Section 5.2 presents the results under different preference assumptions.

5.1 Unit-linked Product Specification

For all products, we assume an annual premium $P$ paid in advance and a contract duration of $T$ years. Again, $A_t$ denotes the account value at time $t$ and $c_t^p$ the percentage of the premium proportional charges reducing the invested premium which are assumed to be equal as for the TPLI contracts. For unit-linked products additional account proportional charges $\gamma^a$ are deducted on an annual basis from the account value. These consist of fund charges $\gamma^F$ and, if applicable, guarantee fees $\gamma^g$. For all unit-linked products we set $\gamma^F = 1\%$. Moreover, denote with $Perf_{t,t+1}$ the performance of the underlying investment from $t$ to $t+1$. At the beginning, the account value is $A_0 = P(1 - c_0^p)$. The account value at the end of the year is then derived in two steps: First, all account proportional charges, denoted as $\gamma^a$, are deducted from the projected value, that is, $A_{t-} = A(t-1) + Perf_t, t+1(1 - \gamma^a)$. Second, if applicable, an annual guarantee or terminal guarantee is taken into account to derive $A_t$ and $A_T$, respectively. While $t < T$ the account value at the beginning of the next year (after payment of the premium) is

\textsuperscript{40}This is in line with studies which show that investors tend to consider rather small samples of investment choices isolated from other choices or future opportunities, cf. Kahneman & Lovallo (1993).
given by $A_{t+} = A_t + P(1 - c_t^p)$.

**Unit-linked products without guarantee**

The case without guarantee is represented by a balanced fund investing a fixed part $\theta \in [0, 1]$ in a risky asset and $(1 - \theta)$ in a less risky asset. The risky asset is modeled by the stock investment $S$ and the less risky asset by a rolling bond investment $R$ (cf. appendix A for details). We assume daily rebalancing to achieve the desired equity portions.

**Unit-linked products with guarantee**

In addition to the simple product without guarantee, we consider also different products equipped with a guarantee which ensure that the policyholder receives at maturity at least the accumulated savings premiums, that is, $G_T := P\left(\sum_{t=0}^{T-1}(1 - c_t^p)\right)$.

Firstly, we consider common types of guarantees offered in the segment of variable annuities (VA). For the sake of simplicity we assume that the VA products implement a suitable hedging strategy to generate the guaranteed amount. To finance the hedging, an account proportional guarantee fee $\gamma_g$ is charged. The remaining part is invested in an underlying balanced fund with stock ratio $\theta \in [0, 1]$. The payoff at maturity of the VA product is given by $A_T = \max(A_{T-}, G_T)$. We only consider fair contracts, that is, we derive the fair guarantee fee numerically such that the fair value of the embedded option coincides with the present value of the future guarantee fees. Besides the pure money-back VA product we also consider a VA product with an additional annual protection in form of a cliquet-style (year-to-year) guarantee $G_t = d_{pl} \cdot A_{(t-1)+}$ with protection level $d_{pl}$. We consider products with a protection level $d_{pl}$ of 90% and 98%, that is, the account value cannot decrease by more than 10% or 2%, respectively, within one year. Similar as for the pure-money back guarantee we can derive fair

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41This is motivated by unit-linked products actually offered in the market, cf. also Graf et al. (2012).

42In the market there are various different variants of VA products. A complete consideration of all variants would exceed the scope of this paper. Hence, we restrict the analyses to VA products with some basic guarantee features. We refer to Bauer et al. (2008) for a detailed description and a framework for valuation of VA products and to www.annuityfyi.com for information on types of VA products currently offered in the US market.

43Cf. table 5 in appendix A for the fair guarantee fees depending on the underlying balanced fund.

44That is, $E^Q\left[e^{-\int_T^0 r_s ds}\max(G_T - A_T, 0)\right] - \sum_{t=1}^T E^Q\left[e^{-\int_T^t r_s ds} \gamma_g A_{(t-1)+} Perf_{t-1,t}\right] = 0.$
contract fees $\gamma_g$. We restrict the analysis to products with reasonable guarantee fees $\gamma_g \leq 1\%$.\cite{footnote45}

Secondly, we consider constant proportion portfolio insurance (CPPI) products which achieve a certain target amount by dynamically investing in riskless and risky assets, cf. Black & Perold (1992). Since continuous rebalancing is not possible in practice, we assume a daily reallocation of the underlying asset structure. In our case, we assume that a CPPI product invests at time $t$ a fraction $x_t$ in the risky stock $S$ and the remaining part $y_t = A_{t+} - x_t$ in zero bonds. Moreover, we assume that leveraging more than the current account value is not possible. Each day, the asset allocation for the client’s account is determined by $x_t = \max(0, \min(A_t, m(A_{t+} - F_t)))$ and $y_t = A_{t+} - x_t$, where $m$ denotes the multiplier and $(A_{t+} - F_t)$ the cushion with floor $F_t$.

Note that CPPI products without further protection are exposed to shortfall risks, that is, the probability that the account value falls below the target amount exceeds zero.\cite{footnote46} In reality, most providers (at least partially) hedge this risk. As we analyze the products from a clients perspective, we refrain from implementing hedging strategies and assume an additional account proportional charge $\gamma_g^{CPPI}$.

Again, we consider two types of guarantees: The first type (pure money-back guarantee) applies a dynamic strategy to pay at least $G_T$ at maturity and invests $y_t$ in zero bonds with maturity $T$ and price $p_t(T)$. The floor is given by $F_t = G_t \cdot \frac{p_t(T)}{(1-\gamma_g)}$ with $G_t = P(1-c_{t}^p) + \mathbb{1}_{\{1 \leq t \leq T\}} \cdot G_{t-1}$.

The second type has an embedded cliquet-style guarantee with an annual guarantee $G_t^{Cli}$ and therefore in each period $[t, t+1)$ for $t \in \{0, \ldots, T - 1\}$ invests the amount $y_t$ in zero bonds with maturity $t + 1$ and price $p_t([t] + 1)$. The floor is then given by $F_t = G_t^{Cli} \cdot \frac{p_t([t] + 1)}{(1-\gamma_g)}$ with $G_t^{Cli} = P(1-c_{t}^p) + \mathbb{1}_{\{1 \leq t \leq T\}} \cdot A_{[t]}$. We analyze products with multiplier $m = 3$ and additional fee $\gamma_g^{CPPI} = 0.1\%$ as well as $m = 4$ and $\gamma_g^{CPPI} = 0.2\%$.\cite{footnote108}
Figure 6: Percentiles of the terminal value of selected unit-linked products and TPLI product A and D.

5.2 Results

Again, we illustrate at first the distributions of the terminal value and the annual changes in the account value of the different product types before analyzing preferences of different investors.

5.2.1 Key Figures

Figure 6 displays the percentiles of the terminal value of selected unit-linked products and the TPLI contracts A and D. The displayed specifications (stock ratio and risk multiplier) have been chosen to illustrate exemplarily the distributions of the terminal value for different unit-linked products. The results show that compared to most unit-linked products the upside potential of the TPLI contracts is very limited (comparable with the upside potential of unit-linked products investing in a low-risk balanced fund with stock ratio $\approx 10\%$). However, products with a higher upside potential perform significantly worse in bad scenarios. Especially for the CPPI products it can be observed that the distributions are very right-skewed and there is a rather large probability that the terminal value is only the guarantee, cf. Graf et al. (2012).

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45 Consequentely, we only allow stock ratios $\theta \in [0, 0.6]$ for $d^{pl} = 90\%$ and $\theta \in [0, 0.1]$ for $d^{pl} = 98\%$. All fair guarantee fees are provided in table 6 in appendix B.

46 This includes overnight risk, that is, the risky asset loses more than $\frac{1}{m}$ during one period, as well as the risk of a changing floor due to interest rate changes.

47 Note that Figure 6 displays only a small sample of the analyzed product specifications to illustrate the most important differences between the distributions of the terminal value of the different product types.
Figure 7 shows the percentiles of the annual changes in the account value of selected unit-linked products. The upper left panel illustrates that balanced funds have a significant risk for rather high annual losses (the higher the stock ratio the higher the risk). The upper right panel shows that the VA money-back guarantee only slightly changes the distributions of the annual changes. In particular, the risk for rather high annual losses is virtually identical as for the underlying balanced fund. Further analyses show that also the VA Cliquet products have similar distributions but with significantly lighter tails. The CPPI money-back product shows very extreme annual changes with high up- and downside potential (lower left panel). The CPPI Cliquet product generates almost no annual losses. In contrast to the TPLI contracts, the distribution of the annual changes of the CPPI Cliquet product are more right-skewed (higher upside potential, but also significantly lower median which is very close to zero).
5.2.2 CPT and MCPT Analysis

Similar to Section 4.2 we analyze the preferences of different investors under the same parameter setting assumptions to compare common unit-linked products with the TPLI contracts A–D.

Figure 8 shows the certainty equivalent returns \( r^{CE} \) as a function of loss aversion (\( \lambda \)) for a CPT-investor with (left panel) and without (right panel) probability distortion. The results show that all unit-linked products dominate the TPLI contracts. Hence, investors who solely focus on the terminal value prefer unit-linked products in all cases. The most attractive type is either a balanced fund or a VA product with pure money-back guarantee. In particular, typical CPT-investors (with loss aversion \( \lambda \approx 2 \) and probability distortion \( \gamma = 0.65 \) as displayed in the left panel) prefer the pure money back VA product. This confirms existing results for CPT-investors, cf., e.g., Ebert et al. (2012), who show that CPT cannot explain the popularity of products with cliquet-style guarantees as in the case of TPLI contracts.

Figure 9 shows the results for an MCPT-investor. The upper left panel shows that for a typical MCPT investor (with loss aversion \( \lambda \approx 2 \) and probability distortion \( \gamma = 0.65 \) as displayed in the left panel), all considered TPLI contracts are preferred over all other products. The TPLI
contract is even preferred in the case of a significant collective malus (contract A). Comparing the two panels illustrates the different impact of the probability distortion on the $r^{CE}$ for the different product types. While for the TPLI contracts probability distortion has almost no impact (almost no annual losses and only moderate upside potential), for the balanced fund and the VA products we find that the $r^{CE}$ is heavily reduced in combination with loss aversion. This is due to overweighting of small probabilities of rather high annual losses. Interestingly, for the CPPI products we find that probability distortion significantly increases the $r^{CE}$ due to the very right-skewed distributions (very low probability events with very high gains). The effect is particularly strong in case of a cliquet-style guarantee due to the limited losses. Overall, the results show that the consideration of the subjective utility of the annual changes in the form of the MCPT is able to explain the popularity of TPLI contracts.\footnote{It is worth noting that for unit-linked products we can also confirm the result of Ruß & Schelling (2018) within this more realistic framework, that is, for most loss-averse MCPT-investors products with a cliquet-style (year-to-year) guarantee are more attractive than products without or with a terminal guarantee only.}

Figure 10 shows the $r^{CE}$ in the combined model as a functions of the weight $s$ that is assigned to annual changes. The results show that typical loss averse MCPT investors ($\lambda = 2$) prefer TPLI contracts with a moderate collective malus over other products if the weight assigned to the annual changes is above roughly 50\% (lower left panel). For $s > 80\%$ all TPLI contracts,
Figure 10: $r^{CE}$ in the $CPT^{com}$ case depended on $s$ for different settings of $\lambda$ and $\gamma = 0.65$ and $a = 0.88$. Plot displays for unit-linked products the maximum $r^{CE}$ of all considered parameter settings (stock ratio and risk multiplier).

that is, even with a significant bonus malus, are preferred over all other products. The other panels illustrate the impact of loss aversion. Without loss aversion (upper left panel) and even in case of a low loss aversion of $\lambda = 1.5$ the results show that TPLI contracts are less appealing than alternative products. Conversely, for MCPT investors with a rather high loss aversion of $\lambda = 3$ a rather small weight $s \approx 35\%$ is sufficient to make the TPLI contracts more appealing than the alternative products.

Last, we discuss some of the assumptions. We have tried to chose all parameters carefully such that the analyzed products are modeled as realistic as possible. However, in particular the numerous management rules of the life insurer for TPLI contracts allow for a large degree
of freedom. The consideration of TPLI contracts A–D as well as the fictitious contracts E and F in Section 4 provide insights into the impact of some of the key aspects (reserve ratio, guaranteed rates of the insurance portfolio, smoothing and risk sharing elements). Another important aspect is the asset allocation of the insurer. In all cases we have assumed that the corridor for the stock ratio for the stochastic (future) projection is given by $[10\%, 17.5\%]$.

To analyze the impact of the asset allocation we also consider the results for lower corridor with $q_{\min} = 7.5\%$ and $q_{\max} = 15\%$, that is, we assume that the corridor is equal to the corridor used for the deterministic (past) scenario. Moreover, we also consider the cases that the stock ratio in the stochastic (future) projection is held constant at 10% and 15%, respectively.

Figure 11 displays the results in the combined model with loss aversion $\lambda = 2$ for TPLI contract A (left panel) and contract C (right panel) subject to the adjusted stock ratios (all other assumptions being equal) as well as for the unit-linked products. The results illustrate that a slightly higher (lower) stock ratio of the underlying collective investment is perceived as more (less) attractive by a long-term investor. Moreover, we find that in the case of a lower stock ratio in combination

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49 In case of contract A (C) the average stock ratio of the insurer (at the beginning of the year) over all scenarios is 11.08% (10%).

50 In case of contract A (C), the average stock ratio of the insurer (at the beginning of the year) over all scenarios is 8.61% (7.5%).

51 Note that the PVFP of the insurer remains positive in all cases.
with a significant bonus malus (TPLI contract A)\textsuperscript{52}, unit-linked products with a clique-style guarantee are slightly preferred. In case of a higher stock ratio the TPLI contact A is again preferred for \( s \geq 0.5 \). Moreover, if the contract is only exposed to a moderate collective malus (TPLI contract C), we find that even in the case of a lower stock ratio the TPLI contract is preferred if the weight assigned to the annual changes is above 50\% - 60\%.

Summarizing, we have shown that MCPT can explain the popularity of TPLI contracts and that this remains true even if annual changes only partly impact the investor’s subjective utility.

\section{Conclusion and Outlook}

In this paper, we have analyzed smoothing and risk sharing elements provided by life insurers from a long-term investor’s perspective. We have also considered various unit-linked products to analyze the popularity of TPLI contracts compared to other common investment choices.

We have shown that return smoothing elements based on a collective investment of a life insurer can heavily stabilize annual returns without significantly changing the risk-return characteristics of the terminal value compared to an unsmoothed investment in the same assets. However, the results under CPT show that investors who focus solely on the terminal value prefer the unsmoothed investment. This and other existing results cast doubt that CPT applied in its standard form describes actual decision making of long-term investors sufficiently. In contrast to CPT-investors, MCPT-investors also gain utility from potential annual changes in the account value. For these investors products with smoothed returns are highly attractive. Moreover, for MCPT-investors TPLI contracts with smoothing and risk sharing elements are typically more attractive than common unit-linked products (with and without embedded guarantees) and this is also true in the case of a (moderate) ex-ante collective malus and even if the subjective utility is only partly influenced by potential annual changes. Hence, in contrast to

\textsuperscript{52}Note that the collective malus depends on the stock ratio. \textit{CB}\% is for contract A between \(-5.7\%\) (15\% stock ratio) and \(-6.7\%\) (10\% stock ratio) and for contract C between \(-1.4\%\) (15\% stock ratio) and \(-2.6\%\) (stock ratio between 7.5\% and 15\%).
standard approaches, MCPT is able to explain the preference of many long-term investors for smoothed returns and the popularity of TPLI contracts. Combined with the results from Ruß & Schelling (2018) and Graf et al. (2018), this gives strong evidence that long-term investors consider potential annual changes already when making the investment decision and that this has an important impact on long-term investment choices, in particular, in the segment of retirement savings.

Understanding the drivers of actual decision making is essential to design products which fit the needs and are at the same time attractive for customers. The results in this paper show that smoothing and risk sharing elements provided by life insurers are highly attractive for long-term investors while at the same time provide the investor with a terminal value that is very similar to an unsmoothed investment in the same assets. However, high year-to-year guaranteed rates force life insurers to invest in low-risk investments which is rather suboptimal for long-term investors with regard to the terminal value. The findings presented in this paper strongly indicate that participating products which make use of smoothing and risk sharing elements of a collective investment without or with rather low guaranteed rates (e.g., applied at maturity only) seem very promising in providing an objectively superior distribution of terminal value while at the same subjectively being attractive for the customer (as well as for the insurer due to the reduced risk, cf. Reuß et al. (2015)).

While MCPT provides an explanation for the popularity of many long-term investment products, the decision making process of long-term investors is not yet fully understood. MCPT is based on the assumption that long-term investors (consciously or subconsciously) already consider future utility or disutility stemming from interim changes when making the investment decision. Future experimental and empirical studies are necessary to improve our understanding of this assumption. Further, future research should address how we can help long-term investors to make better decisions to improve their retirement savings and to ensure a desired

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53It is noteworthy that providing customers with appropriate information on these elements is essential. Participating products should therefore ideally be based on more transparent management rules for smoothing and risk sharing which are more easily to communicate to customers.
A Appendix - Financial Market Model

For the purpose of pricing, we consider a filtered probability space \((\Omega, \mathcal{F}, \mathbb{F}, \mathbb{Q})\) on a finite time horizon \([0, T]\), \(T < \infty\) under the risk-neutral measure \(\mathbb{Q}\) satisfying the usual conditions with \(\mathbb{F} = (\mathcal{F}_t)_{0 \leq t \leq T}\) and \(\mathcal{F}_t\) the \(\sigma\)-algebra containing the available information at time \(t\). The financial market model is given by a stock process \(S\) following a geometric Brownian motion and a short rate process \(r\) described by the Vasicek model, cf. Vasicek (1977). More precisely, \(dS_t^Q = S_t^Q \left( r_t^Q \, dt + \sigma_S \, d\tilde{W}_t^S \right)\) and \(dr_t^Q = \kappa (\xi^Q - r_t^Q) \, dt + \sigma_r \, d\tilde{W}_t^r\) with \(\sigma_S, \kappa, \xi^Q, \sigma_r > 0\) and \(d\tilde{W}_t^S \, d\tilde{W}_t^r = \rho \in [-1, 1]\). Furthermore, we define a rolling bond investment \(R\) based on zero bonds with term to maturity \(T_B < \infty\) years. The dynamic is given by \(dR_t^Q = R_t^Q \left( r_t^Q \, dt - \sigma_r B(t, t + T_B) \, d\tilde{W}_t^r \right)\) with \(B(t, t + T_B) = \frac{1}{\kappa} \left( 1 - e^{-\kappa T_B} \right)\).

The dynamics under the real word measure \(\mathbb{P}\) are given by \(dS_t^P = S_t^P \left( (r_t^P + \lambda_S) \, dt + \sigma_S dW_t^S \right)\) with constant risk premium \(\lambda_S > 0\), \(dr_t^P = \kappa (\xi^P - r_t^P) \, dt + \sigma_r dW_t^r\) with \(\xi^P = \xi^Q + \frac{\sigma_r \lambda_S}{\kappa}\) and \(\lambda_r\), the price of the interest risk, and \(dR_t^P = R_t^P \left( (r_t^P - \lambda_r \sigma_r B(t, t + T_B)) \, dt - \sigma_r B(t, t + T_B) \, dW_t^r \right)\). Moreover, \(dW_t^S = \tilde{W}_t^S - \frac{\lambda_r}{\sigma_S} dt\) and \(dW_t^r = \tilde{W}_t^r - \lambda_r dt\) and therefore \(dW_t^S \, dW_t^r = \rho \, e^{-\int_0^t r_u \, du}\) is used as numeraire.

The parameters have been chosen in accordance with the European money market and recent literature (cf. Graf et al. (2011) or Hieber et al. (2016)). More precisely, we assume \(\sigma_S = 20\%\), \(\sigma_r = 1.5\%\), \(\lambda_r = -23\%\), \(\kappa = 30\%\), \(\rho = 15\%\) and mean-reversion level \(\xi^Q = 4.2\%\) (and therefore \(\xi^P = 3.05\%\)). Moreover, the risk premium is \(\lambda_S = 4\%\). Due to the current low interest rate environment we use a negative initial short rate \(r_0 = -0.06\%\). Further, we use \(T_B = 10\) for the rolling bond investment. Additionally, Table 5 and 6 display the fair

\footnote{We can derive closed formulas of the processes and the zero bond prices, cf., e.g., Brigo & Mercurio (2007).}

\footnote{This value is also used by the German product contact point for old-age provision (Produktinformationstelle Altersvorsorge) to generate legally prescribed risk-return profiles for old-age provision products, cf. PIA (2016).}

\footnote{The value of \(r_0\) has been chosen to match the average value of the three-month EURIBOR rates of the last 6 months of 2015, cf. Deutsche Bundesbank (2016).}
Table 5: Pure money back VA fair guarantee fees $\gamma^g$ rounded to three decimals for different stock ratios $\theta$.

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1</th>
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<td>$\gamma^g$ (%)</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
<td>0.015</td>
<td>0.050</td>
<td>0.107</td>
<td>0.182</td>
<td>0.269</td>
<td>0.365</td>
<td>0.467</td>
<td>0.573</td>
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</tbody>
</table>

Table 6: VA cliquet-style fair guarantee fees $\gamma^g$ rounded to three decimals for protection levels $d^{pl}$ and different stock ratios $\theta$.

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma^g$ (%) for $d^{pl} = 90%$</td>
<td>0.001</td>
<td>0.007</td>
<td>0.006</td>
<td>0.052</td>
<td>0.199</td>
<td>0.491</td>
<td>0.941</td>
</tr>
<tr>
<td>$\gamma^g$ (%) for $d^{pl} = 98%$</td>
<td>0.334</td>
<td>0.491</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

guarantee fees $\gamma^g$ for the VA products derived under the risk-neutral measure $Q$.

**B Appendix - Life Insurance Figures**

Table 7 shows selected values of the deterministic scenario based on historical data from Deutsche Bundesbank (2016) and DAV (2017) which was used to derive the initial insurance portfolio of the insurance company.

Table 8 gives an overview of key figures of the profits of the fictitious insurance company resulting from the stochastic simulation in the different settings (A–D). In the base case A the present value of the insurer’s future profits\(^{57}\) amounts to $PVFP_0 = \sum_{t=1}^{20} E^Q \left[ e^{-\int_0^t r_u du} IR_t \right] = 3,358 \€$ and the average insurer’s future profit per year is given by $\overline{TR}_t = 462 \€$. This indicates that the future business of the insurance company is on average profitable. The Value-at-Risk (99.5%) of the insurer’s future profits $IR_t$ is -7,948\€ which is −4.03% of the corresponding balance sheet total $BS_t$. The maximal loss amounts to -19,246\€ which is -10.31% of the corresponding balance sheet total $BS_t$. Further, we can observe the asymmetry of the surplus distribution: the average loss (-3,477\€) that has to be borne by the insurer is higher than the average gain (703\€). However, the probability that $IR_t$ becomes negative is only 5.77%.

\(^{57}\)Cf., e.g., Burkhart et al. (2015) for details.
### Table 7: Selected values (% p.a.) of the deterministic scenario based on historical data from Deutsche Bundesbank (2016) and DAV (2017).

<table>
<thead>
<tr>
<th>year t</th>
<th>-29</th>
<th>-28</th>
<th>-27</th>
<th>-26</th>
<th>-25</th>
<th>-24</th>
<th>-23</th>
<th>-22</th>
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<td>1Y spot rate</td>
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<td>8.90</td>
<td>8.63</td>
<td>6.27</td>
<td>5.48</td>
<td>4.60</td>
<td>3.36</td>
</tr>
<tr>
<td>10Y spot rate</td>
<td>6.48</td>
<td>6.82</td>
<td>7.02</td>
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<td>8.31</td>
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<td>7.03</td>
<td>8.82</td>
<td>8.40</td>
<td>7.71</td>
<td>6.40</td>
<td>7.06</td>
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<td>3.50</td>
<td>3.50</td>
<td>3.50</td>
<td>3.50</td>
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<table>
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<tbody>
<tr>
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<td>3.58</td>
<td>3.10</td>
<td>4.69</td>
<td>3.98</td>
<td>3.39</td>
<td>2.26</td>
<td>2.22</td>
<td>2.27</td>
<td>3.34</td>
</tr>
<tr>
<td>10Y spot rate</td>
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<td>4.76</td>
<td>5.35</td>
<td>4.98</td>
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<td>4.23</td>
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<td>4.65</td>
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<td>4.82</td>
<td>4.16</td>
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<td>3.84</td>
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<td>4.00</td>
<td>4.00</td>
<td>4.00</td>
<td>3.25</td>
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<td>2.75</td>
<td>2.75</td>
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<tr>
<td>stock perf.</td>
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<td>17.71</td>
<td>39.10</td>
<td>-7.54</td>
<td>-19.79</td>
<td>-43.94</td>
<td>37.08</td>
<td>7.33</td>
<td>27.07</td>
<td>21.98</td>
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<table>
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<tr>
<th>year t</th>
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<tr>
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<tr>
<td>10Y spot rate</td>
<td>4.31</td>
<td>4.16</td>
<td>3.59</td>
<td>2.97</td>
<td>2.84</td>
<td>1.65</td>
<td>1.69</td>
<td>1.23</td>
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<tr>
<td>coupon rate</td>
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<td>4.10</td>
<td>3.46</td>
<td>2.86</td>
<td>2.76</td>
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<td>1.63</td>
<td>1.20</td>
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<td>-</td>
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<tr>
<td>$i^q_t$</td>
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<td>2.25</td>
<td>2.25</td>
<td>2.25</td>
<td>2.25</td>
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<td>1.75</td>
<td>1.75</td>
<td>1.25</td>
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</tr>
<tr>
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<td>23.85</td>
<td>16.06</td>
<td>-14.69</td>
<td>29.06</td>
<td>25.48</td>
<td>2.65</td>
<td>9.56</td>
<td>-</td>
</tr>
</tbody>
</table>

A higher (terminal) reserve ratio (case B) can be used to offset moderate losses which is reflected e.g., in a lower probability for losses (3.93%) and a higher $PVFP_0 = 5,281\text{ }\mathcal{E}$. If additionally the average guaranteed rates of the insurance portfolio are lower (case C) then there are almost no losses which have to be borne by the insurer (only 0.11% of the profits are negative). The $PVFP_0$ is in this case significantly higher (10,405\text{ }\mathcal{E}). Increasing the surplus participation in this setting to $\alpha^{Sp} = 97\%$ (case D) reduces the $PVFP_0$ to 3,178\text{ }\mathcal{E} but (virtually) without increasing the probability of losses nor the size of losses. In total, the results show that the fictitious life insurer is in none of the considered cases A–D exposed to excessive (unrealistic) losses and that the future business is in all cases on average profitable.
<table>
<thead>
<tr>
<th>case</th>
<th>(\min(IR_t))</th>
<th>(VaR_{99.5%}(IR_t))</th>
<th>(IR_t)</th>
<th>(\max(IR_t))</th>
<th>(PVFP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-19,246 €</td>
<td>-7,948 €</td>
<td>462 €</td>
<td>4,249 €</td>
<td>3,358 €</td>
</tr>
<tr>
<td>B</td>
<td>-19,690 €</td>
<td>-7,181 €</td>
<td>546 €</td>
<td>4,314 €</td>
<td>5,281 €</td>
</tr>
<tr>
<td>C</td>
<td>-7,295 €</td>
<td>0 €</td>
<td>740 €</td>
<td>4,335 €</td>
<td>10,405 €</td>
</tr>
<tr>
<td>D</td>
<td>-7,384 €</td>
<td>0 €</td>
<td>226 €</td>
<td>1,349 €</td>
<td>3,178 €</td>
</tr>
</tbody>
</table>

Table 8: Key figures of insurer’s future profits in the cases A–D.

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4 When and How Framing Makes Annuitzation Appealing: A Model-Based Analysis

Source:
When and How Framing Makes Annuitization Appealing:
A Model-Based Analysis

Stefan Schelling*

Abstract

Several experimental studies provide evidence that annuities are much more appealing under a consumption frame than under an investment frame. However, due to the complexity of the annuitization decision, the drivers of this result and their interaction are not yet fully clear. We consider a theoretical model to analyze the impact of various determinants. The results suggest that the main driver are the different reference points. Partial annuitization seems attractive under a consumption frame in most cases if the subjective life expectancy is not significantly shorter than the objective average life expectancy and if the aspired standard of living is not already covered by other sources of regular income. However, the impact of other behavioral aspects like loss aversion or subjective risk perception vary for different levels of wealth and regular incomes.

Keywords: Annuities, Framing, Mental Accounting, Prospect Theory, Reference Points, Retirement Savings

JEL: D14, G11, G22, G41, J26, J32

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1 Introduction

The idea of a life annuity is to provide a fixed stream of income for the rest of the life. Life annuities therefore protect against the risk of running out of money in old age, which is called longevity risk. Starting with Yaari (1965) and Fischer (1973), numerous authors incorporated longevity risk in life-cycle models of saving and consumption under standard economic assumptions. While early studies are based on several simplifying assumptions like no bequest motive, complete markets, actuarially fair annuities, etc., more recent studies like Davidoff et al. (2005) or Peijnenburg et al. (2016) examine the annuitization problem also under more realistic assumptions. The results of the vast majority of this literature shows that risk-averse utility maximizers prefer to annuitize significant fractions of their wealth at retirement age. Yet, in reality annuitization rates are often rather low and in particular only few individuals voluntarily purchase life annuities. The discrepancy between theoretically optimal and observed annuitization rates is known as the “Annuity Puzzle”.

There is a wide stream of literature exploring possible explanations for this puzzle.\(^1\) Explanations include the role of family risk sharing and the related bequest motive (cf. Brown & Poterba (1999), Post et al. (2006) or Lockwood (2012)), the role of preannuitized wealth, social security and real estate property (cf. Bernheim (1991), Mitchell et al. (1999) or Dushi & Webb (2004)) as well as incomplete markets, high loadings, asymmetric information and liquidity restrictions (cf., e.g., Friedman & Warshawsky (1990) or Gupta & Li (2007)). The literature shows that all these factors have an important impact on the demand for annuities and are able to explain the lack of full annuitization. Nevertheless, especially for individuals of middle wealth the lack of voluntary annuitization remains puzzling (cf., e.g., Dushi & Webb (2004), Davidoff et al. (2005) or Benartzi et al. (2011)). Additionally, against the background of current challenges like the demographic change and the related consequences for social security systems, voluntary annuitization seems increasingly important for this group to maintain an aspired standard of living in old age (cf., e.g., Wilke (2009) for details in Germany).

\(^1\)We refer to Brown (2007), Benartzi et al. (2011) and Milevsky (2013) for extensive reviews of the literature.
More recent literature suggests that behavioral aspects are crucial in order to understand the annuity puzzle. The concept of narrow framing\(^2\) suggests that individuals tend to focus on specific investments without considering many other options. Moreover, individuals with this bias tend to lose sight of the big picture, evaluate the investment standalone and overlook for example interactions with other already existing investments and/or the impact of their investment on their future consumption. Studies confirm this by showing that annuities are often considered as a gamble on a long life rather than a protection against longevity risk. By doing so, the risk of an early death and therefore of losing the annuitized wealth dominates the longevity risk (cf. Gazzale & Walker (2009)). Furthermore, behavioral biases like loss aversion with respect to the annuitized wealth (which serves as reference point),\(^3\) time preference, and overweightig of small probabilities suggest that annuitization becomes even less appealing. Under this so-called *investment frame* individuals focus solely on the investment risk and return characteristics and not on maintaining an aspired standard of living. As Hu & Scott (2007) point out, this investment frame provides an explanation for low voluntary annuitization rates.

Results from the field of psychology show that many decisions depend on how choices are presented – the so-called framing effect (cf. Tversky & Kahneman (1981) and Thaler (1985)). By means of an online survey Brown *et al.* (2008) find evidence that annuities are much more appealing when presented under a so-called *consumption frame*, where individuals focus on maintaining an aspired standard of living expressed through consumption. Also, Goedde-Menke *et al.* (2014), who have conducted and analyzed a representative survey among German consumers, as well as Nolte & Langer (2016), who used laboratory experiments, conclude that this framing effect has a strong impact on annuitization. However, Brown *et al.* (2013) provide evidence that even under the consumption frame individuals typically make suboptimal decisions according to standard life-cycle models. These findings raise the following questions: "How do individuals evaluate annuitization under the consumption frame?" and "What are the impacts of typical behavioral biases like reference points, loss aversion, and subjective probabilities as


\(^3\)Loss aversion suggests that individuals are much more sensitive to losses than to gains measured to a reference point (cf. Kahneman & Tversky (1979)).
well as different levels of financial means?” As empirical data is thin, we contribute to this newly emerging literature by analyzing the impact of various determinants within a theoretical model framework. In doing so, we do not analyze whether partial annuitization is optimal from a specific point of view, but rather focus on how individuals actually perceive the annuitization decision under different frames.

The results illustrate that while under an investment frame annuitization is not appealing for most individuals, under a consumption frame partial annuitization is often preferred. We disentangle the impact of different drivers like loss aversion or probability distortion under the consumption frame dependent on the level of liquid wealth and regular income provided by social security. The presented insights improve our understanding of decision making in the context of old-age provision. Moreover, we find that a rather short subjective life expectancy reduces annuitization rates significantly. Therefore, to increase annuitization rates and protection against longevity risk, information on the life expectancy and longevity seems essential.

The remainder of this article is organized as follows. Section 2 describes and motivates the framework and the framing perspectives which includes the investment frame as well as the consumption frame. Moreover, we specify the main model assumptions and the considered parametrization of the model for the numerical analysis. In Section 3 we present the results of the numerical analysis. Section 4 summarizes and gives an outlook for future research.

2 Methodology

The aim of this chapter is to propose a model that attempts to describe how individuals actually perceive and (possibly subconsciously) evaluate the annuitization decision under different frames. We consider an individual at retirement age $x$ at time $t_0 = 0$ who deals with the

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4 This is also closely related to framing issues. For example Payne et al. (2013) give experimental evidence that framing strongly affects annuitization by comparing the subjective life expectancies in a “live-to” with a “die-by” setting. The findings show that the subjective life expectancy is significantly higher in the “live-to” frame than in the “die-by” frame.
question of annuitization which is assumed to be a one-time decision at retirement age.\textsuperscript{5} The random subjective remaining lifetime is given by $\tau_{\text{sub}} \leq \omega$ (in full years) with $\omega$ denoting the maximal remaining lifetime. For simplicity, we assume that death is only considered at the end of the year.\textsuperscript{6} For the individual’s subjective probability of survival from time $t_0$ to time $t$, we use the common actuarial notation $p^\text{sub}_x := P(\tau_{\text{sub}} \geq t)$ for $t \in \{0, \ldots, \omega\}$.

Moreover, we denote the objective average remaining lifetime by $\tau^{\text{obj}} \leq \omega$ and the corresponding objective average probabilities of survival by $p^{\text{obj}}_x$. Differences between the subjective and the objective average probabilities can arise for objective reasons like a better or worse health condition or due to estimation errors caused by cognitive distortions. The actuarially fair annuity factor for a life annuity is then defined by $a^* := \sum_{k=0}^{\omega} k p^{\text{obj}}_x \cdot P(0, k)$ with $P(s, d)$ denoting the fair price of a zero bond with duration $d$ at time $s$.\textsuperscript{7} We assume that effects of adverse selection and other market incompleteness are covered by an expense factor $c^{\text{ann}}$. Therefore, the applied annuity factor can be calculated by $a_x := (1 + c^{\text{ann}}) \cdot a^*_x$. Moreover, we denote the resulting constant annual annuity\textsuperscript{8} payment by $A^{\text{ann}}$. We capture effects from inflation by considering an inflation-adjusted model framework.

Focusing on individuals of middle wealth, we assume that future consumption is only determined by income and liquid wealth and hence independent of illiquid assets (which are therefore not explicitly modeled in the framework).\textsuperscript{9} More precisely, we denote with $W_t$ the liquid wealth

\textsuperscript{5}There is a large literature which analyzes annuitization under settings which allow individuals to adjust annuitization during the whole remaining life-span (cf., e.g., Dushi & Webb (2004), Hainaut & Devolder (2006) or Horneff \textit{et al.} (2008b)). This line of research mainly focuses on optimal investment and annuitization strategies (from a normative perspective). However, in reality most individuals do not adjust their annuitization rate on a regular basis but rather deal with the question of annuitization when approaching the retirement age. As we focus on the question why individuals are attracted by annuities under a consumption frame, the one-time decision problem represents the typical decision problem most individuals are confronted with when approaching retirement age and seems therefore suitable from a descriptive perspective (cf. also Benartzi \textit{et al.} (2011)).

\textsuperscript{6}If the remaining lifetime is for example equal to 0, the individual dies at time $t = 1$.

\textsuperscript{7}We use fair prices of zero bonds which are in line with the financial market described in Section 2.3.

\textsuperscript{8}If not stated otherwise, we use the term “annuity” for life annuity.

\textsuperscript{9}Note that there is large empirical evidence that consumption is mainly driven by income and liquid assets and only minor by other rather illiquid assets, cf. for example Skinner (1996) or also Levin (1998). Also, the results of Venti & Wise (2004) suggest that elderly do typically not plan to use home equity to support general nonhousing consumption. Moreover, they also show that even in bad states, housing equity is largely preserved while other assets are consumed. This is also related to Shefrin & Thaler (1988) and their behavioral life-cycle hypothesis which assumes that due to mental accounting, not all assets are considered as fungible.
of the individual at time $t$ and the annuitization rate at time $t_0$ by $\theta^{ann} \in [0, 1]$. We assume that the remaining liquid wealth is invested in a balanced fund with constant stock ration $\theta^S \in [0, 1]$. The fraction invested in the balanced fund is denoted by $\theta^{bal} = 1 - \theta^{ann}$ and the return in period $[t-1, t)$ by $R^{bal}_t$. Furthermore, we assume that the individual receives predefined regular constant livelong social security benefits $A^{soc}$ at the beginning of each year. The total secure income of the individual at the beginning of the period $[t, t+1)$ is therefore $I_t := A^{soc} + A^{ann}$ for $t \in \{0, 1, \cdots, \tau^{sub}\}$.

As we aim to model and analyze actual decision making, we refrain from deriving consumption patterns that maximize preference functions, but rather restrict the analysis to several consumption plans based on typical recommendations (for example by financial advisers). Hence, we assume that the individual plans the future consumption $c^{act}_t$ at time $t \in \{0, \cdots, \tau^{sub}\}$ for period $[t, t+1)$ dependent on income and liquid wealth according to the following rule:

$$c^{act}_t := \begin{cases} 
\min (I_t + W_t, c^{mg}), & I_t + R^{bal}_t < c^{mg} \text{ and } W_t < k_t (c^g - I_t)^+ \\
\max (c^{mg}, \min (I_t + \frac{W_t}{k_t}, c^{max})), & \text{else}
\end{cases} \quad (1)$$

where $c^{mg}$ represents the minimal consumption goal per year needed to maintain a desired standard of living. This includes basic needs like housing, energy and food, which represents a minimal requirement, as well as expenses for comforts of everyday life (like for a car or for leisure activities). A consumption below this level leads to cuts in the standard of living. $c^g$ represents the aspired consumption goal per year that is sufficient to additionally meet further aspirations (for example traveling during the retirement period). Further, we assume that the individual does not plan to spend more than $c^{max}$ per year. The liquid wealth at time $t$ is determined by $W_t := W_{t-1} + I_{t-1} - c^{act}_{t-1} + R^{bal}_t$ for $t \in \{1, \cdots, \tau^{sub}\}$. Moreover, $k_t$ denotes an age-depending withdrawal rule which affects the planned spending of the liquid wealth. In the base case we consider the remaining life expectancy rule which is based on the remaining subjective life expectancy of the individual.\footnote{We consider in Section 3 also the effect of other common withdrawal rules like a fixed rule and a limiting age rule.} Consequently, the withdrawal
rate increases over time with decreasing remaining subjective life expectancy. Additionally, we require that $k_t$ is at least three, which serves as a safety cushion. More precisely, we set $k_t = \max\left(\mathbb{E}(\tau_{\text{sub}}|\tau_{\text{sub}} \geq t) - t, 3\right)$. The individual applies the withdrawal rate $k_t$ in case of adequate liquid financial means, that is, the income together with the return from the fund investment is larger than the minimal consumption goal $c^{mg}$, or the current wealth is sufficient to cover the aspired consumption goal $c^g$ for at least $k_t$ periods. Otherwise, that is, if the income together with the return from the fund investment is less than $c^{mg}$ and the liquid wealth is not sufficient to cover $c^g$ for at least $k_t$ periods, the individual consumes $c^{mg}$ as long as possible to avoid cuts in the standard of living.

### 2.1 Investment Frame

Under the investment frame we assume that the individual evaluates the annuitization decision solely on the investment risk and return characteristics of the outcome and isolated from implications on the future consumption. Under this frame the individual compares the total outcome of the investment with the lump sum that is invested (in our case $W_0$). Hence, to model the subjective utility under the investment frame, we assume that the individual considers $W_0$ as reference point. The return of the outcome with respect to this reference point is defined by $11$ $X^i := (1 - \theta^{ann}) \cdot W_0 + \sum_{t=0}^{T_{\text{sub}}} (A^{ann} + R^{bal}_{t+1}) - W_0 = \sum_{t=0}^{T_{\text{sub}}} (A^{ann} + R^{bal}_{t+1}) - \theta^{ann} \cdot W_0$. A positive value of the outcome $X^i$ defines a gain and a negative value a loss. We follow Tversky & Kahneman (1992) and assume that the individual’s subjective utility under the investment frame is driven by an S-shaped value function $v(\cdot)$ defined by $v(X^i) := (X^i)^{\alpha} \cdot \mathbb{1}\{X^i > 0\} - \lambda |X^i|^{\alpha} \cdot \mathbb{1}\{X^i \leq 0\}$, where $\lambda > 0$ is the loss aversion parameter (loss aversion if $\lambda > 1$) and $\alpha > 0$ determines the risk appetite. Finally, we assume that, the individual’s preference is based on the outcome $X^i$ evaluated according to Cumulative Prospect Theory (CPT) by

$$V^{\text{CPT}}(X^i) := \int_{-\infty}^{0} v(x) d\left(\mathbb{w}(F(x))\right) + \int_{0}^{\infty} v(x) d\left(-\mathbb{w}(1 - F(x))\right)$$

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11It is worth noting that one can think of various ways on how to define $X^i$ in the investment frame. We refrain from considering discounting etc. by using the most simple definition. The results presented in the later sections qualitatively remain for reasonable other definitions of $X^i$.

12Note that focusing on the case $\theta^{ann} = 1$, can also be interpreted as an isolated evaluation of the annuity product under a narrow frame.
with \( F(s) = \mathbb{P}(X \leq s) = \int_{-\infty}^{s} d\mu_X \) with \( \mu_X \) the probability measure given by the random variable \( X \) and \( w(\cdot) \) the probability distortion function \( w(p) := \frac{p^\gamma \gamma + (1-p)\gamma}{p^\gamma + (1-p)\gamma} \) with \( \gamma \in (0.28, 1] \), where the lower boundary for \( \gamma \) is chosen, such that \( w(p) \) is strictly monotonically increasing for \( p \in [0, 1] \).\(^{13}\)

### 2.2 Consumption Frame

Several studies like Brown et al. (2008) suggest that explaining annuities in the context of consumption protection in old age changes subconsciously the reference points and the evaluation. This motivates that under the consumption frame the individual evaluates the subjective utility based on future consumption as described by formula (1), where the minimal and the aspired consumption goal serve as reference points. Several studies suggest that individuals consider multiple reference points like minimal requirements, the status quo, aspirations or goals (cf. Koop & Johnson (2012) or Knoller (2016)). Hence, we assume that the individual is also aware of his or her minimal consumption requirement \( c_{mr} \) which is needed to cover the basic needs (for example for housing, energy and food). A consumption below this level leads to harsh cuts in the standard of living. Consequently, following the Tri-Reference Point Theory introduced by Wang & Johnson (2012), we assume that the individual considers three reference points when making the annuitization decision: the minimal consumption requirement \( c_{mr} \), the minimal consumption goal \( c_{mg} \) and the aspired consumption goal \( c^g \).

Depended on the future consumption \( c_{act} \), the outcome can therefore fall into four different functional regions determined by the three reference points:

- functional region 1 (full success: \( c_{act} \geq c^g \)): The individual considers the outcome as full success if future consumption is equal or above \( c^g \). Since this is a region of gains the individual is risk averse.

- functional region 2 (on target: \( c_{mg} \leq c_{act} < c^g \)): The individual’s consumption is on target if future consumption is between \( c_{mg} \) and \( c^g \). However, not reaching \( c^g \) as well as

\(^{13}\)Note that we refrain from a different treatment of gains and losses with respect to probability distortion and that \( \gamma = 1 \) represents the case without probability distortion.
the cushion with respect to $c^{mr}$ trigger a risk seeking behavior in this region. Moreover, because $c^g$ is not reached, the positive subjective utility is assumed to be slightly reduced by loss aversion with respect to $c^g$.

- functional region 3 (below target: $c^{mr} \leq c^{act}_t < c^{mg}$): The individual considers the outcome as below target if future consumption is above $c^{mr}$ but below $c^{mg}$. Contrary to region 2, the individual is risk averse in this region because of the small cushion to $c^{mr}$. Due to the shortfall with respect to $c^{mg}$, the subjective utility is reduced by loss aversion.

- functional region 4 (total failure: $c^{act}_t < c^{mr}$): The individual considers the outcome as a total failure if the future consumption is below $c^{mr}$. Therefore, the individual is risk-seeking in this region and the subjective utility is heavily reduced by loss aversion because of not reaching $c^{mr}$.

The four functional regions can be translated in a function of $X_t := (c^{act}_t - c^{mg}) \cdot \mathbb{1}\{t \leq \tau^{sub}\}$ and
modeled by the following double S-shaped value function (cf. Figure 1 for an illustration):  

\[
v_{TRP}(X_t) := \begin{cases} 
(X_t - (c^g - c^{mg}))^\alpha + \lambda_1 (c^g - c^{mg})^\alpha, & X_t > c^g - c^{mg}, \\
-\lambda_1 (|X_t - (c^g - c^{mg})|^{\alpha} - (c^g - c^{mg})^{\alpha}), & 0 < X_t \leq c^g - c^{mg} \\
\lambda_2 (|X_t + (c^{mg} - c^{mr})|^{\alpha} - |c^{mg} - c^{mr}|^{\alpha}), & c^{mr} - c^{mg} < X_t \leq 0 \\
-\lambda_3 |X_t + (c^{mg} - c^{mr})|^{\alpha} - \lambda_2 |c^{mg} - c^{mr}|^{\alpha}, & X_t \leq c^{mr} - c^{mg} 
\end{cases}
\]  

(3)

for \( t \leq \omega \) and with \( \lambda_1, \lambda_2 \) and \( \lambda_3 > 0 \) denoting the loss aversion parameters with respect to the corresponding consumption levels and \( \alpha > 0 \) the risk appetite parameter. It is worth noting that the three loss aversion parameters capture the different impact of the loss aversion in the different functional regions. We assume that the impact of the loss aversion increases from region 2 to region 4, that is, we require \( \lambda_1 \leq \lambda_2 \leq \lambda_3 \).

Similar to the different consumption levels, we assume that the individual considers also three levels of bequest: \( b^g \) denotes the aspired bequest goal, \( b^{mg} \) the minimal bequest goal, and \( b^{mr} \) the minimal bequest requirement. The subjective utility of bequest is assumed to be based on the liquid wealth at time of death,\(^{15}\) that is, \( W_{(\tau_{sub} + 1)_{-}} := W_{\tau_{sub}} + I_{\tau_{sub}} - c^{act}_{\tau_{sub}} + R^{bal}_{\tau_{sub}} \). Again, we model the subjective utility by a double S-shaped value function \( v^{TRP}(\cdot) \) with corresponding reference points. The considered bequest outcome is defined by \( X_t^{beq} := \min \left\{ W_{(\tau_{sub} + 1)_{-}} - b^{mg}, b^{max} \right\} \cdot 1 \{ t = \tau_{sub} \} \). We assume that bequest generates only additional subjective utility until a certain threshold \( b^{max} \) which represents the maximal planned amount of bequest. This is motivated by studies which indicate that many (and in particular high liquid) bequests are not on purpose (cf. Hurd (1989) or Benartzi et al. (2011)).

In total, based on the double S-shaped value functions the preference function for the annuitization decision with outcome \( X^{con} = \{ X_0, \cdots, X_\omega, X_0^{beq}, \cdots, X_\omega^{beq} \} \) is then determined

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\(^{14}\)This value function is also closely related to the value function proposed by Tversky & Kahneman (1992). Note that by setting \( c^g = c^{mg} = c^{mr} \) the value function reduces to the typical S-shaped value function used in CPT with loss aversion \( \lambda_3 \).

\(^{15}\)Note that we have assumed that death occurs at the end of the year.
by

\[ V^{\text{con}}(X^{\text{con}}) := \sum_{t=0}^{\omega} \rho^t \cdot (1 - s) \cdot V^{\text{TRP}}(X_t) + s \cdot \rho \cdot V^{\text{TRP}}(X_{\text{beq}}^t) \] (4)

with

\[ V^{\text{TRP}}(X) := \int_{-\infty}^{0} v^{\text{TRP}}(x) d(w(F(x))) + \int_{0}^{\infty} v^{\text{TRP}}(x) d(-w(1 - F(x))) \],

where \( s \in [0, 0.5] \) controls the impact of the bequest motive.\(^{16}\) We assume that the probability distortion function \( w(\cdot) \) is the same as in the CPT case. Moreover, \( \rho \) denotes a time preference discounting factor which captures the subjective time preference of the individual.

### 2.3 Model Assumptions and Parametrization

#### Financial Market Model

The financial market model is based on a stock process \( S \) described by a geometric Brownian motion and a short rate process \( r \) described by the Vasicek model (cf. Black & Scholes (1973) and Vasicek (1977)). The parameters have been chosen in accordance with the European money market and recent literature (cf. appendix A for more details). In the presented base case we restrict our analysis to a balanced fund\(^ {17} \) with a stock ratio \( \theta^S = 60\% \). The fraction of the balanced fund is chosen to be in line with typical “rules of thumb” often recommended by financial advisers (cf. for example Polyak (2005) or Whitaker (2005)). It therefore appears reasonable that many individuals consider stock ratios in this magnitude when comparing an annuity product with a withdrawal plan based on an investment fund.\(^ {18} \) The expected inflation-adjusted return of the considered balanced fund is 3.3\% with standard deviation 12.5\% (in the

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\(^{16}\)Note that we only consider values of \( s \) up to 0.5. Values above 0.5 imply that the bequest motive dominates the consumption motive and seem therefore not reasonable under a consumption frame. In fact, under a consumption frame rather small values of \( s \) seem appropriate for most individuals. Further, note that our analysis is restricted to liquid wealth and that other illiquid assets can also meet the bequest motive.

\(^{17}\)The balanced fund invests a constant fraction \( \theta^S \) in a stock investment and the remaining part in a pension fund based on a “rolling” zero bond investment. We apply daily rebalancing.

\(^{18}\)Moreover, behavioral heuristics like mental accounting (Thaler (1985)) indicate that individuals do not compare the whole variety of possible investment choices but rather focus on small samples.
long run).\textsuperscript{19}

**Survival Probabilities and Annuity Factor**

We use objective average survival probabilities \( t_p^{obj} \) with \( x = 65 \) based on the German Federal Statistical Office’s cohort mortality tables with trend (V2) for males\textsuperscript{20} born in the year 1952 (cf. Federal Statistical Office (2017)). For the simulation we consider \( \omega = 120 \). The mortality tables of the Federal Statistical Office have a cut-off age of 100 years. Hence, we extrapolated the mortality tables until an age of 120 years using a Kannisto model approach starting at an age of 80 years (cf. Wilmoth \textit{et al.} (2007)).

To analyze the question of how the subjective survival probabilities influence the annuitization decision, we consider different specifications of \( t_p^{sub} \) (cf. Figure 2). In the base case, we assume that the subjective survival probabilities correspond with the objective average survival probabilities for males, that is, \( t_p^{sub} = t_p^{obj} \). Additionally, we consider lower and higher subjective survival probabilities. This can be due to better (or worse) than average health conditions or other objective reasons, but also due to behavioral biases like the anchor effect. In this context

\textsuperscript{19}It is noted that the results presented in Section 3 depend on the parametrizations of the financial market and the choice of the balanced fund. However, numerous sensitivity analyses show that under reasonable assumptions, the structure of the results and the described impacts remain very similar (cf. also Section 3).

\textsuperscript{20}On average male individuals have a shorter remaining life expectancy than females, that is, the longevity risk is lower. In the European Union unisex annuity rates are applied for females and males as a result of the European Union Gender Directive (cf. Council Directive 2004/113/EC described in European Union (2004)) prohibiting any gender-based discrimination. This implies that under otherwise identical conditions annuities are on average more attractive for females than for males.
the anchor effect suggests that many individuals use the age at death of the generation of their parents or grandparents as an anchor when estimating their own remaining lifetime. By doing so, the individuals do not account for the fact that the life expectancy has increased steadily in the last century (cf., e.g., Oeppen & Vaupel (2002)) and tend to underestimate their life expectancy (cf. Bucher-Koenen & Kluth (2012)).\textsuperscript{21} We reduce subjective survival probabilities by multiplying the objective average probabilities of death of the corresponding cohort by a factor, such that the subjective life expectancy at age 65 is exactly 3 years (respectively 7) shorter than the objective average life expectancy (which is 19 years or age 84).\textsuperscript{22} To model individuals with a longer subjective life expectancy (for example particularly healthy or female individuals), we consider the case where the subjective survival probabilities correspond to the objective average survival probabilities for females. The subjective life expectancy of these individuals is 22.4 years (age 87.4).

To calculate the fair annuity factors $\tilde{a}_x$, we use the objective average survival probabilities for males and fair prices of zero bonds which are in line with the financial market model. Further, we assume that the applied annuity factors are reduced by the expense factor $c^{ann} = 15\%$ which captures also adverse selection effects.\textsuperscript{23} Note that based on these assumptions the fair annuity factor for a life annuity for a male individual results in $\tilde{a}_x^* = 18.61$ and the applied annuity factor in $\tilde{a}_x = 21.41$. That is, the yearly fixed nominal annuity payout at the beginning of each year equals \(4.67\, \text{€} \text{ per } 100\, \text{€} \text{ premium.}\textsuperscript{24}

\textsuperscript{21}Note that while lower annuitization rates are rational if lower probabilities are due to objective reasons, this is not true if lower probabilities are due to behavioral biases.

\textsuperscript{22}Cf. Vaupel \textit{et al.} (1979) for a precise description.

\textsuperscript{23}This value has been chosen such that the annuity payments are in line with (unisex) annuity rates in the German annuity market in 2017.

\textsuperscript{24}In reality, many annuity products are surplus participating. Since we consider an inflation-adjusted model, we refrain from considering any effects from surplus participation or similar mechanisms, which in reality can be used to compensate (at least partially) losses of purchasing power due to inflation.
Consumption and Subjective Utility

We consider individuals with the following annual consumption characteristics: The minimal requirement is assumed to be $c^{mr} = 12,000€$, the minimal goal is $c^{mg} = 18,000€$, the aspired goal $c^g = 24,000€$, and the maximal consumption is set to $c^{max} = 36,000€$. Based on these assumptions, we consider individuals who have different financial means, which is expressed through the social security benefits and the initial liquid wealth.

The social security benefit is varied between 6,000€, 12,000€ and 18,000€ per year. The considered values describe three fundamentally different initial situations: An individual with low social security benefits of only 6,000€ per year faces a risk of harsh cuts in the standard of living (not reaching the minimal requirement $c^{mr}$). For individuals with a medium social security benefit of 12,000€ per year, the minimal requirement consumption $c^{mr}$ is already fully covered by the social security benefit. However, for consumption beyond the minimal requirement, in particular to reach the aspired goal, additional resources (either from a withdrawal plan or an annuity) are needed. In the light of current demographic trends and the fact that in most countries social security benefits are implemented as a layer of the old-age provision system which provides only basic income, these two initial situation seem particularly relevant when considering individuals of middle wealth. Nevertheless, we also consider individuals, whose minimal consumption goal is already covered by social security benefits, that is, $A^{soc} = 18,000€$.

For the initial liquid wealth $W_0$ we consider 50,000€, 100,000€, 200,000€, and 500,000€. Figure 3 shows exemplarily the fundamentally different structures of the future consumption in the case of a social security benefit of 12,000€ without annuitization ($\theta^{ann} = 0$) for the different levels of initial liquid wealth. Individuals with a rather a low initial liquid wealth of

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25 The values have been chosen to represent typical individuals of middle wealth at retirement age in Germany and are based on empirical data from Germany between 2013 and 2017 (cf. for example Federal Statistical Office (2013) and Deutsche Rentenversicherung - German Statutory Pension Insurance Scheme (2017)).

26 The maximal consumption has been chosen to be reasonable for individuals of middle wealth and in relation to the other consumption goals. Moreover, it is noted that the structure of the presented results remains even without the restriction of a maximal consumption.

27 For example the German federal ministry of labor and social affairs notes in a current report that in future the German statutory pension insurance will in general not be sufficient to maintain the accustomed standard of living, cf. German federal ministry of labor and social affairs (2017), p. 12.
Figure 3: Percentiles of the future consumption in case of a social security benefit of 12,000 € and different levels of initial liquid wealth without annuitization.

50,000 € (100,000 €) are only able to maintain the minimal consumption goal $c^{mg}$ until age 70-75 (80-85). A higher initial liquid wealth of 200,000 € (500,000 €) is sufficient to cover $c^{mg}$ ($c^{max}$) until the age of roughly 95 (90) in most cases.

Further, we assume that all individuals require a non negative bequest, that is, $b^{nr} = 0$ €. The bequest motive is further driven by the minimal bequest goal, the aspired bequest goal, and the maximal planned bequest, which are, $b^{mg} = 0.1 \cdot W_0$, $b^g = 0.2 \cdot W_0$, and $b^{max} = 0.3 \cdot W_0$.

In the base case, we assume that the individual’s subjective utility is based on the following parametrization: The risk appetite parameter is set to $\alpha = 0.88$ and the probability distortion is set to $\gamma = 0.65$. This parametrization is chosen to be in line with Tversky & Kahneman (1992). For the investment frame we set the loss aversion parameter in the base case to $\lambda = 2.4$. In the consumption frame, the parameters for the loss aversion reflect the different impact of the loss aversion depending on the functional region (cf. Section 2.2). If not stated otherwise, we use $\lambda_1 = 1.2$, $\lambda_2 = 2.4$, and $\lambda_3 = 4.8$. Therefore, $\lambda_1$ only slightly reduces the subjective utility in case of on target compared to the case of full success. $\lambda_2$ and $\lambda_3$ applied in the the region of below target and total failure have been chosen such that the loss aversion ratio $\lambda_r := \frac{\lambda_2}{\lambda_1} = \frac{\lambda_3}{\lambda_2}$.

Note that under the considered framework, this is always fullfilled.
equals to two. A total failure which leads to harsh cuts in the standard of living is punished by a significantly higher loss aversion. Last, in the base case we refrain from considering subjective time preference, that is, $\rho = 1$. We investigate the impact of subjective time preference as well as other behavioral biases and model assumptions in Section 3.2.

### 3 Results

This Section presents the results of the numerical analysis based on Monte Carlo Simulations with 500,000 trajectories. The main goal of the analyses is to improve our understanding of the impact of framing and other behavioral biases when making the annuitization decision. Therefore, we focus on the structure of the results and note that precise numbers of course depend on the respective assumptions.

#### 3.1 Comparing the Impact of the Frames

At first, we compare the results under the different frames for $A^{soc} = 12,000\,€$. The impact of the social security benefit is analyzed subsequently in Section 3.2.

**Investment Frame**

The left panel of Figure 4 displays the certainty equivalents ($CE$) (in % of the initial wealth) that an individual under the investment frame would consider as equally desirable as the outcome of the corresponding annuitization decision. If the value is below one the outcome is considered as not attractive. For $\theta^{ann} = 1$ the left panel of Figure 4 displays the case where the investor evaluates the annuity isolated from other products. In this case we find that the annuity product results in a $CE < 1$ (0.86), that is, 14% lower than the price of the annuity product. Hence, the annuity product is considered as an immediate loss. One main reason is the risk of high losses in case of an early death. Loss aversion and probability distortion (overweighting the small probability of an early death) intensify this result. Further analyses show that even in

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29 Consequently, the negative utility generated by a moderate loss amounts to twice the positive utility generated by a moderate gain in the same magnitude, which is in line with Tversky & Kahneman (1992).

30 Note, in this regard, that we consider an inflation-adjusted framework.

31 For the calculation of the certainty equivalent under CPT we refer to Ebert et al. (2012).
the case without loss aversion ($\lambda = 1$) and without probability distortion ($\gamma = 1$), the annuity product has a $CE < 1$ (0.95). In case of a shorter subjective life expectancy (3 or even 7 years shorter), the annuity product has only a $CE$ of 0.79 or 0.67, respectively. One explanation for this result is the expense factor of 15% which increases the applied annuity factor and limits the chances of positive or even high investment returns of the annuity product significantly. Nevertheless, even without expenses ($c_{ann} = 0$), the annuity product has a $CE > 1$ only in the case of a (very) low loss aversion ($\lambda < 1.25$ with probability distortion and $\lambda < 1.45$ without probability distortion). For an individual with a longer subjective life expectancy, we find that the annuity product has a $CE > 1$, only in cases without probability distortion and with a rather low loss aversion ($\lambda < 1.75$) as well as with probability distortion ($\gamma = 0.65$) and a very low loss aversion ($\lambda < 1.3$).

If we consider all values of $\theta_{ann}$, we find that the $CE$ decreases strictly monotonically in the annuitization rate. Hence, no annuitization is preferred. This holds for all considered levels of initial wealth. The reasons are the same as in the case $\theta_{ann} = 1$. The results remain also true without loss aversion and probability distortion, other (reasonable) financial market conditions and balanced funds, in the case without expenses, longer and shorter subjective life expectan-
cies as well as for all levels of social security benefits.

In total, we can conclude that under an investment frame annuitization (full or partial) is not appealing for most individuals. Moreover, the results suggest that this is mainly driven by the nature of the reference point considered under the investment frame.

**Consumption Frame**

The right panel of Figure 4 displays the certainty equivalent consumption \( c^{CE} \) under the consumption frame in the case without bequest \( (s=0) \) depending on the annuitization rate \( \theta^{ann} \), where the vertical lines indicate the preferred annuitization rates. The certainty equivalent consumption \( c^{CE} \) is defined as the fixed future consumption that is equally desirable under the consumption frame as the corresponding future consumption \( c^{act} \).

Under the consumption frame without bequest, we find that annuitization of a significant fraction of the initial wealth is optimal for all considered individuals. For an initial wealth of 50,000 \( \epsilon \), 100,000 \( \epsilon \) and 500,000 \( \epsilon \) the preferred annuitization rates are above 80%. For an initial wealth of 200,000 \( \epsilon \) the preferred annuitization rates equal 40%. Interestingly, the reasons for the high annuitization rates differ.

For an initial wealth of 50,000 \( \epsilon \), consumption is always below or equal to \( c^{mg} \). Without annuitization consumption for the first roughly 10 years is equal or at least very close to this goal. However, at an age of around 75 the individual runs out of liquid wealth. Therefore, the individual very likely faces annual “losses” in the magnitude of \( c^{mg} - c^{mr} = 6,000 \epsilon \) thereafter. A higher annuitization rate results in a higher regular income – which is, however, still significantly lower than \( c^{mg} \) – and therefore reduces the amount of the annual losses in later years. But, in many cases it reduces consumption in early years. Hence, independent of the annuitization rate, the individual faces a high risk of running out of liquid wealth. However, due to the small cushion to \( c^{mr} \), the individual is assumed to be risk averse in functional region

\[32 \text{The value can be derived numerically, for example, by means of a regula falsi method.}\]
3 and therefore prefers a high annuitization rate (certain but smaller losses with respect to $c^{mg}$).

Also an initial wealth of 100,000 € is not sufficient to provide an annuity such that the regular income covers $c^{mg}$. However, with full annuitization the annual losses with respect to $c^{mg}$ can be reduced significantly when running out of liquid wealth. Hence, due to loss aversion the individual prefers a high annuitization rate.

An initial wealth of 200,000 € is sufficient to provide an annuity such that the regular income fully covers $c^{mg}$. But, a high annuitization rate significantly reduces the potential for even higher consumption. Moreover, in this case also a self-managed withdrawal plan is able to cover the consumption goal for many years (in case without annuitization the future consumption until age 85 is at least equal to $c^{mg}$). In contrast to individuals with lower initial wealth, the individual has a considerable financial cushion and is therefore less affected by single years with negative returns of the balanced fund. Nevertheless, without any annuitization the individual is still facing a significant risk of running out of liquid wealth at an age of 95 (probability of more than 10%). In this case, without annuitization, the individual has to reduce consumption to the level of the social security benefits. Hence, due to loss aversion, the individual is attracted by the annuity. Nevertheless, as we assume that the individual is risk seeking in functional region 2, the individual prefers only a moderate annuitization rate of 40%.

An initial wealth of 500,000 € can afford an annuity such that the regular income is only slightly below $c^{max}$. Without any annuitization even these individuals face the risk of running out of liquid wealth at an age of around 100, and even more important, with a moderate annuitization rate, for example of 40%, the future consumption drops below $c^g$ at an age around 95 in more than 50% of the cases. Also in younger ages (starting with age 75) consumption is significantly below $c^{max}$ with a probability of roughly 5%-25%. Since individuals are assumed to be risk averse for positive outcomes above $c^g$ (functional region 1) and since consumption is limited by $c^{max}$, high annuitization rates are also preferred by these individuals.
3.2 Detailed Analyses under the Consumption Frame

While the consideration of consumption goals as reference points is the main driver of the higher annuitization rates under the consumption frame, the impacts of other factors depend on the characteristics of the individual. Next, we analyze impacts of different factors in detail.

The Impact of Social Security Benefits and the Bequest Motive

Figure 5 displays the preferred annuitization rates under the consumption frame for $A^{soc} = 6,000\,€$, $12,000\,€$ and $18,000\,€$ depending on the bequest motive $s \in [0, 0.5]$.

In case of $A^{soc} = 6,000\,€$, very high annuitization rates are preferred by all considered individuals. Annuitization of a significant fraction of the initial liquid wealth is also preferred in case of a moderate bequest motive (cf. Figure 5 with $s > 0$). This is mainly due to loss aversion and the risk of not reaching the consumption goals (particularly $c_{mr}$). For an initial liquid wealth of $50,000\,€$ and $100,000\,€$, the bequest motive has only a minor impact as financial means are needed to reach the minimal consumption requirement. For individuals with initial liquid wealth of $200,000\,€$ and $500,000\,€$ preferred annuitization rates decrease in $s$. Nevertheless, even in case of a rather strong bequest motive ($s \approx 0.5$) high annuitization rates remain...

\footnote{For these individuals the subjective utility from additional consumption is higher than from leaving a bequest. Therefore, these individuals leave in most scenarios no bequest.}
tractive.

For $A_{soc} = 12,000 \, \text{€}$, that is the basic needs are fully covered by social security benefits, the bequest motive has a stronger impact on the preferred annuitization rates. For an initial liquid wealth of 50,000 € a moderate bequest motive ($s < 0.2$) reduces the preferred annuitization rates only slightly because maintaining consumption close to $c^{mg}$ as long as possible generates more subjective utility than leaving a bequest in case of an early death. For an initial liquid wealth of 100,000 €, the preferred annuitization rate is below 40% for $s > 0.1$. Without annuitization, these individuals have a high probability to meet their bequest goals if they die before age 85 (note that the probability of death before 85 is around 50%). A higher annuitization rate reduces this probability significantly. For an initial liquid wealth of 200,000 €, the bequest motive has almost no impact on the preferred annuitization rate. One reason is that individuals draw no subjective utility from leaving liquid wealth above $b^{max}$. Without annuitization these individuals meet $b^{max}$ in most cases until age 87. However, in bad states (worst 5%) the bequest can already fall below $b^g$ at an age of around 80. In particular, in states with negative investment returns, annuitization (which is not exposed to capital market risk) can even result in a higher bequest. Furthermore, the higher annuity reduces the withdrawal required to maintain the aspired standard of living. Therefore, these individuals prefer partial annuitization. Also, for an initial liquid wealth of 500,000 € annuitization rates of over 50% are preferred for $s < 0.2$. Only if the bequest motive becomes stronger ($s > 0.2$), preferred annuitiation rates drop down to around 20%. Again, the results are based on the assumption that individuals draw no subjective utility from leaving liquid wealth above $b^{max}$ which is achieved in most scenarios even in the case of partial annuitization.

In the case of $A_{soc} = 18,000$, that is, individuals are not exposed to any losses, annuitization is avoided by most individuals (initial liquid wealth between 50,000 € and 200,000 €) as they are risk seeking in the functional region 3. Contrary, individuals with an initial liquid wealth of 500,000 € prefer high annuitization rates (risk averse in the functional region 1). For these individuals a high annuitization rate eliminates the risk of not reaching the consumption goals.
at old age (without reducing the consumption in early years due to the maximal consumption goal $c^{max}$). As the remaining liquid wealth is still sufficient to cover the bequest motive in almost all cases, preferred annuitization rates remain high also in the case of a bequest motive.

In total, we can conclude that the level of the social security benefits (in relation to the consumption goals) has a significant impact. Individuals with social security benefits below the minimal consumption goal prefer high annuitization. Moreover, for these individuals annuitization of a significant fraction of the initial liquid wealth remains attractive even with a moderate bequest motive.

Figure 6: Preferred annuitization rate for $A^{soc} = 6,000\, \text{€}$ (a) and $12,000\, \text{€}$ (b) depending on the bequest motive $s$ for different levels of initial wealth under various specifications of the loss aversion. Left panel: no loss aversion. Middle panel: $\lambda_1 = 1.2$ and $\lambda' = 1.25$. Right panel: $\lambda_1 = 1.2$ and $\lambda' = 1.5$.

4 When and How Framing Makes Annuitization Appealing Research Papers
The Impact of Loss Aversion

Figure 6 displays the impact of loss aversion on the preferred annuitization rates under the consumption frame in the case of $A^{soc} = 6,000\,€ (a)$ and $A^{soc} = 12,000\,€ (b)$. The left panels show the case without loss aversion, that is, all loss aversion parameters are set equal to one. The middle panel and the right panels display the results for lower loss aversion ratios $\lambda^r$ (1.25 and 1.5, respectively) with $\lambda_1 = 1.2$.

In the case of low social security benefits and initial liquid wealth of up to 200,000\,€, we find that loss aversion has an important impact. Without loss aversion no annuitization is preferred. One reason is that individuals are risk seeking if the future consumption is below the minimal requirement $c^{mr}$ (functional region 4). However, the results also show that a rather low level of loss aversion (middle and right panels) is sufficient to make annuitization attractive in many cases. For individuals with initial liquid wealth of 500,000\,€, loss aversion has almost no impact on the annuitization rate because the consumption of these individuals is mainly in the region of gains above $c^g$. We find similar results for $A^{soc} = 12,000\,€$. In these cases, loss aversion has only a noticeable impact for an initial liquid wealth of 100,000\,€ and 200,000\,€. For these individuals, the consumption is fluctuating between positive outcomes (functional region 1 and 2) and negative outcomes (functional region 3 and 4), while for individuals with initial liquid wealth of 50,000\,€ and 500,000\,€, the consumption is almost only in the region of negative and positive outcomes, respectively.

The Impact of Probability Distortion

To analyze the impact of probability distortion, we set $\gamma = 1$. We find that switching off probability distortion has a significant impact on the annuitization rate. Even in the case without bequest motive the preferred annuitization rates reduce to 10% to 20% for individuals with social security $A^{soc} = 12,000\,€$ and initial liquid wealth of up to 200,000\,€. The reason is that with probability distortion, the probability of becoming very old and running out of liquid wealth in old age is overweighted. Moreover, to increase the consumption in old age, these
individuals have to reduce their consumption in early years. This is not the case for individuals with an initial liquid wealth of 500,000 €. For these individuals, annuitization only reduces the risk of running out of liquid wealth in old age without reducing consumption in early years (due to $c^{max}$). Hence, annuitization rates are less affected by probability distortion. However, for individuals whose basic needs are not covered by social security benefits ($A^{soc} = 6,000$ €) the impact is weaker. Due to loss aversion with respect to $c^{mr}$, preferred annuitization rates remain rather high (in the magnitude of 40% - 50%).

The Impact of Time Preference

Figure 7 displays the preferred annuitization rate under the consumption frame with time discounting $\rho = 0.98$. In particular individuals with a lower initial liquid wealth are much less attracted by annuitization. These individuals prefer to maintain $c^{mg}$ as long as possible. A high annuitization rate, however, reduces the consumption already in young ages to a level below $c^{mg}$ in order to finance consumption in old age which is now valued lower. If more initial wealth is available, the impact becomes smaller, since individuals can maintain a higher consumption for a longer period.
The Impact of the Subjective Life Expectancy

Figure 8 shows the preferred annuitization rates for individuals with different subjective life expectancies. All other parameters are equal to the base case. The results show that a reduction of 7 years dramatically reduces the annuitization rates. For $A^{soc} = 12,000\, \text{€}$ (lower panels) and initial liquid wealth of 50,000\, \text{€}, 100,000\, \text{€}, and 200,000\, \text{€}, the annuitization rates are below 10% even in the case without bequest. For $A^{soc} = 6,000\, \text{€}$ (upper panels), we find the same results for an initial liquid wealth of 50,000\, \text{€} and 100,000\, \text{€}. The simple reason is that these individuals perceive a much smaller subjective risk of not reaching the consumption goals in

\[34\text{If we additionally switch off loss aversion, even partial annuitization becomes unfavorable.}\]
older ages. However, if the subjective life expectancy is only 3 years shorter, annuitization of a significant fraction of the initial liquid wealth is preferred in many cases. For individuals with a longer subjective life expectancy (right panels), high annuitization rates are typically preferred.

The Impact of the Consumption Rule

So far, all presented results are based on the assumption that individuals plan consumption according to the remaining life expectancy rule. This rule leads in many cases to a hump-shaped consumption pattern over time (upper left panel of Figure 9). To investigate the impact of the consumption rule, we also consider other often recommended withdrawal rules.$^{35}$

Firstly, we consider a limiting age rule, which defines the age-depending withdrawal by the difference between a limiting age (in our case set to 100 years) and the age at time $t$. Formally, the 100 year rule is defined by $k_t = \max(100 - 65 - t, 3)$. Using a limiting age of 100 years leads to a consumption pattern which typically increases when approaching the age of 100. Since most of the liquid wealth is depleted at an age of 100, for older ages than 100 consumption

$^{35}$The considered rules are, e.g., in line with Horneff et al. (2008a).
Figure 10: Preferred annuitization rate depending on the social security $A^{soc}$, the initial wealth $W_0$, the bequest motive $s$ and the consumption rule. Left, middle and right panels: $A^{soc} = 6,000 \, €$, $12,000 \, €$, and $18,000 \, €$.

decreases rapidly (cf. upper right panel of Figure 9).

Secondly, we implement a simple (age-independent) fixed rule, that is, the withdrawal is determined by setting $k_t$ to a fixed value $k \in \mathbb{R}_+$. We consider two versions: (a) The Live “Forever” Rule by setting $k$ to the actual applied annuity factor, that is, $k = \hat{a}_x = 21.41$. Consequently, the initial withdrawal equals the payout of a life annuity that can be afforded by the initial liquid wealth. This is a very cautious rule since the wealth at time $t$ is always budgeted for another roughly twenty years (cf. lower left panel of Figure 9). Nevertheless, this rule is closely related to the very common $\approx 4\%$ rule which is often recommended for drawing income from
Figure 11: Preferred annuitization rates depending on the social security $A^{soc}$, the initial wealth $W_0$ and the bequest motive $s$ if alternative investment is given by a pension fund. Left panel: $A^{soc} = 6,000\, \text{€}$. Middle panel: $A^{soc} = 12,000\, \text{€}$. Right panel: $A^{soc} = 18,000\, \text{€}$.

a self-managed retirement portfolio. (b) The Live “Now” Rule by setting $k = 5$, which results in a rapidly decreasing withdrawal plan. Individuals following this rule can be interpreted as having a high preference for consumption in early years (cf. lower right panel of Figure 9).

Figure 10 displays the preferred annuitization rates under the different consumption rules for various levels of initial liquid wealth and social security benefits, while the other parameters are set equal to the base case. In total, we find that the structure of the results is rather similar for individuals with social security benefits of 6,000€ and 12,000€. For these individuals the annuitization of a significant fraction of the initial liquid wealth is preferred under all considered consumption rules. For individuals with a social security benefit of 18,000€, annuitization is preferred for an initial liquid wealth of 500,000€. Moreover, for the 100 year rule and live “forever” rule, annuitization is also preferred by some individuals with an initial liquid wealth below 500,000€. Of course, the impacts of the different behavioral biases can vary, for example, the impact of time preference in case of the live “now” rule is stronger. Nevertheless, most results hold qualitatively for all considered consumption rules.

**The Impact of the Investment Alternative**

Last, we investigate the impact of a different investment alternative, namely a pension fund with an inflation-adjusted long-term return of roughly 2% and a very low standard deviation of
There are two main reasons why we consider this very conservative alternative: Firstly, numerous studies show that most individuals prefer rather save investments. Secondly, there are several studies which show that especially individuals with lower financial literacy have a tendency to underestimate possible returns from funds, cf. Lusardi & Mitchell (2007), Lusardi & Mitchell (2011) or Jappelli & Padula (2013). Hence, it seems reasonable that many individuals compare the annuity with such an investment. Figure 11 shows that the structure of the results is very similar to the base case. However, in most cases, the preferred annuitization rates are at least slightly higher. Preferred annuitization rates increase heavily for individuals with a high social security benefits of \( A^{soc} = 18,000 \) and initial liquid wealth between 50,000€ and 200,000€. This is due to the lower investment return and the resulting significantly higher probability of running out of liquid wealth in old age.

Further analyses show that the structure of the results is also very similar if we assume that the individual considers all possible balanced funds, that is, all levels for the stock ratio between 0% and 100%. Individual with an initial liquid wealth of up to 200,000€ prefer in most cases a rather high annuitization rate combined with a pure stock investment. Only individuals whose basic needs are not covered by social security benefits and with a rather low initial liquid wealth of up to 100,000€ prefer high annuitization rates in combination with a pension fund. One main reason is the strong loss aversion below \( \epsilon^{mr} \).

Summarizing, we can conclude that the structure of the results and the described impacts remain very similar in case of other (simple) investments alternatives.

## 4 Conclusion and Outlook

In this paper, we have modeled the annuitization decision under an investment frame as well as under a consumption frame. Furthermore, we have analyzed the impact of various determinants on the annuitization decision under both frames focusing on behavioral aspects.

\(^{36}\)Note that the results are qualitatively remain for balanced funds with a rather low stock ratio (below 30%).
Under the investment frame we have shown that partial annuitization is only appealing for individuals with a significantly higher subjective life expectancy than the average objective life expectancy and only in combination with a rather low level of loss aversion. The results show that annuitization rates will remain low as long as individuals evaluate the annuitization under the investment frame. Other behavioral biases like loss aversion additionally intensify the annuity aversion. These results are in line with previous studies (cf. Hu & Scott (2007)).

The main contribution of this work is that we are able to model and to disentangle impacts of various determinants on the annuitization decision under the consumption frame. The results show that under the consumption frame most individuals are attracted by partial annuitization if their subjective life expectancy is not significantly shorter than the objective average life expectancy. However, due to the fact that most individuals tend to underestimate their subjective life expectancy, framing the annuity as protection for consumption seems only promising in combination with education programs and trustworthy information on longevity (which is a non-trivial task, cf., e.g., Teppa et al. (2015)). Furthermore, while the main driver of the higher annuitization rates is the consideration of consumption goals as reference points, we show, that other determinants can play a crucial role. We find in almost all cases that especially individuals with social security benefits below the minimal consumption goal prefer high annuitization rates. Moreover, already a low level of loss aversion increases annuitization rates significantly in most cases. The impact of loss aversion is particularly strong for individuals whose basic needs are not fully covered by social security. The overweighting of the small probabilities of reaching old ages (probability distortion) increases annuitization rates particularly for individuals with lower levels of initial liquid wealth and whose basic needs are covered by social security. Individuals equipped with a rather high subjective time preference prefer significantly lower annuitization rates – particularly, in case of lower initial liquid wealth.

In total, the main results are in line with experimental and empirical findings (cf. Brown et al. (2008), Goedde-Menke et al. (2014) or Brown et al. (2013)) and suggest that framing
can significantly increase voluntary annuitization. Due to the simplifying model assumptions and the complexity of the annuitization decision, we do not claim that the model captures all aspects of decision making in this context. However, the results show that if individuals (subconsciously) consider multiple reference points when making the annuitization decision – as suggested by several studies – then behavioral aspects can have very diverse impacts on the decision. This paper provides helpful insights on these impacts and their interactions from a theoretical point of view. By doing so, it points out promising directions for future research to improve our knowledge on actual decision making in the context of old-age provision.

A Appendix - Financial Market Model

We consider a filtered probability space \((\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})\) on a finite time horizon \([0, T]\), \(T \in \mathbb{N}\), under the real-world measure \(\mathbb{P}\) satisfying the usual conditions. \(\mathbb{F} = (\mathcal{F}_t)_{0 \leq t \leq T}\) with \(\sigma\)-algebra \(\mathcal{F}_t\) containing the available information at time \(t\). The financial market model is based on a stock process \(S\) described by a geometric Brownian motion and a short rate process \(r\) described by the Vasicek model (cf. Black & Scholes (1973) and Vasicek (1977)). The dynamics are given by

\[
dS_t = S_t \left( (r_t + \lambda_S)dt + \sigma_S dW_t^S \right) \quad \text{with} \quad S_0 > 0
dr_t = \kappa(\xi - r_t)dt + \sigma_r dW_t^r \quad \text{with} \quad r_0 \in \mathbb{R}, \quad \sigma_S, \kappa, \xi, \sigma_r > 0
dW_t^S dW_t^r = \eta \in [-1, 1], \quad \text{that is,} \quad W_t^S = \eta W_t^r + \sqrt{1 - \eta^2} W_t^* \quad \text{with} \quad W^* \text{ and } W^r \text{ independent Brownian motions}^{37}\text{ under } \mathbb{P}. \quad \text{Moreover, } \lambda_S > 0 \text{ denotes the constant equity risk premium. Furthermore, we assume that the considered balanced fund invests the constant fraction } \theta^S \text{ in a stock investment and the remaining part in a “rolling” bond investment (pension fund) based on zero bonds with term to maturity } T_B = 5 \text{ years.}^{38}
\]

The simulation of the financial market is done on a daily basis assuming 252 days per year.\(^{39}\)

The parameters have been chosen in accordance with the European money market and recent literature (cf. Graf et al. (2011) or Hieber et al. (2016)), that is, \(\sigma_S = 20\%, \quad \sigma_r = 1.5\% , \kappa = 30\%, \quad \eta = 15\%\). For the sake of simplicity, we consider a model without inflation and adjust the

\(^{37}\)Note that the random remaining lifetime is assumed to be independent of the financial market.

\(^{38}\)Note that we can derive closed formulas for the dynamics of the described processes and the price of zero bonds \(P(0, t)\) in the Vasicek model, cf. Brigo & Mercurio (2007).

\(^{39}\)We also apply a daily rebalancing between stock and bonds of the considered balanced funds.
the mean-reversion level by 2% and set it to $\xi = 1.05\%$. Moreover, the stock risk premium is assumed to be $\lambda_S = 3\%$ and due to the current low interest rate environment we use a negative initial short rate $r_0 = -0.33\%$.\footnote{Note that under the assumption that the price of the interest risk $\lambda_r$ equals $-23\%$, this corresponds to a mean-reversion level (with inflation) under the risk-neutral measure of $\xi^R = 4.2\%$ and is therefore also in line with the ultimate forward rate under Solvency II in 2017 (cf., e.g., Eiopa (2017)).}

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\footnote{The value of $r_0$ has been chosen to match the average value of the three-month EURIBOR rates of the first 6 months of 2017 (cf. Deutsche Bundesbank (2017)).}


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