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ORIGINAL RESEARCH PAPER

From intertemporal smoothing to intergenerational risk sharing: The effects of different return smoothing mechanisms in life insurance

Alexander Kling[1] · **Timon Kramer**[1] · **Jochen Ruß**[1]

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Abstract

In traditional life insurance, typically return smoothing mechanisms are used to reduce the volatility of policyholders' returns and provide risk sharing between policyholders. By analyzing two illustrative smoothing mechanisms, we demonstrate that different smoothing mechanisms may have different effects. We find that mechanisms that are purely based on average historical asset returns can significantly reduce pathwise volatility (intertemporal smoothing) but have hardly any impact on the standard deviation of terminal wealth. In contrast, mechanisms using buffers that are built up "in good years" in order to increase returns "in bad years" can—when properly designed—reduce the standard deviation of terminal wealth without reducing the ex ante expected return by means of intergenerational risk sharing.

We conclude that simple generic mechanisms that are often used in academic papers may not fully cover the effects resulting from return smoothing. Our results indicate that—when properly designed—intergenerational risk sharing mechanisms can improve risk-return profiles but at the price of increased complexity and potentially lower surrender values. A strong regulatory focus on simple products and sufficiently high surrender values might disincentivize products with intergenerational risk sharing despite their positive effects.

Keywords Return Smoothing · Intergenerational Risk Sharing · Participating Life Insurance · Retirement Savings

✉ Timon Kramer
t.kramer@ifa-ulm.de

Alexander Kling
a.kling@ifa-ulm.de

Jochen Ruß
j.russ@ifa-ulm.de

[1] Universität Ulm and Institut für Finanz- und Aktuarwissenschaften (ifa), Max-Born-Str. 12, 89081 Ulm, Germany

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1 Introduction

In traditional (i.e., not unit-linked) life insurance, it is common practice that many policyholders participate in the same pool of assets. As a consequence, an individual policyholder's return does not directly depend on the assets that were bought from her premiums, but rather on the return of the total asset pool which already provides some implicit risk sharing between policyholders. Moreover, certain mechanisms are typically explicitly implemented to further reduce the volatility of the individual policyholder's return. E.g., in Germany, buffers exist on both sides of the balance sheet[1] that can be used to reduce the volatility of returns of traditional life insurance contracts. In typical UK-with-profit products, often a return is credited to policyholders that depends on some average performance of the corresponding assets over the last years (cf. Haberman et al. [14]).

It is worth noting that these mechanisms can vary heavily from country to country. Moreover, whilst there exists regulation (limiting the size of certain buffers or demanding that money that is taken from such buffers must be credited to policyholders' accounts) often some management discretion remains.

As a consequence of the above, there exists a very large variety of different return smoothing/risk sharing mechanisms, some of which are rather complex since they may depend on complex accounting rules as well as management rules.

We argue in this paper, that the concrete design of a smoothing mechanism can have a significant effect on the resulting product. We illustrate our argument by analyzing two examples of return smoothing mechanisms based on two different approaches. Our main result is that a mechanism that is purely based on crediting average historical asset returns is very good at reducing pathwise volatility of a contract but has hardly any impact on the standard deviation of terminal wealth (and hence of the annualized return over the whole contract term). Hence, such mechanisms primarily lead to an intertemporal smoothing of a contract's annual returns but have only a very low impact on a contract's terminal wealth distribution. The reason is that such mechanisms result in very little intergenerational risk sharing (i.e., risk sharing between policyholders who enter or leave the system at different points in time). In contrast, a mechanism that uses buffers that are built up "in good years" and are used to increase returns "in bad years" results in a much higher degree of intergenerational risk sharing and hence has a larger effect on a contract's distribution of terminal wealth. This effect is even larger if buffers are not only used to increase returns in "bad years" but rather to increase the value of contracts that have a "poor value" at maturity.

This result has important consequences: First, simple generic mechanisms that are often used in academic papers may not fully cover the effects resulting from

[1] These buffers are hidden reserves on the asset side and a balance sheet item called "Rückstellung für Beitragsrückerstattung" on the liability side. Cf., e.g., Alexandrova et al. [1] for more information on the latter.

return smoothing in practice.[2] Second, if mechanisms prevailing in certain countries are modeled in detail, the results may not be applicable to other countries where other return smoothing regimes prevail.

Various smoothing mechanisms are considered in the existing literature on participating life insurance products. However, the design and effects of smoothing mechanisms, as well as the differences between different smoothing mechanisms are typically not in the focus of such papers. The two commonly observed approaches of smoothing mechanisms involve either crediting a return that is based on historical returns or utilizing buffer accounts in order to smoothen credited returns or a combination of both.

Rather simple mechanisms that use the average return over the last, e.g., three, years can be found in Haberman et al. [14], Korn and Wagner [19], and Ruß et al. [24]. Haberman et al. [14] develop valuation techniques for with-profit life insurance policies in the UK, where such smoothing designs are common. Korn and Wagner [19] apply this smoothing to participating insurance policies to classify state-subsidized private German pension products into risk-return classes. Ruß et al. [24] use insights from behavioral economics to explain the subjective attractiveness of smoothed products. They already allude to one key theme of our paper by stating that purely rule based smoothing algorithms that are based on historical returns predominantly reduce pathwise volatility with little effect on the distribution of terminal wealth. However, they only look at one smoothing mechanism and contrast their results to the effects of buffer-based mechanisms.

Smoothing using buffer accounts is considered in several papers. The concrete models differ heavily, depending on, among other things, the regulation of the country that is approximated and the level of detail (i.e., the degree of simplification) of the considered model. Grosen and Jørgensen [12] investigate the fair pricing of traditional life insurance contracts. The amount that is given from a buffer account to the policyholders' accounts and hence also the smoothed returns depend only on the difference between the company's reserve ratio and a target ratio. Hansen and Miltersen [15] extend this smoothing mechanism by adding annual fees and a terminal bonus which resembles the smoothing used by most Danish life insurance companies. Hieber et al. [16] expand the smoothing mechanism of Grosen and Jørgensen [12] to multiple cohorts with different minimum return guarantees. Kling et al. [18] and Ruß and Schelling [22] both model surplus distribution and return smoothing within the German regulatory framework. Kling et al. [18] measure the impact of interest rate guarantees of traditional insurance policies on the risk of an insurance company. Their rather simple smoothing mechanism distributes some target interest rate as long as the reserve ratio (i.e., a buffer account) remains within a given range and increases (decreases) the credited rate when the buffer account becomes too large (small). Ruß and Schelling [22] examine the subjective attractiveness of return smoothing for long-term investors. Again, they only look at one smoothing model that builds on the model from Burkhart et al. [5] which closely models the specifics of German regulation and hence uses both approaches, i.e. historical returns and buffer accounts.

[2] We will come back to this in Sect. 4.

In recent years, a notable shift from defined benefit (DB) to defined contribution (DC) pension plans could be observed in many countries (cf. OECD [20]). Many DC plans use a collective approach which is often called collective defined contribution (CDC) plan (cf. Boado-Penas et al. [4]). In such plans, policyholders participate in a collective investment using certain smoothing mechanisms as a substitute for guarantees. For example, in Dutch CDC pension plans, as modeled by Van Bilsen et al. [26], and in the proposed pension design of Boado-Penas et al. [4], a buffer account (also referred to as solidarity reserve) is funded by parts of the premiums and by excess returns in years with high market returns. These reserves are then used during periods of low market returns to maintain stability. Goecke [11] analyzes the effect of a similar collective buffer in collective pension saving products demonstrating that these products offer a comparative advantage over fully individualized mutual funds. There also exist CDC schemes without buffer accounts, such as the Royal Mail CDC scheme in the UK. Here, some degree of risk-sharing between generations can be achieved through annual pension increases that depend on predictions of future returns (cf. Armstrong et al. [2]). Guillen et al. [13] analyze an individual pension saving product marketed by a Danish life insurer. For this product yet another smoothing mechanism is implemented to reduce the risk resulting from the absence of a guarantee. Here, the return on the individual account is smoothed using a transparent mathematical formula depending on past returns. However, they conclude that smoothing is an accounting phenomenon, and with respect to fair market values, the contracts are as volatile as some direct financial market investment.

There is only very little literature comparing different smoothing mechanisms. For instance, Zemp [27] conducts a comparative analysis with regard to risk valuation on participating life insurance contracts. Cummins et al. [6] compare the probability distribution of the future payoff from contracts with guaranteed interest rates.

The focus of our paper is on the comparison of different smoothing mechanisms, for the sake of simplicity in contracts without interest rate guarantees. We focus on effects from the perspective of the policyholder. By using a pool of contracts starting at different points in time, not only intertemporal (between different calendar years within one contract) but also intergenerational (between different generations of policyholders) risk sharing can be studied. We consider two illustrative return smoothing mechanisms; first, a mechanism purely based on crediting average historical asset returns, similar to Korn and Wagner [19]; and second, an adaptation of the mechanism from Boado-Penas et al. [4] which utilizes a collective buffer account for return smoothing. These mechanisms are intentionally kept generic to serve as representative examples of their respective approaches. This allows for an illustration of the hugely different effects that different categories of smoothing mechanisms can have on pathwise volatility and the uncertainty of terminal wealth, respectively.

The remainder of this paper is organized as follows. In Sect. 2, we introduce the capital market model, the considered insurance contracts, as well as the asset and liability model of the insurance company based on two different return smoothing mechanisms. In Sect. 3, we analyze the two return smoothing mechanisms from the policyholders' perspective and—for the sake of comparison—a

product that simply credits unsmoothed returns. This is followed by some sensitivity analyses. Section 4 summarizes the main findings, derives implications for insurance companies as well as regulators and provides an outlook for future research.

2 The model

In this section, we present the general structure of our model starting with the capital market model followed by a description of an individual contract as well as the generations of contracts making up our considered book of business. We then present the model for the insurance company and the development of the policyholders' accounts under the different smoothing mechanisms.

2.1 The capital market model

For our capital market model, we consider a filtered probability space $(\Omega, \mathcal{A}, \mathbb{F}, \mathbb{P})$ on a finite time horizon $\mathbb{T}$ under the real-world measure $\mathbb{P}$ with natural filtration $\mathbb{F} = \left(\mathcal{F}_t\right)_{t\in\mathbb{T}} = \sigma\left(W_s, s \le t\right)$ generated by the $\mathbb{P}$ -Brownian motion W_t. Our model consists of one risky asset, a fund F. The fund process $F = (F_t)_{t\in\mathbb{T}}$ follows a geometric Brownian motion and satisfies the stochastic differential equation

$$dF_t = F_t(\mu dt + \sigma dW_t)$$

with constant drift μ and volatility σ.

This rather simple setup is sufficient since our focus is on the effects of return smoothing on the distribution of annual returns and terminal wealth (and not on the properties of annual returns and terminal wealth pre-smoothing). The inclusion of, e.g., stochastic interest rates would increase the complexity of our model without providing substantial additional insights on the effects of smoothing mechanisms on random distributions.

2.2 Individual insurance contract and book of business

In order to focus on the main effects, we model a pure savings contract and ignore any other aspects like death benefits and charges. The policyholder pays a single premium at the start of the contract. The insurer invests the single premium and credits interest to the policyholder's account according to the rules explained in Sects. 2.4 and 2.5. After T years, the contract matures and the policyholder's account is paid to the policyholder.

We then model a book of business based on different generations of policyholders buying the same contract but starting at different points in time. Each year, a new generation buys a new contract, again with a maturity of then T years. Our analysis focuses on the contract of one generation while the company is in a going-concern

Table 1 Balance sheet overview

Assets	Liabilities
$A_t^{-/+}$	$E_t^{-/+}$
	$B_t^{-/+}$
	$L_t^{-/+}$

state.[3] We denote the point in time where the observed generation enters their contract with $t = 0$ and assume that before that, already T generations have entered the company at times $t \in \{-T, \dots, -1\}$. The generation we focus on is thus labeled as generation 0. During the term of this contract, T additional generations enter the company at $t \in \{1, \dots, T\}$. Hence, the set of all generations is given by $\mathcal{H} = \{-T, \dots, T\}$ and the total time period modeled is $\mathbb{T} = [-T, T]$. The time period is divided into the starting period where the company builds up a block of business (i.e., the time period before generation 0 enters the company) and the observation period (i.e., the time period during the life cycle of the contract of generation 0).

2.3 The insurance company

The insurance company's accounts are considered at each annual point in time $t \in \mathcal{T} = \{-T, \dots, T\}$ before and after the payments from generations joining and generations leaving the company are settled (indicated with a superscript $-$ and $+$, respectively). We use notation with respect to the insurance company and the contract in the style of Døskeland and Nordahl [7]. The corresponding balance sheet is illustrated in Table 1.

For any $t \in \mathcal{T}$, all assets are combined in one asset account $A_t^{-/+}$. The company's assets are invested into the fund F. The company's equity at time t is labelled $E_t^{-/+}$, whereas $B_t^{-/+}$ is the collective buffer account that belongs to the policyholders on a collective basis and is designed to be completely redistributed to the policyholders in case of runoff. Details with respect to the buffer account depend on the specific smoothing mechanism (cf. Section 2.5). Each generation h has their specific individually allocated technical reserve account $L_t^{h,-/+}$ at time t. The sum of the individual technical reserve accounts is called the technical reserve $L_t^{-/+}$.

2.4 Development of the account values

This section illustrates the development of the account values in the observation period and in the starting period.

[3] In particular, we do not want our results to be confounded by effects that are caused if certain accounts systematically shrink towards zero in an insurance company without new business.

2.4.1 Observation period

At time $t = 0$, we assume that T generations have already entered the company during the years $\{-T, \dots, -1\}$ and that A_0^-, E_0^-, B_0^-, L_0^-, and $L_0^{h,-}$ for $h \in \{-T, \dots, -1\}$ are given.

In each year $t \in \{0, \dots, T\}$, the generation $h = t$ buys a contract by paying a single premium P to the company. Depending on the smoothing mechanism, a share $\alpha \in [0,1]$ of this single premium payment is credited to its individually allocated technical reserve account $L_t^{h,+}$ while $(1-\alpha) \cdot P$ is added to the collective buffer account B_t^-.

In addition, at any time point $t \in \{0, \dots, T\}$, the contract of generation $h = t - T$ matures. The terminal benefit I_t^h of this contract is given by the sum of its individually allocated technical reserve account $L_t^{h,-}$, and a terminal bonus payment TB_t^h from the collective buffer account, that is

$$I_t^h = L_t^{h,-} + TB_t^h.$$

The calculation of TB_t^h depends on the smoothing mechanism and is described in Sect. 2.5.

From time t^- to time t^+ for $t \in \{0, \dots, T\}$, the accounts are updated by premium payment of the joining generation and the benefit payment to the leaving generation as follows:

$$\begin{aligned}
A_t^+ &= A_t^- + P - L_t^{t-T,-} - TB_t^{t-T},\\
L_t^{t,+} &= P\alpha,\\
L_t^{h,+} &= L_t^{h,-}, \forall h \in \mathcal{H} \text{ with } t - T < h < t,\\
L_t^{h,+} &= 0, \forall h \in \mathcal{H} \text{ with } h \le t - T \text{ or } t < h,\\
L_t^+ &= L_t^- + L_t^{t,+} - L_t^{t-T,-},\\
B_t^+ &= B_t^- + P(1-\alpha) - TB_t^{t-T}.
\end{aligned} \tag{1}$$

The equity account remains unchanged, as—again for the sake of simplicity—we do not consider dividend payments to or payments from shareholders,

$$E_t^+ = E_t^-.$$

From time $t - 1^+$ to time t^- with $t \in \{1, \dots, T\}$ the accounts change as follows:
The asset account A_t^- evolves according to the underlying fund F, i.e.,

$$A_t^- = A_{t-1}^+ \cdot \frac{F_t}{F_{t-1}}$$

and we call $r_t^F = \ln(\frac{F_t}{F_{t-1}})$ the fund return.

The individually allocated technical reserve account at time t of an active generation h (i.e., $t - T \le h < t$) grows by the smoothed return $r_t^{h,smooth}$ credited to the policyholders' account in year t. Thus,

$$L_t^{h,-} = L_{t-1}^{h,+} \exp\left(r_t^{h,smooth}\right).$$

The calculation of $r_t^{h,smooth}$ depends on the smoothing mechanisms explained in Sect. 2.5.

For all non-active generations, i.e., generations that have not yet purchased a contract or have already left the company, the individually allocated technical reserve is zero, i.e.,

$$L_t^{h,-} = 0, \forall h.$$

The technical reserve is the sum of the individually allocated technical reserves, given by

$$L_t^- = \sum_{h=-T}^{T} L_t^{h,-}.$$

The evolution of the buffer account B_t^- depends on the smoothing mechanism. The calculation is shown in Sect. 2.5.

E_t^- is determined residually as

$$E_t^- = A_t^- - L_t^- - B_t^-.$$

Note that in the smoothing mechanisms explained below, negative equity can only occur for the product with return averaging and only temporarily. This is discussed in more detail in Sect. 2.5.2.

2.4.2 Starting period

The starting period begins at $t = -T$ with some start equity $E_{-T}^- = E_{Start}$, an empty collective buffer account $B_{-T}^- = 0$, and an empty technical reserve $L_{-T}^- = 0$ because no contract has been sold yet. Therefore, $A_{-T}^- = E_{Start}$.

For $t \in \{-T, \ldots, -1\}$, every year a new generation $h = t$ buys a contract. The accounts change from time t^- to t^+ as described in (1). But since no contract has expired yet, $L_t^{t-T,-} = 0$ and $PB_t^{t-T} = 0$. From time $t - 1^+$ to t^- with $t \in \{-T + 1, \ldots, 0\}$, the accounts evolve as stated above.

2.5 Smoothing mechanisms

In the remainder of the paper, we consider a contract without smoothing as well as two different illustrative smoothing mechanisms. As mentioned above, they are deliberately not based on existing smoothing mechanisms (e.g. based on the rules of a particular country). We rather use generic approaches that serve as a representative example of each category.

2.5.1 No smoothing

The single premium P of each generation h is completely allocated to the technical reserve account $L_t^{h,+}$, i.e., $\alpha = 1$. The collective buffer account $B_t^{-/+}$ is not used and hence always amounts to zero. Consequently, there is no terminal bonus payment from the collective buffer, i.e.,

$$B_t^- = 0, B_t^+ = 0, TB_t^h = 0 \; \forall t \in \mathcal{T} \, and \, h \in \mathcal{H}.$$

As no smoothing is applied, the smoothed return coincides with the return of the underlying fund F at time $t \in \{-\mathrm{T}+1, \ldots, \mathrm{T}\}$, i.e.,

$$r_t^{h,smooth} = r_t^F \; \forall h \in \mathcal{H} \; with \; t - T \leq h < t.$$

The payoff at maturity of the contract for generation $h \in \mathcal{H}$ at time $t = h + T$ is given by the individually allocated technical reserve account, i.e.,

$$I_t^h = L_t^{h,-} = P \cdot \frac{F_t}{F_{t-T}}.$$

The distribution of the overall return of the unsmoothed product of generation 0 conditional on $\mathcal{F}_{-T}$ is obviously given by (cf. Appendix)

$$\ln\left(\frac{I_T^0}{P}\right)|_{\mathcal{F}_{-T}} \sim N(T(\mu - \frac{\sigma^2}{2}), T\sigma^2), \; for \; T > 0.$$

2.5.2 Return averaging

The single premium P of each generation h is completely allocated to the technical reserve account $L_t^{h,+}$, i.e., $\alpha = 1$. The collective buffer account $B_t^{-/+}$ is not used and hence always amounts to zero. Consequently, there is no terminal bonus payment from the collective buffer, i.e.,

$$B_t^- = 0, B_t^+ = 0, TB_t^h = 0 \; \forall t \in \mathcal{T} \, and \, h \in \mathcal{H}.$$

For any time $t \in \mathcal{T}$, the smoothed return is given by the mean of the return of the underlying fund F over the last $n \in \mathbb{N}$ years and is the same for all active generations h, i.e.,

$$r_t^{h,smooth} = \frac{1}{n} \sum_{i=0}^{n-1} r_{t-i}^F, \forall h \in \mathcal{H} \text{ with } t - T \leq h < t.$$

The payoff at maturity of the contract for generation $h \in \mathcal{H}$ at time $t = h + T$ is given by the individually allocated technical reserve account, i.e.,

$$I_t^h = L_t^{h,-}.$$

We show in the Appendix that the distribution of the overall return of generation 0 conditional on $\mathcal{F}_{-T}$ is given by

$$\ln\left(\frac{I_T^0}{P}\right)|_{\mathcal{F}_{-T}} \sim N\left(T\left(\mu - \frac{\sigma^2}{2}\right), \left(T - \frac{1}{3}\left(n - \frac{1}{n}\right)\right)\sigma^2\right), \text{ for } T \geq n-1.$$

Hence, the return averaging changes only the variance of the overall return. However, since the payoff has a log-normal distribution the expectation is reduced.

In practice, such a smoothing mechanism would require the use of some buffer account to cover the difference between the unsmoothed and the smoothed value. For the sake of simplicity, we follow the methodology of Korn and Wagner [19] and do not explicitly model such a buffer account as it adds complexity without providing more insight into the effects of return averaging. Instead, we allow that equity (which is used to cover the difference between the market return and the smoothed returns) can become negative. Note that any shortfall would only be temporary and recovered in the following years. This is because of our assumed book of business in the going-concern state and that the expected payoff on premiums invested in the fund exceeds that from return averaging at maturity.[4]

2.5.3 Collective buffer smoothing

The policyholder's initial payment P is allocated between the individually allocated technical reserve account $L_t^{h,-}$ and the collective buffer account. We assume that the share $\alpha \in [0,1]$ is allocated to the individual technical reserve and that α is the same for all generations.

At time $t \in \{-T+1, \ldots, T\}$, the smoothed return $r_t^{h,smooth}$ for generation $h \in \mathcal{H}$ with $t - T \leq h < t$ is calculated based on the following idea: We first assume that the return on the fund r_t^F is completely credited to the individually allocated technical reserve account. Then, for each contract generation, we look at the annualized return from the start of the contract until t. If this return lies within a predefined target range, no payments are made to or from the buffer account. If, however, this return lies above (below) the target range, a share $p \in [0,1]$ ($q \in [0,1]$) of the excess is paid to (taken from) the buffer. Note that in our numerical analyses in Sect. 3, the lower bound of the target range depends on the contract term; in particular, we

[4] In practice, surrender rates of existing business and demand for new contracts might depend on the difference between unsmoothed and smoothed returns, since policyholders might anticipate future credited returns based on past returns and exploit favorable or avoid unfavorable future smoothing outcomes. This would require the consideration of appropriate surrender charges (sometimes referred to as market value adjustments) and the use of different smoothed returns in the first $n-1$ years after entering, respectively. Both are common practices in some markets. We have additionally analyzed an alternative way to calculate smoothed returns in the first years: We have used the expected fund return ($\mu - \sigma/2$) for the years before the policyholder entered the contract in the calculation of the average return. As this had almost no effect on the results, we refrain from showing these results, and only show results for the simpler model explained in this section.

use a higher lower bound for contracts that are close to maturity. This ensures that contracts that will mature soon and have achieved a bad return so far will profit more from the collective buffer than others.

This intuitive idea is formalized as follows: Let $l_\tau, u_\tau \in [-1,1]$ denote the lower and upper bound of the target range for the contract year $\tau = \mathrm{t} - \mathrm{h} \in \{1, \ldots, T\}$. Then, the amount $P_t^h = \xi_t IP_t^h 1_{IP_t^h > 0} + IP_t^h 1_{IP_t^h < 0}$ is taken from or paid to the collective buffer of generation $h \in \mathcal{H}$ with $t - T \leq h < t$ based on the intended payment

$$IP_t^h = \begin{cases} p\left(P\alpha \exp\left(u_\tau\right)^\tau - L_{t-1}^{h,+} \exp\left(r_t^F\right)\right), & \ln\left(\frac{L_{t-1}^{h,+} \exp\left(r_t^F\right)}{P\alpha}\right)^{\frac{1}{\tau}} > u_\tau \\ 0, & \ln\left(\frac{L_{t-1}^{h,+} \exp\left(r_t^F\right)}{P\alpha}\right)^{\frac{1}{\tau}} \in \left[l_\tau, u_\tau\right] \\ q\left(P\alpha \exp\left(l_\tau\right)^\tau - L_{t-1}^{h,+} \exp\left(r_t^F\right)\right), & \ln\left(\frac{L_{t-1}^{h,+} \exp\left(r_t^F\right)}{P\alpha}\right)^{\frac{1}{\tau}} < l_\tau. \end{cases}$$

as well as a factor $\xi_t \in [0,1]$ ensuring that intended payments from the buffer are only made in full if there is enough money in the buffer:

$$\xi_t = \min\left(\frac{B_{t-1}^+ \exp\left(r_t^F\right) - \sum_{h \in \mathcal{H}, t < h \leq t+T, IP_t^h < 0} IP_t^h}{\sum_{h \in \mathcal{H}, t < h \leq t+T, IP_t^h > 0} IP_t^h}, 1\right).$$

Note that intended payments to the buffer are always made in full.

The smoothed return at time t of generation h is then given by

$$r_t^{h,smooth} = \ln\left(\frac{L_{t-1}^{h,+} \exp\left(r_t^F\right) + P_t^h}{L_{t-1}^{h,+}}\right).$$

The collective buffer changes due to the return from the underlying fund and the payments from and to the generation to

$$B_{\mathrm{t}}^- = B_{t-1}^+ \exp\left(r_t^F\right) - \sum_{\substack{h \in H \\ t < h \leq t+T}} P_t^h.$$

In addition, at contract expiry the leaving generation h at time $t = h + T$ receives a terminal bonus from the collective buffer account. The amount of the terminal bonus depends on the share of the individually allocated technical reserve $L_t^{h,-}$ in the total technical reserve account L_t^- and a terminal bonus parameter $\vartheta \in [0,1]$, i.e.,

$$TB_t^h = \frac{L_t^{h,-}}{L_t^-} \vartheta B_t^-.$$

The payoff at maturity of the contract for generation $h \in \mathcal{H}$ at time $t = h + T$ is the sum of the individually allocated technical reserve account and the terminal bonus, i.e.,

$$I_t^h = L_t^{h,-} + TB_t^h.$$

3 Analyses and results

In this section, we analyze different products from the policyholders' perspective. We consider three different smoothing mechanisms: one utilizing return averaging, another employing the collective buffer smoothing, and, for the sake of comparison, a product crediting unsmoothed returns.

3.1 Parameters in the base case

We perform a Monte Carlo Simulation using 10,000 scenarios. Our focus is purely on the effect of the smoothing mechanism on the contract of a generation of contracts once the system has reached a somewhat steady state, i.e. systematic effects that may result from the building up of the insurer's portfolio are no longer relevant. We denote the observed generation by "generation $h = 0$". We observe these contracts during the observation period and want to avoid confounding effects that might result from different starting conditions in each sample path. Hence, we want all our 10,000 scenarios of the observation period to have the same starting values.[5] For this, we first simulate the starting period with 10,000 Monte Carlo simulation paths. Then, we take the average of the account values A_0^-, E_0^-, B_0^-, L_0^-, and $L_0^{h,-}$ for $h \in \{-T, \dots, 1\}$ as starting values for the observation period. The observation period is then simulated based on those starting values.[6] The same set of parameters is used in the starting and the observation period.

The contract maturity is set to $T = 20$ years, the single premium to $P = 10{,}000$. The starting equity is $E_{Start} = 10{,}000$. For the underlying fund F, we assume a drift of $\mu = 4\%$ and a volatility of $\sigma = 10\%$ representing a moderate risk-return-profile of the underlying asset pool.

For the product with return averaging, we use smoothing over three years, i.e., $n = 3$ as it is done in Korn and Wagner [19]. Note that we use the market returns r_t for $t \in \{-2, -1\}$ simulated in each scenario in the observation period to calculate the smoothed returns in the first two years.[7]

The parameters used for the collective buffer smoothing are shown in Table 2. The parameters were chosen to ensure that the CBS mechanism leads to intergenerational

[5] Subsequent generations will not have the same starting values, especially the size of the buffer will vary. This is inherent in such products.

[6] We analyze generation 0 in a going-concern state without interfering effects of different starting conditions. Since results for later generations are similar, as mentioned in Sect. 3.5, we focus on generation h = 0 only.

[7] The values for the product with no smoothing and the product with return averaging could be calculated directly. However, for the sake of consistency we derive all values using simulations.

Table 2 Parameters for the product with collective buffer smoothing

l_τ $\tau \in \{1,\dots,18\}$	l_{19}	l_{20}	u_τ $\tau \in \{1,\dots,20\}$	α	ϑ	p	q
0.03	0.035	0.04	0.065	0.9	0.703	0.5	1

risk sharing. The lower and upper bounds were selected reasonable given drift and volatility of the underlying funds.

Note that the terminal bonus parameter ϑ for the payment from the collective buffer account has been chosen such that the buffer account B_0^+ at the beginning of the contract of generation 0 coincides with the expected buffer account B_T^+ after the contract of generation 0 expired. Hence, ex ante, the observed contract does neither systematically profit from nor systematically suffer from the existence of other generations.

3.2 Annual returns and pathwise volatility

Figure 1 shows different percentiles (5, 10, 25, 75, 90 and 95%) as well as median and expected value of the annual return for a contract of generation 0 in year $\tau \in \{1,\dots.,20\}$ for the three considered smoothing mechanisms. The bars without

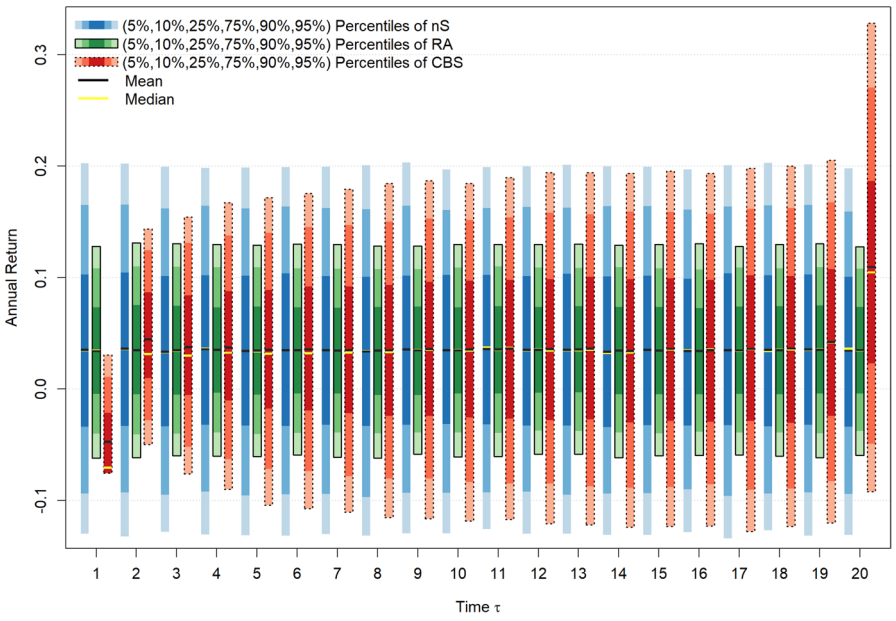

Fig. 1 Distributions of the annual returns of the different smoothing mechanisms: no smoothing (nS), return averaging (RA), collective buffer smoothing (CBS)

a border (on the left of each group of three) show the percentiles of the product with no smoothing (nS), the bars with a solid border (in the middle of each group of three) the percentiles of product with return averaging (RA), and the bars with a dotted border (on the right of each group of three) the percentiles of the product with collective buffer smoothing (CBS).

For the unsmoothed product and the product with return averaging, we observe that the annual return distribution is very stable over time. The effect of the return averaging on the annual returns is fairly strong: The distance between an upper and the corresponding lower percentile is reduced by roughly 40% for each percentile shown. E.g., the distance of the 95th percentile and the 5th percentile is 33 percentage points for the unsmoothed product and 19 percentage points for the product with return averaging. The standard deviation of the annual return in contract year 10, for example, is 9.9% for the unsmoothed product and 5.8% for the product with return averaging. A look on pathwise volatility confirms this: The pathwise volatility of the unsmoothed product is almost twice as high (10.3%) as the pathwise volatility of the product with return averaging (5.6%). At the same time, the expected returns of the two products coincide. This means that return averaging heavily reduces the uncertainty of annual returns without reducing the expected return.

The annual return distribution of the product with collective buffer smoothing is structurally different. First, the return distribution changes heavily over time. In particular in the first year, annual returns are comparatively low since a part of the premium is invested into the collective buffer. In addition, the collective buffer smoothing leads to a clear reduction of volatility in the first years. Very good and very bad performances directly are smoothed by use of the collective buffer. This effect gets smaller over time and the distribution of annual returns more and more approaches the distribution of annual returns of the unsmoothed product. During the second half of the contract, these two return distributions are almost identical. The standard deviation of the annual return in contract year 10, for example, is 9.9% for the unsmoothed product and 9.2% for the product with collective buffer smoothing. Finally, the return in the last contract year comes with a significant upside potential since in the last year the lower bound of the target range is increased and the terminal bonus is credited to the policyholders allowing for a noticeably high annual return in certain scenarios. As a consequence of these different effects, the pathwise volatility over the whole contract term of the CBS (9.6%) is only slightly lower than for the unsmoothed product (10.3%).

In summary, an isolated look at the annual returns suggests that return averaging is extremely effective in reducing risk without reducing expected return: We observe a significant reduction in annual risk, a significant reduction of pathwise volatility and hardly any effect on the expected return. Collective buffer smoothing in contracts appears to have a much smaller and somewhat erratic effect on the returns.

In a next step, we challenge this first impression by a closer look on the probability distributions of the development of the contract value and the corresponding annualized returns.

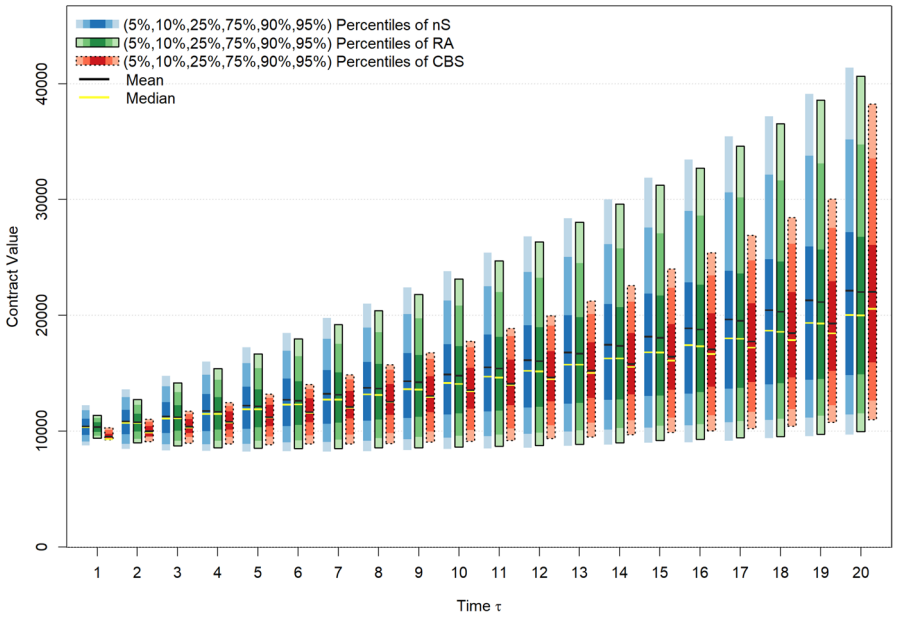

Fig. 2 Distributions of the annual contract values of the different smoothing mechanisms: no smoothing (nS), return averaging (RA), collective buffer smoothing (CBS)

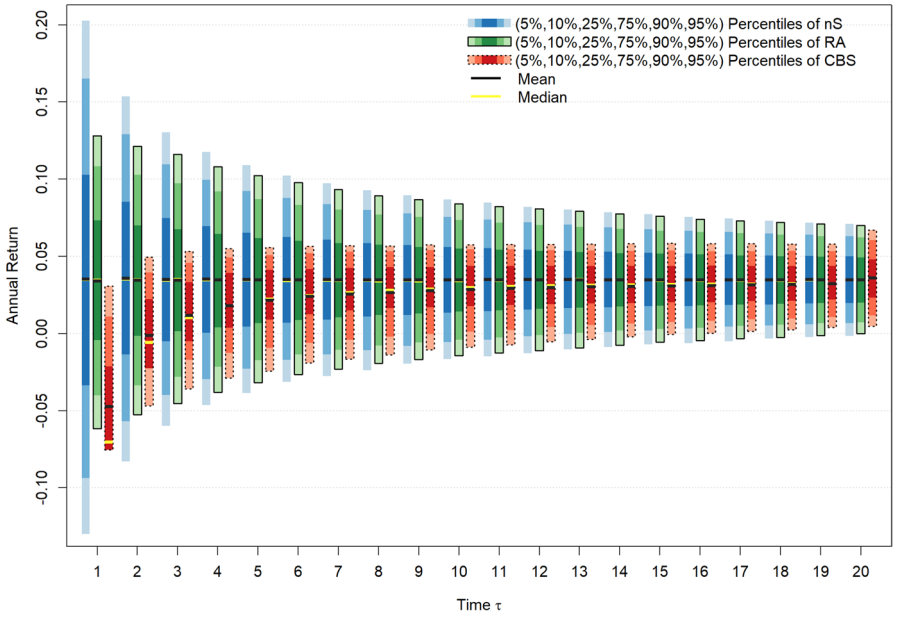

Fig. 3 Distributions of annualized returns of the different smoothing mechanisms: no smoothing (nS), return averaging (RA), collective buffer smoothing (CBS)

3.3 Contract value and annualized return

Figure 2 shows different percentiles (5, 10, 25, 75, 90 and 95%) as well as median and expected value of the contract value for a contract of generation 0 in year $\tau \in \{1, \ldots, 20\}$ for the three considered smoothing mechanisms. The bars without a border show the percentiles of the unsmoothed product, the bars with a solid border the percentiles of product with return averaging, and the bars with a dotted border the percentiles of the product with the collective buffer smoothing.

Figure 3 provides the same information by showing different percentiles (5, 10, 25, 75, 90 and 95%) as well as median and expected value of the annualized return up to time t for a contract of generation 0 in year $\tau \in \{1, \ldots, 20\}$ for the three considered smoothing mechanisms. The bars without a border show the percentiles of the unsmoothed product, the bars with a solid border the percentiles of product with return averaging, and the bars with a dotted border the percentiles of the product with the collective buffer smoothing.

A look on the first contract years seems to confirm the impression from the analysis of annual returns. The product with return averaging seems to be very effective in terms of reducing the risk without reducing expected performance. Over the time of the contract, however, the distribution of the contract value (and the corresponding annualized return) more and more converges to the respective distribution of the unsmoothed product. At maturity, both probability distributions are almost identical meaning that return averaging has next to no effect on the risk and return profile of the whole contract, i.e., the probability distribution of terminal wealth.

In contrast, the collective buffer smoothing results in structurally different effects. While the initial payment to the buffer account leads to significantly lower contract values, particularly in the first contract years, the terminal bonus payment compensates for this leading to a similar expected value and median at maturity. At the same time, the risk of the product (with respect to the distribution of the terminal value) is considerably reduced: The distribution of annualized returns is narrower than for both other products, the standard deviation of the maturity value, e.g., is given by 8,338 for the collective buffer smoothing while it is above 10,000 for both other products. Therefore, looking at the probability distribution at maturity (which is the relevant figure if the contract is used for retirement saving), the collective buffer smoothing turns out to be the most effective: It reduces uncertainty of the terminal value without reducing expected return over the whole contract term.

We investigate this a little further by having a closer look into the mutual dependencies between the annualized returns and annual returns in single years between the different smoothing mechanisms.

3.4 Pathwise comparison of smoothing mechanisms

Figure 4 compares the annualized returns over the whole contract term (left) and annual returns in single years (right) between the unsmoothed product and the different smoothing mechanisms for all 10,000 scenarios in scatterplots. The line in each case represents the identity function. In the two figures on top (bottom), the

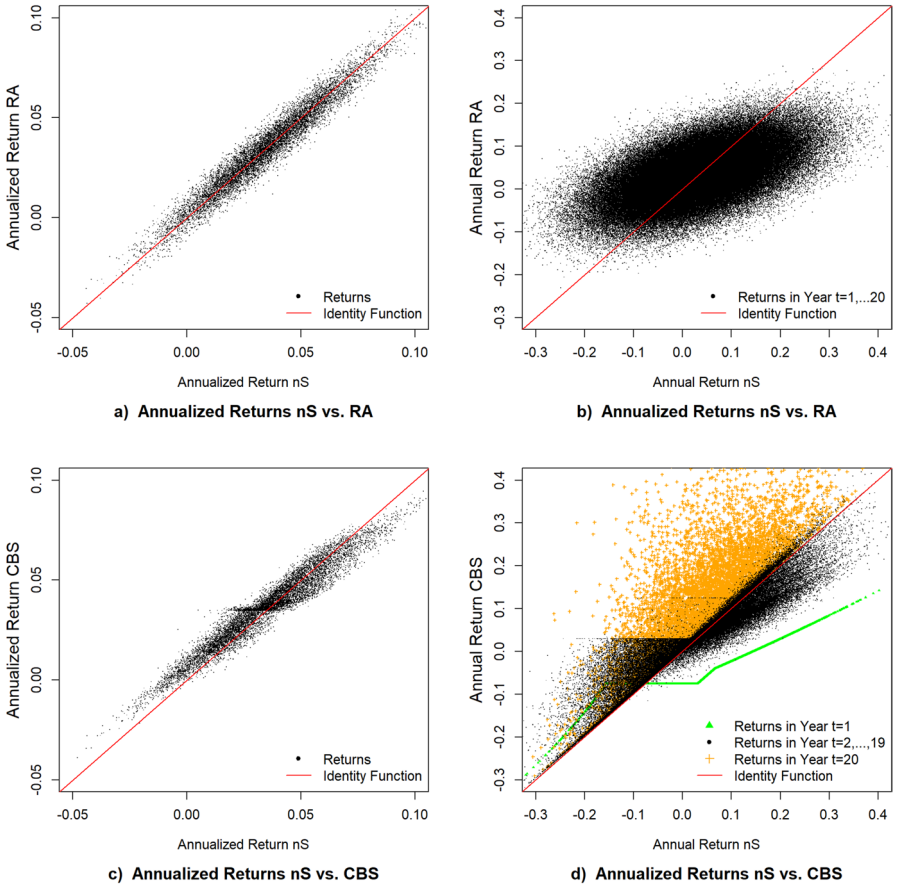

Fig. 4 Mutual dependencies between the annualized returns over the whole contract term (left) and annual returns in single years (right) between the different smoothing mechanisms: no smoothing (nS), return averaging (RA), collective buffer smoothing (CBS)

returns of the product with unsmoothed returns are compared with the returns of the product with return averaging (with the returns of the product with collective buffer smoothing). Note that in the left figures the return over the whole contract term in each of the 10,000 scenarios is one dot, resulting in 10,000 dots, whereas in the right figures, the annual return in each of the 20 years in each of the 10,000 scenarios is shown leading to 200,000 dots. In the bottom right figure, the triangles represent the first contract year $\tau = 1$, the crosses represent the last contract year $\tau = 20$, and the dots represent the remaining contract years.

The results of the scatter plot provide further insights into the effects of the different smoothing mechanisms. We start with a comparison of return averaging and unsmoothed returns (upper part of Fig. 4). The annualized return (left chart) of the smoothed product is highly correlated with the respective unsmoothed product. There is no systematic reduction of risk visible for the smoothed annualized

Table 3 Sensitivity of the share of individual premium allocation for the product with collective buffer smoothing

l_τ $\tau \in \{1, \dots, 18\}$	l_{19}	l_{20}	u_τ $\tau \in \{1, \dots, 20\}$	α	ϑ	p	q
0.03	0.035	0.04	0.065	1.0	0.79	0.5	1
0.03	0.035	0.04	0.065	0.9	0.703	0.5	1
0.03	0.035	0.04	0.065	0.8	0.7215	0.5	1

returns. Annual returns in single years (right chart), however, come with a systematic risk reduction. Smoothed returns tend to be below the identity function for high unsmoothed annual returns and above the identity function for low unsmoothed annual returns. We still see a positive correlation between unsmoothed and smoothed returns, but this correlation is much smaller than for the annualized returns. In particular, the plot scatters much more from left to right (uncertainty of the annual returns of the unsmoothed product) than from the bottom to the top (uncertainty of the smoothed product). This leads to the results observed above.

Comparing the collective buffer smoothing with unsmoothed returns (lower part of Fig. 4) shows an entirely different pattern. The annualized return of the smoothed product (left chart) still shows a significant correlation with the unsmoothed product. However, a systematic reduction of risk can be observed: In scenarios where the unsmoothed product comes with a high annualized return, the smoothed product typically comes with a lower return and vice versa. Hence, scenarios with high (low) annualized returns sponsor (benefit from) the collective buffer. This is the effect of intergenerational risk sharing since it can transfer money between different policyholders of such products. Finally, a look at annual returns in single years (right chart) shows a structurally different pattern for different years. While we observe the effect of a systematic payment into the buffer and a functional relationship between unsmoothed and smoothed returns in the first year, the terminal bonus payment and the effect of an increase in the corridor in the last year leads to systematic returns above the respective unsmoothed returns in the last year. In all other years, a very clear structure of the collective buffer smoothing can be observed: While positive returns in single years are reduced by payments into the buffer, negative single returns are systematically subsidized.

3.5 Sensitivity analyses

In this section, we provide sensitivity analyses for the product with collective buffer smoothing with respect to the share α allocated to the individual policyholder account and with respect to the size of the collective buffer at $t = 0$. We further performed sensitivity calculations with respect to the assumptions for

the capital market parameters μ and σ, the time to maturity T, and the number of years for the return averaging. The latter did not show any additional insights. Therefore, we refrain from showing the corresponding results.

In addition, we extended the observation period and the number of generations to be able to analyze subsequent generations ($h \in \{5, 10, 20\}$) while the company is still in a going-concern state. These generations did not have the same starting values. However, the results were very similar to generation $h = 0$ showing the model is in a steady state.

3.5.1 Contribution to the buffer account

In the first step, we change the parameter α which defines the share of the premium payment invested into the individual mathematical reserve account of the CBS. The remaining part $(1 - \alpha)$ is invested into the buffer account.

We increase α to 1.0 and decrease it to 0.8. In both cases, we adjust the terminal bonus parameter ϑ such that $B_0^+ = E(B_{20}^+)$. The remaining parameters are set as in the base case. Note that it is not obvious how a change in the initial payment into the collective buffer $(1 - \alpha)$ affects ϑ since changes in the parameters are also applied for previous generations and thus affect the size of the initial buffer at $t = 0$. The first and last line of Table 3 show the parameters used for both sensitivities (the second line shows the base case for comparison).

Figure 5 shows different percentiles (5%, 10, 25, 75, 90 and 95%) as well as median and expected value of the contract value for a contract of generation 0 in

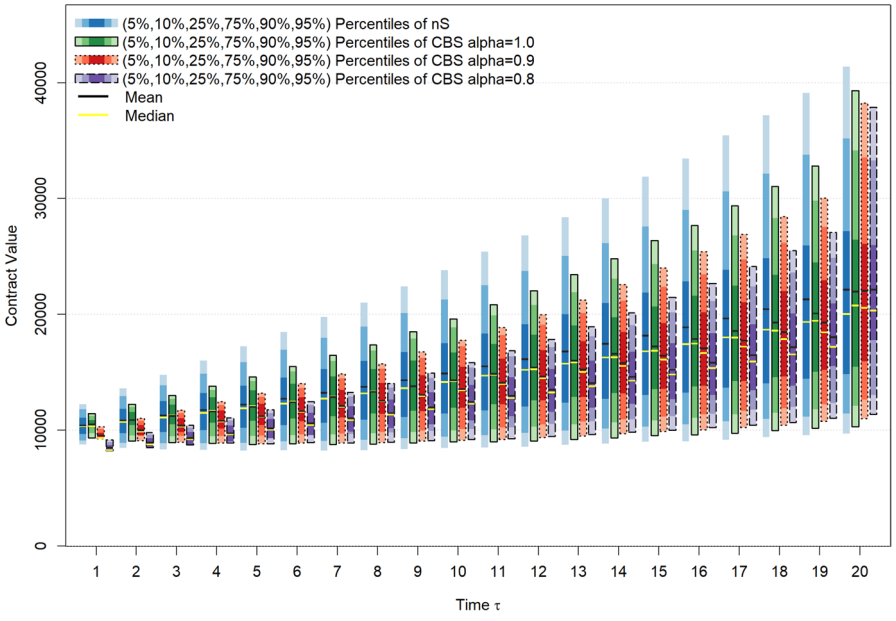

Fig. 5 Distributions of the annual contract values of the different smoothing mechanisms and different vaules of α: no smoothing (nS), collective buffer smoothing (CBS) with $\alpha = 0.8$, $\alpha = 0.9$, $\alpha = 1.0$

year $\tau \in \{1, \ldots, 20\}$ for the considered smoothing mechanisms. The bars without a border (on the left of each group of four) show the percentiles of the product with no smoothing, the bars with a solid border (second from the left in each group of four) for the product with $\alpha = 1$, the bars with a dotted border (second from the right in each group of four) the base case with $\alpha = 0.9$, and the bars with a dashed border (on the right of each group of four) the case with $\alpha = 0.8$.

The probability distribution of the contract value in the first year, of course, is highly affected by the initial allocation into the individual policyholder account. If only a share of $\alpha = 0.8$ is allocated to the individual policyholder account, the contract value after 1 year shows a fairly low expected value and only very little volatility. This is a clear result of a high allocation into the collective buffer account. If the premium is fully invested into the individual account ($\alpha = 1$), on average we observe a positive account development already in the first year of the contract.

Our interest lies more in the development of the account value over time, and particularly in the probability distribution of the contract value at maturity. The lower the investment α into the individual reserve account, the more effective the collective buffer smoothing proves to be at maturity. Also, for a fairly high initial investment of 20% into the collective buffer account ($\alpha = 0.8$), the expected account value is practically identical to the expected account value of the unsmoothed product. The standard deviation of the maturity value, however, is reduced, and the whole probability distribution shows less variation, i.e., upper percentiles are smaller and lower percentiles are larger.

3.5.2 Size of the initial buffer account

A similar observation can be made for a sensitivity of the amount in the initial buffer account B_0^-. For the base case, the initial buffer at the end of the starting period (at $t = 0$) is given by 10.3% of the mathematical reserve L_0^-. We performed two sensitivity analyses where we doubled or cut in half the initial buffer account B_0^- at the beginning of the observation period. In both cases, the same parameters as in the base case are used. This means that, in contrast to the previous sensitivity, we do not adjust the terminal bonus parameter ϑ. Therefore, $B_0^+ = E(B_{20}^+)$ only holds for the base case, but not for the sensitivities with decreased or increased buffer.

Figure 6 shows different percentiles (5, 10, 25, 75, 90 and 95%) as well as median and expected value of the contract value for each contract of generation 0 in year $\tau \in \{1, \ldots, 20\}$ for the considered smoothing mechanisms. The bars without a border show the percentiles of the product with no smoothing, the bars with a solid border for the product with decreased buffer, the bars with a dotted border the base case, and the bars with a dashed border the product with increased buffer.

We observe that with increasing buffer, all percentiles and the expected account value are strictly increasing throughout the whole time period. The effects are particularly pronounced at maturity $\tau = 20$. If the initial buffer is increased from 10.3 to 20.6%, the average maturity benefit increases by roughly 6%. This corresponds to an increase in the average return of roughly 30 bp. If the initial buffer is cut in half (reduced from 10.3% to 5.15%), the average maturity benefit decreases by roughly

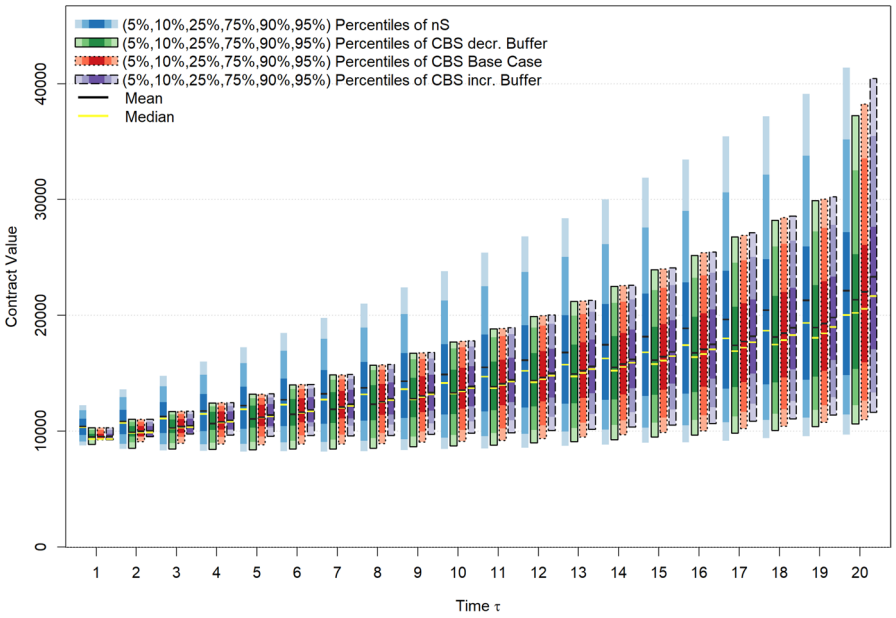

Fig. 6 Distributions of the annual contract value of the different smoothing mechanisms for different initial buffers: no smoothing (nS), collective buffer smoothing (CBS) with decreased buffer, base case buffer, increased buffer

3% corresponding to a decrease in the average return by roughly 15 bp. The effects are slightly less pronounced on the upper percentiles and stronger on the lower percentiles.

The result is quite intuitive. A higher initial buffer increases the performance potential of the CBS product, i.e., policyholders on average profit from higher buffers. The effect is especially pronounced for adverse capital market scenarios. This means that a higher buffer not only increases the return potential of the product, but also further reduces the risk of the product from a policyholder's perspective.

In addition to the sensitivity on the initial buffer shown, we conducted sensitivity analyses with decreased and increased buffer including a repricing after the change in buffer, i.e., we adjusted the terminal bonus parameter ϑ such that $B_0^+ = E(B_{20}^+)$. In this case, the results are as expected: We observe almost no effect on expected account value but the reduction of risk is higher for larger initial buffers.

4 Summary and outlook

In this paper, we have analyzed different return smoothing mechanisms. We found that mechanisms that are primarily based on crediting average historical returns of the underlying assets (i.e., intertemporal smoothing) can significantly reduce the

standard deviation of annual returns and hence pathwise volatility without affecting the expected return. The standard deviation of terminal wealth (and hence of the annualized return over the whole contract term) is however hardly reduced. On the other hand, smoothing mechanisms that use some collective buffer account can be designed in a way that money is transferred from "good states" to "bad states" not only intertemporally but also intergenerationally, i.e., between different (generations of) policyholders. This leads to an effect that we call intergenerational risk sharing. Such mechanisms—when properly designed—can reduce the standard deviation of terminal wealth (and hence of the annualized return over the whole contract term) again without reducing ex ante expected return.

Our findings have important consequences for academics, practitioners, and regulators. First and foremost, all parties need to be aware that results about smoothed products that were derived using some generic smoothing model may not be applicable to all smoothed products. If the results were achieved using a generic model that results primarily in intertemporal smoothing, these results may not or only to a small extent be applicable to products using a smoothing algorithm that also comes with intergenerational risk sharing. To give just one example: Surplus distribution mechanisms under German regulation leads to smoothing effects that have elements of both, intertemporal smoothing and intergenerational risk sharing. Government subsidized old age provision products in Germany must be classified in risk/return-classes using a pre-described model (similar to the model used in Korn and Wagner [19]). The classification only looks at risk/return characteristics of terminal wealth. The algorithm implemented in the model is a pure intertemporal smoothing model without intergenerational risk sharing. Hence, effects of smoothing on the distribution of terminal wealth are probably significantly underestimated.

It also seems worth noting that appropriately designed intergenerational risk sharing can reduce the relevant risk for retirement saving, i.e., the uncertainty of terminal wealth without reducing expected return. This essentially results in more efficient retirement savings products, whereas intertemporal smoothing primarily increases the subjective attractiveness (cf. Ruß et al. [24]). However, this efficiency comes at the price of high complexity and lack of transparency. This is particularly relevant in the context of recent and upcoming European regulation for retirement savings products. On the one hand, the regulator puts a high focus on "value for money". Appropriately designed intergenerational risk sharing mechanisms can increase risk adjusted return, which can be seen as a positive contribution to value for money. In contrast, value for money considerations of the European Commission, the European Insurance and Occupational Pensions Authority (EIOPA), and some local regulators put a strong focus on a sufficient level of surrender values for customers who surrender their contracts early (cf. BaFin [3]; EIOPA [8–10]; IVASS [17]). When parts of the premiums are used to build up buffer accounts, intergenerational risk sharing might decrease surrender values in early years which can be seen as a negative contribution to value form money. Moreover, complex products are seen particularly critical by the regulator resulting in a higher degree of product

governance requirements for manufacturers under the insurance distribution directive (IDD, cf. Directive, EU, 2016/97).[8] In particular, measures to be taken under the product approval process are higher for products with a higher level of complexity (cf. Article 4 of Delegated Regulation, EU, 2017/2358).[9]

In summary, this regulation puts pressure on the product design of insurance-based investment products and long term savings products in general and limits flexibility of the products. A strong focus on the surrender value disincentivizes products with intergenerational risk sharing even though such products would lead to a more efficient risk-return-profile at maturity. This should be considered by regulators when value for money benchmarks are discussed.

In future research it would be interesting to analyze aspects of consumer demand: Which smoothing mechanism result in high (objective) utility? Which mechanisms result in high subjective attractiveness for consumers that behave according to theories of behavioral economics, e.g., Cumulative Prospect Theory (cf. Tversky and Kahneman [25]) or versions of it that have recently been proposed for long term savings processes (cf. Ruß and Schelling [21]). It would also be interesting to see if smoothed products that contain elements of intertemporal smoothing as well as intergenerational risk sharing can be used to find "good compromises" in the sense of Ruß et al. [23], i.e., products that offer a high (albeit not the highest possible) objective utility while at the same time offering a high (albeit not the highest possible) subjective attractiveness to consumers.

Appendix: distribution of the overall return for nS and RA

In the model from Sect. 2.1, the distribution of the annual fund return is given by

$$r_t^F \sim N\left(\mu - \frac{\sigma^2}{2}, \sigma^2\right) i.i.d. \forall t \in \{T-1, \ldots, T\}.$$

For the overall return of the unsmoothed product of generation 0 conditional on $\mathcal{F}_{-T}$ we get

$$\ln\left(\frac{I_T^h}{P}\right) = r_{h+1}^F + \cdots + r_{h+T}^F \sim N\left(T\left(\mu - \frac{\sigma^2}{2}\right), T\sigma^2\right), \text{ for } T > 0.$$

For the product with return averaging of generation 0 conditional on $\mathcal{F}_{-T}$ we get (for $T \geq n-1$)

[8] https://eur-lex.europa.eu/legal-content/en/TXT/?uri=CELEX%3A32016L0097

[9] https://eur-lex.europa.eu/legal-content/EN/TXT/HTML/?uri=CELEX:32017R2358

$$\ln\left(\frac{I_T^h}{P}\right) = r_{h+1}^{h,smooth} + \ldots + r_{h+T}^{h,smooth} = \frac{1}{n}\left(\sum_{i=0}^{n-1} r_{h+1-i}^F + \ldots + \sum_{i=0}^{n-1} r_{h+T-i}^F\right)$$
$$= \frac{1}{n}(1\, r_{h+1-(n-1)}^F + 2r_{h+1-(n-2)}^F + \ldots + (n-1)r_h^F) + \sum_{i=1}^{T-(n-1)} r_{h+i}^F$$
$$+\frac{1}{n}\left((n-1)r_{h+T-(n-1)+1}^F + ... + 1r_{h+T}^F\right).$$

Since, $\ln\left(\frac{I_T^h}{P}\right)$ is the sum of independent normally distributed random variables, it is also normally distributed with expected value

$$E\left(\ln\left(\frac{I_T^h}{P}\right)\right) = \left(\mu - \frac{\sigma^2}{2}\right)\left(\frac{2}{n}(1+2+\cdots+(n-1)) + T - (n-1)\right) = \left(\mu - \frac{\sigma^2}{2}\right)T$$

and variance

$$Var\left(\ln\left(\frac{I_T^h}{P}\right)\right) = \sigma^2\frac{1}{n^2}\left(1^2 + \ldots + (n-1)^2 + n^2(T-(n-1)) + (n-1)^2 + \ldots + 1^2\right)$$
$$= \sigma^2\left(T - (n-1) + \frac{2}{n^2}\left(1^2 + \ldots + (n-1)^2\right)\right)$$
$$= \sigma^2\left(T - (n-1) + \frac{2}{n^2}\left(\frac{n^3}{3} - \frac{n^2}{2} + \frac{n}{6}\right)\right)$$
$$= \sigma^2\left(T - \frac{1}{3}\left(n - \frac{1}{n}\right)\right).$$

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Declarations

Conflict of interest The authors have no competing interests to declare that are relevant to the content of this article.

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