Degarching the Commodity time series

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1 R code

1.1 Load Data

Load the libraries needed in the following.

```
> library(zoo)
> library(fGarch)
> library(rugarch)
> library(strucchange)
```

First, we load the data and separate dates from prices.

```
> data ← read.table("../commdata.dat",sep=":",header=T)
> dates ← data[,1]
> prices ← data[,2:12]
> T ← nrow(prices)
> n ← ncol(prices)
```

Linearly interpolate missing values and determine the log returns.

```
> interpol ← na.approx(prices)
> logret ← apply(interpol,2,function(x) diff(log(x)))
```

1.1.1 Time Series Plots

Plot the prices (red lines represent the times where an interpolation was necessary), the logreturns and the empirical residuals in a loop for every $i = 1, \ldots, n$

```
> # prices/interpol
> plot.ts(cbind(interpol[,i],prices[,i]),plot.type="single",col=c(2,1),
+ main=colnames(prices)[i],ylab="prices",xlab="",axes=F)
> axis(2); box(); axis(1,at=ticks,lab=dates[ticks])
> # logrets
> plot.ts(logret[,i],ylab="logreturns",xlab="",axes=F)
> axis(2); box(); axis(1,at=ticks,lab=dates[ticks])
> # empires: garch(1,1)
> container ← logret[,i]
> empiresR.padded ← replace(container,!is.na(container),empiresR[[i]])
> plot.ts(empiresR.padded,main="Garch(1,1)",ylab="empires",axes=F)
> axis(2); box(); axis(1,at=ticks,lab=dates[ticks])
> # empires: ar(1)-egarch(1,1)
> Eempires.padded ← replace(container,!is.na(container),Eempires[[i]])
> plot.ts(Eempires.padded,main="AR(1)-EGARCH(1,1)",ylab="empires",axes=F)
> axis(2); box(); axis(1,at=ticks,lab=dates[ticks])
```

1.1.2 GARCH(1,1)-Model Fit

First, fit a very basic GARCH(1,1)-model with a constant mean for every univariate time series and determine the standardized residuals.

```
> models ← apply(logret,2,
+ function(x) garchFit(~garch(1,1),data=as.zoo(na.omit(x)),trace=F))
> empires ← lapply(models,function(x) residuals(x,standardize=T))
```

Then some goodness-of-fit tests and diagnostic plots for the model fit can be given.

```
> # diagnostic output of model fits
> summary(models[[i]])

> # diagnostic plots of garch(1,1)-models
> plot(models[[i]],which=2:13)
```

1.1.3 AR(1)-EGARCH(1,1)-Model Fit

Secondly, a more sophisicated AR(1)-EGARCH(1,1)-model according to Chevallier (2012) can be fitted, the empirical residuals are retrieved as well.

```
> spec←ugarchspec(variance.model=list(model="eGARCH",garchOrder=c(1,1)),
+ mean.model=list(armaOrder=c(1,0)))
> Emodels ← apply(logret,2,function(x) ugarchfit(spec,data=as.zoo(na.omit(x))))
> Eresults ← sapply(Emodels,as.data.frame)
> Eempires ← apply(Eresults,2,function(x) x[[3]]/x[[4]])
```

Again some goodness-of-fit tests and diagnostic plots for the model fit can be given.

```
> # diagnostic output of model fits
> show(Emodels[[i]])

> # diagnostic plots of ar(1)-egarch(1,1)-models
> plot(Emodels[[i]],which="all")
```

1.1.4 Empirical Fluctuation Processes

Based on the conditional standard deviation in the models in section 1.1.3 set up a constant mean model, i.e.

```
> csdM ← function(data) data ~ 1
```

The empirical residuals from this model are considered. On the one hand, we distinguish between two estimation schemes: first, the ordinary least squares estimates (`ols`); second, recursive estimates (`rec`). On the other hand, two different summation schemes to obtain the empirical fluctuation process are applied: first, cumulative sums (`CUSUM`); second, moving sums (`MOSUM`). Furthermore, an empirical fluctuation process based on the estimates themselves is considered. The estimation scheme is either recursive (`RE`) or moving (`ME`).

```
> eGsigma ← apply(eGresults,2,function(x) x$sigma)
> Cmodels.ols ← lapply(eGsigma,function(x) efp(csdM(x),type="OLS-CUSUM"))
> Cmodels.rec ← lapply(eGsigma,function(x) efp(csdM(x),type="Rec-CUSUM"))
> Cmodels ← list(ols=Cmodels.ols,rec=Cmodels.rec)
> Mmodels.ols ← lapply(eGsigma,function(x) efp(csdM(x),type="OLS-MOSUM"))
> Mmodels.rec ← lapply(eGsigma,function(x) efp(csdM(x),type="Rec-MOSUM"))
> Mmodels ← list(ols=Mmodels.ols,rec=Mmodels.rec)
> Emodels.mov ← lapply(eGsigma,function(x) efp(csdM(x),type="ME"))
> Emodels.rec ← lapply(eGsigma,function(x) efp(csdM(x),type="RE"))
> Emodels ← list(mov=Emodels.mov,rec=Emodels.rec)
```

Then the respective plots of the empirical fluctuation processes with the respective boundaries (to the default significance level $\alpha = 5\%$) can be given.

```
> plot(Cmodels$ols[[i]],axes=F,xlab="",ylab="efp")
> axis(2); box(); axis(1,at=ticks2,lab=dates[ticks])
> plot(Cmodels$rec[[i]],axes=F,xlab="",ylab="")
> axis(2); box(); axis(1,at=ticks2,lab=dates[ticks])
```

```
> plot(Mmodels$ols[[i]],axes=F,xlab="",ylab="efp")
> axis(2); box(); axis(1,at=ticks2,lab=dates[ticks])
> plot(Mmodels$rec[[i]],axes=F,xlab="",ylab="")
> axis(2); box(); axis(1,at=ticks2,lab=dates[ticks])
> plot(Emodels$mov[[i]],axes=F,xlab="",ylab="efp")
> axis(2); box(); axis(1,at=ticks2,lab=dates[ticks])
> plot(Emodels$rec[[i]],axes=F,xlab="",ylab="")
> axis(2); box(); axis(1,at=ticks2,lab=dates[ticks])
```

A formal significance test in order to test the null hypothesis of no structural change (against no specific alternative) can be conducted by `sctest(...)` viz

```
> Csctest.ols ← lapply(Cmodels.ols,sctest)
> Csctest.rec ← lapply(Cmodels.rec,sctest)
> Msctest.ols ← lapply(Mmodels.ols,sctest)
> Msctest.rec ← lapply(Mmodels.rec,sctest)
> Esctest.mov ← lapply(Emodels.mov,sctest)
> Esctest.rec ← lapply(Emodels.rec,sctest)
```

The results will be given in tabular form (each model in a row) for each univariate time series.

Finally, an F-Test to test the null hypothesis against the specific alternative of (an optimal amount of) shifts is performed. The optimal number of shifts is computed simultaneously, such that the test statistic is derived on every "homogeneity" interval. These values are then aggregated either by sup, by mean or by exp in order to test the null hypothesis.

```
> Fstat ← lapply(eGsigma,function(x) Fstats(csdM(x)))
> Ftest.sup ← lapply(Fstat,function(x) sctest(x,type="supF"))
> Ftest.ave ← lapply(Fstat,function(x) sctest(x,type="aveF"))
> Ftest.exp ← lapply(Fstat,function(x) sctest(x,type="expF"))
```

The results of this test can be checked visually, either by depicting the F-statistic (here with the boundaries for the aveF-test and the mean of the F-statistic as dotted line) or the respective p-values.

```
> plot(Fstat[[i]],axes=F,xlab="",aveF=TRUE)
> axis(2); box(); axis(1,at=ticks2,lab=dates[ticks])
> plot(Fstat[[i]],axes=F,xlab="",pval=T)
> axis(2); box(); axis(1,at=ticks2,lab=dates[ticks])
```

Again the formal results for the F-Test with different functional is given in tabular form.

In order to test a multiple regimes alternative against the null hypothesis of homogeneous parameters, an optimal (with respect to the BIC) partition in $m = 1, \ldots, m_{\max} = \frac{1}{h}$ homogeneity intervals is computed. The procedure is due to Bai & Perron (2003) and implemented in `strucchange` by Zeileis & Hornik (2010). Despite the dynamic programming approach based on the Bellman principle to obtain the trinagular RSS matrix this is quite a time consuming procedure.
The calls of `summary()` and `plot()` give the results for all possible partitions up to $m_{\max}$ segments and graphically the values of the RSS and the BIC respectively. The component `$breakpoints` of the results of `breakpoints()` contains the numbers of the observation where the changes occured in case of the optimal segmentation[1].

```
> # allows segments of at least 0.1 of the sample size (default: h=0.15)
> bp.empires.1 ← lapply(eGempires,function(x) breakpoints(x ~ 1,h=.1))
> # summary output of breakpoints calculations
> lapply(bp.empires.1,summary)
> # graphically: optimal no of bp = argmin(BIC)
> lapply(bp.empires.1,plot)
```

The coefficients with standard errors in the respective intervals of the optimal segmentation as well as the breakdates with 95% confidence intervals can then be determined. By specifying the option `breaks=n` a different n-breaks partition can be assumed. As pointed out in Zeileis & Kleiber (2004) on page 4:

[1] `NA` indicates that according to BIC an one-segment (i.e. no break) partition is optimal. This will be the case for the empirical data in every univariate component, see Section 2.

> [...] The standard errors are estimated utilizing a kernel HAC estimator with a quadratic spectral kernel, prewhitening using a VAR(1) model and an AR(1) approximation for the automatic bandwidth selection, as implemented in the `kernHAC()` function of the `R` package `sandwich`.

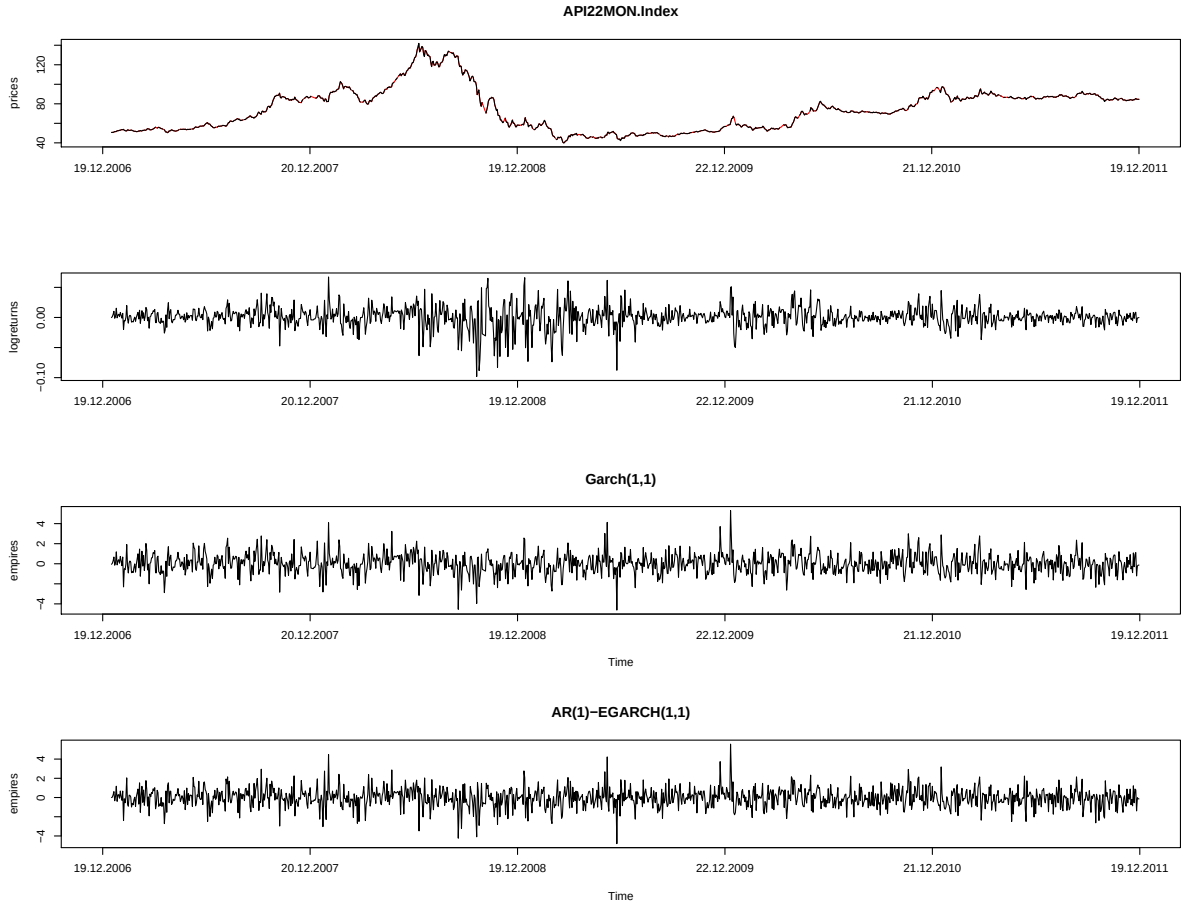
This default option corresponds to `vcov=kernHAC`. If the data is a `time series` object, the breakdates with corresponding confidence intervals can be added to a plot of the data via `lines(bp.confint)`.

```
> bp.coef ← lapply(bp.empires.1,coef)
> bp.std ← lapply(bp.empires.1,vcov,vcov=kernHAC)
> # confidence intervals only make sense if there is a breakpoint
> bp.confint ← lapply(bp.empires.1,confint,vcov=kernHAC,breaks=3)
```

2 Results

2.1 API22MON.Index

2.1.1 Time Series Plots



2.1.2 GARCH(1,1)-Model Fit

```
Title:
 GARCH Modelling

Call:
 garchFit(formula = ~garch(1, 1), data = as.zoo(na.omit(x)), trace = F)

Mean and Variance Equation:
 data ~ garch(1, 1)
<environment: 0xa54e288>
 [data = as.zoo(na.omit(x))]

Conditional Distribution:
 norm

Coefficient(s):
        mu       omega      alpha1       beta1
7.1248e-04  2.5557e-06  1.2701e-01  8.7237e-01
```

```
Std. Errors:
 based on Hessian

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     7.125e-04   3.181e-04    2.240  0.02510 *
omega  2.556e-06   9.303e-07    2.747  0.00601 **
alpha1 1.270e-01   1.733e-02    7.330  2.3e-13 ***
beta1  8.724e-01   1.543e-02   56.541  < 2e-16 ***
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ âĂŹ 1

Log Likelihood:
 3663.397    normalized:  2.853113

Description:
 Wed Jan 11 13:50:13 2012 by user: maggie


Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  206.9611  0
 Shapiro-Wilk Test  R    W      0.9823204 2.031543e-11
 Ljung-Box Test     R    Q(10)  80.84964  3.420597e-13
 Ljung-Box Test     R    Q(15)  87.41621  3.001488e-12
 Ljung-Box Test     R    Q(20)  93.58246  1.740807e-11
 Ljung-Box Test     R^2  Q(10)  9.43363   0.4915032
 Ljung-Box Test     R^2  Q(15)  22.27743  0.1007331
 Ljung-Box Test     R^2  Q(20)  24.4443   0.2235226
 LM Arch Test       R    TR^2   17.31785  0.1380269

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-5.699996 -5.683928 -5.700015 -5.693963
```

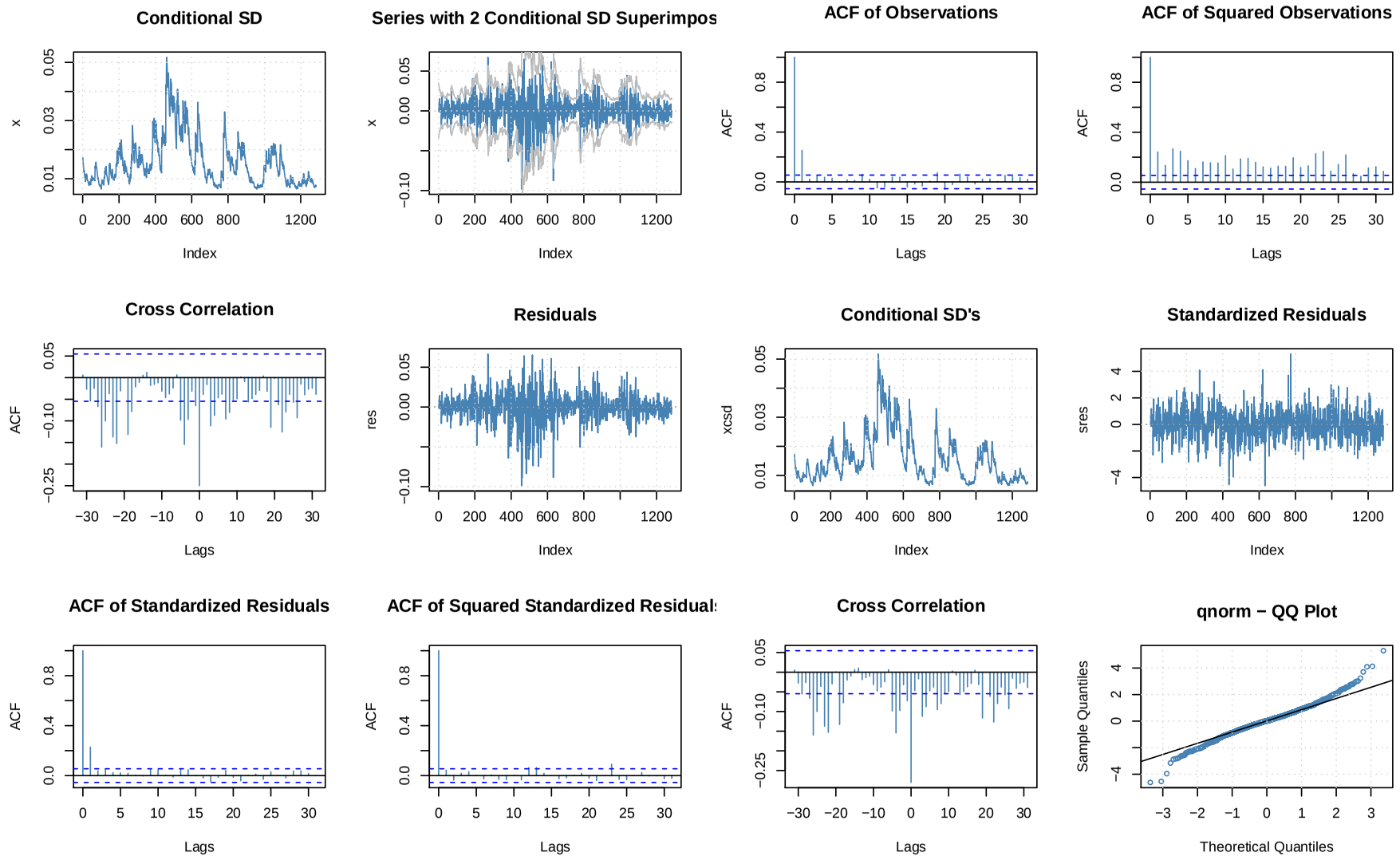


Figure 1: GARCH(1,1) diagnostic plots for API22MON.Index

2.1.3 AR(1)-EGARCH(1,1)-Model Fit

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model : eGARCH(1,1)
Mean Model : ARFIMA(1,0,0)
Distribution : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001179    0.000376   3.1360 0.001713
ar1     0.221276    0.008992  24.6074 0.000000
omega  -0.083907    0.042937  -1.9542 0.050681
alpha1  0.037013    0.013929   2.6572 0.007879
beta1   0.988758    0.005057 195.5264 0.000000
gamma1  0.221908    0.027260   8.1405 0.000000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001179    0.000381   3.0975 0.001952
ar1     0.221276    0.008402  26.3347 0.000000
omega  -0.083907    0.062475  -1.3431 0.179252
alpha1  0.037013    0.023679   1.5631 0.118030
beta1   0.988758    0.007407 133.4940 0.000000
gamma1  0.221908    0.038431   5.7742 0.000000

LogLikelihood : 3686.729

Information Criteria
------------------------------------

Akaike       -5.7332
Bayes        -5.7091
Shibata      -5.7333
Hannan-Quinn -5.7242

Q-Statistics on Standardized Residuals
------------------------------------
      statistic p-value
Lag10     11.73  0.2292
Lag15     18.62  0.1799
Lag20     26.05  0.1288

H0 : No serial correlation

Q-Statistics on Standardized Squared Residuals
------------------------------------
      statistic p-value
Lag10     11.77  0.2267
Lag15     21.27  0.0949
Lag20     23.47  0.2171

ARCH LM Tests
------------------------------------
             Statistic DoF P-Value
ARCH Lag[2]     0.4586   2  0.7951
ARCH Lag[5]     7.7174   5  0.1725
ARCH Lag[10]   11.6510  10  0.3091

Nyblom stability test
------------------------------------
Joint Statistic:  1.2814
Individual Statistics:
mu     0.40083
ar1    0.09460
omega  0.31346
alpha1 0.36715
beta1  0.29666
gamma1 0.02261

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:       1.49 1.68 2.12
Individual Statistic:  0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value    prob sig
Sign Bias           1.7370 0.08263   *
Negative Sign Bias  0.6561 0.51187
Positive Sign Bias  0.6749 0.49988
Joint Effect        3.1356 0.37119


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     65.25    5.555e-07
2    30     92.92    1.287e-08
3    40     94.75    1.534e-06
4    50    119.97    6.937e-08


Elapsed time : 0.872077
```

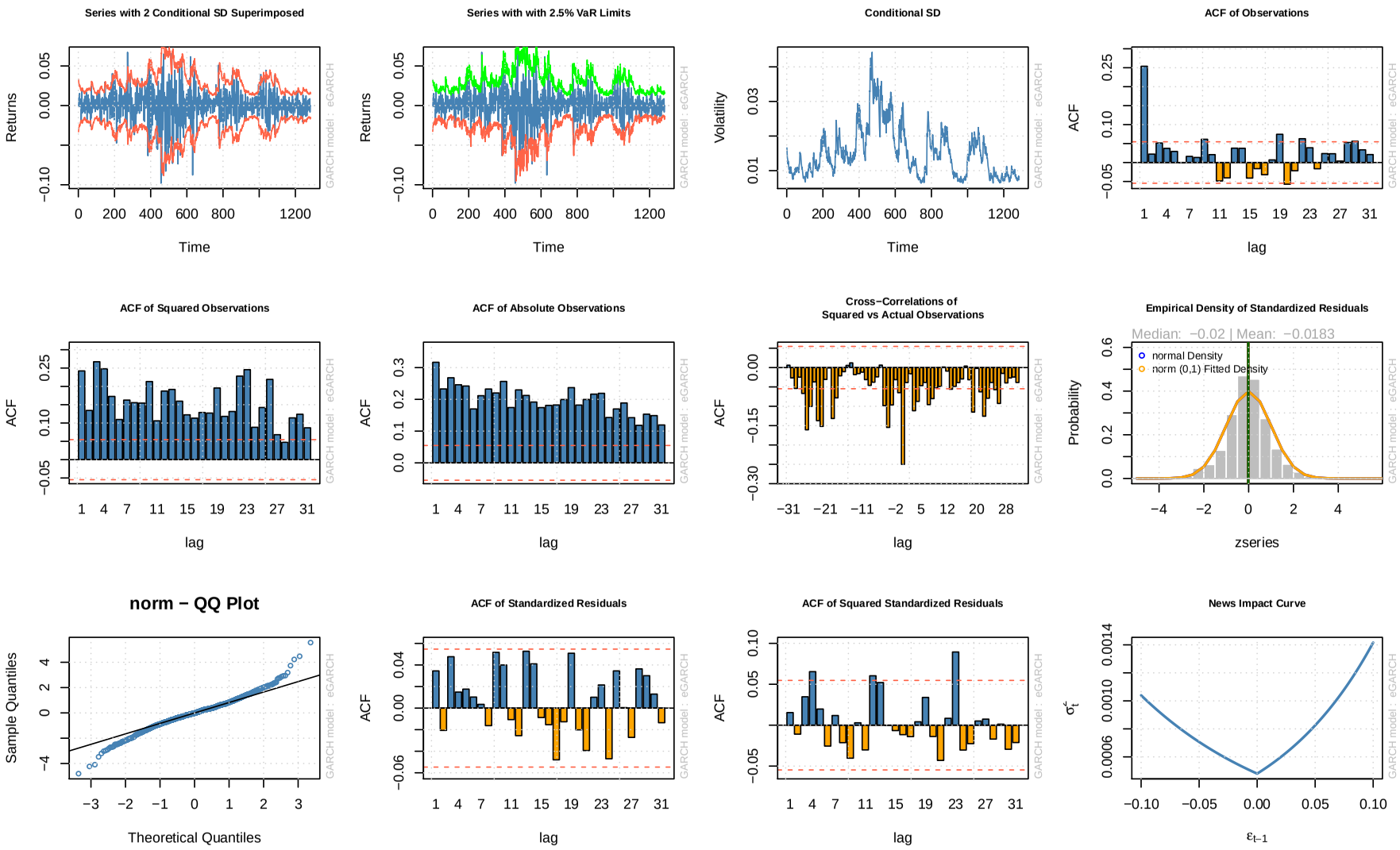


Figure 2: AR(1)-EGARCH(1,1) diagnostic plots for API22MON.Index

2.1.4 Empirical Fluctuation Process

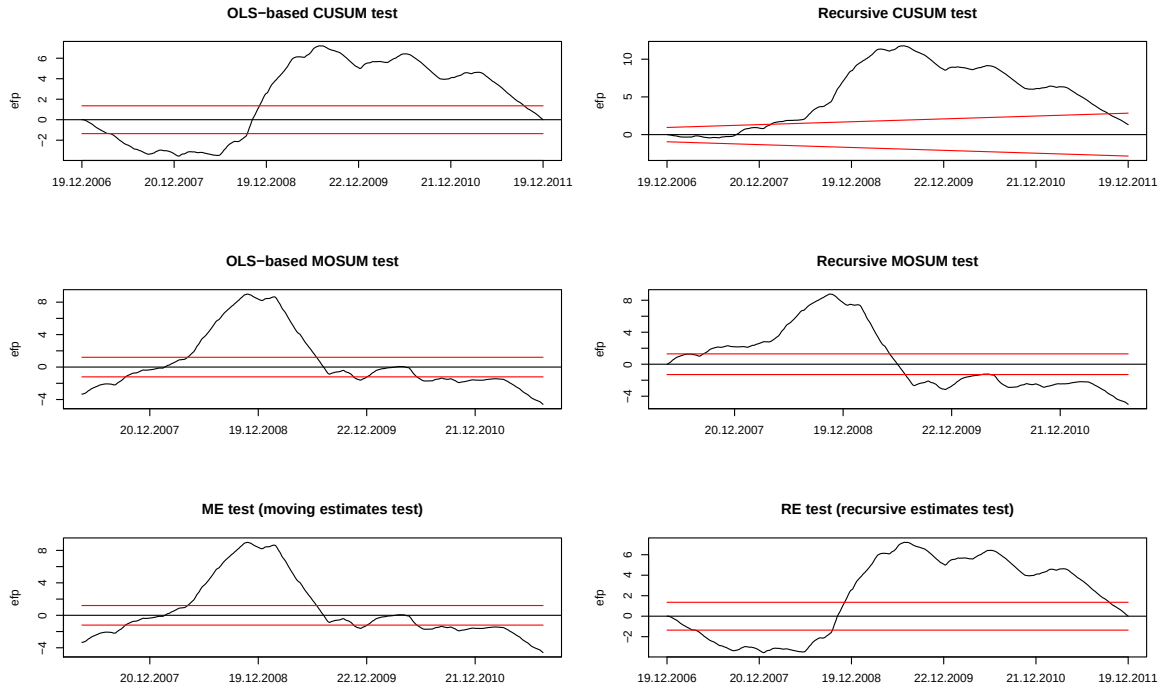


Figure 3: Empirical fluctuation processes for API22MON.Index

	statistic	p-value
OLS-CUSUM	7.21078	0.00000
Rec-CUSUM	5.90159	0.00000
OLS-MOSUM	8.99345	0.01000
Rec-MOSUM	8.76955	0.01000
ME	8.99345	0.01000
RE	7.21078	0.00000

Table 1: General Test for all models for API22MON.Index

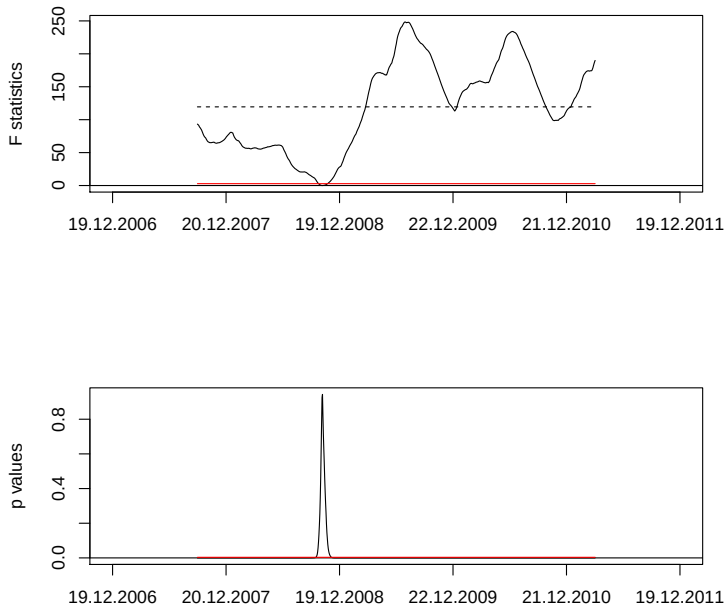


Figure 4: F-test statistic and p-values for API22MON.Index

	statistic	p-value
sup	248.31348	0.00000
ave	119.45943	0.00000
exp	119.78230	0.00000

Table 2: F-Test with different functionals for API22MON.Index

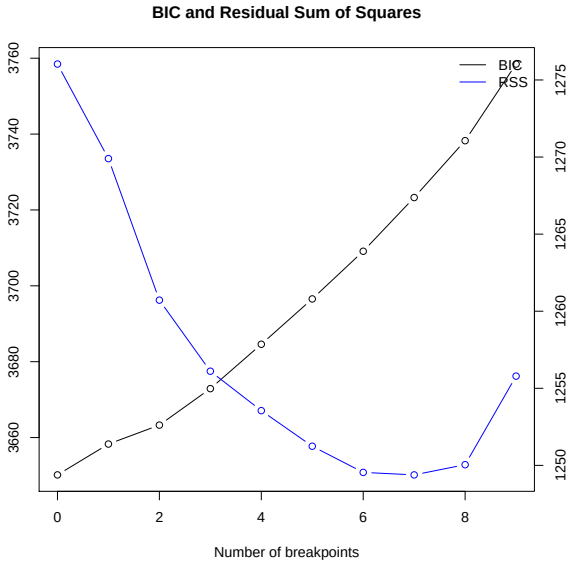


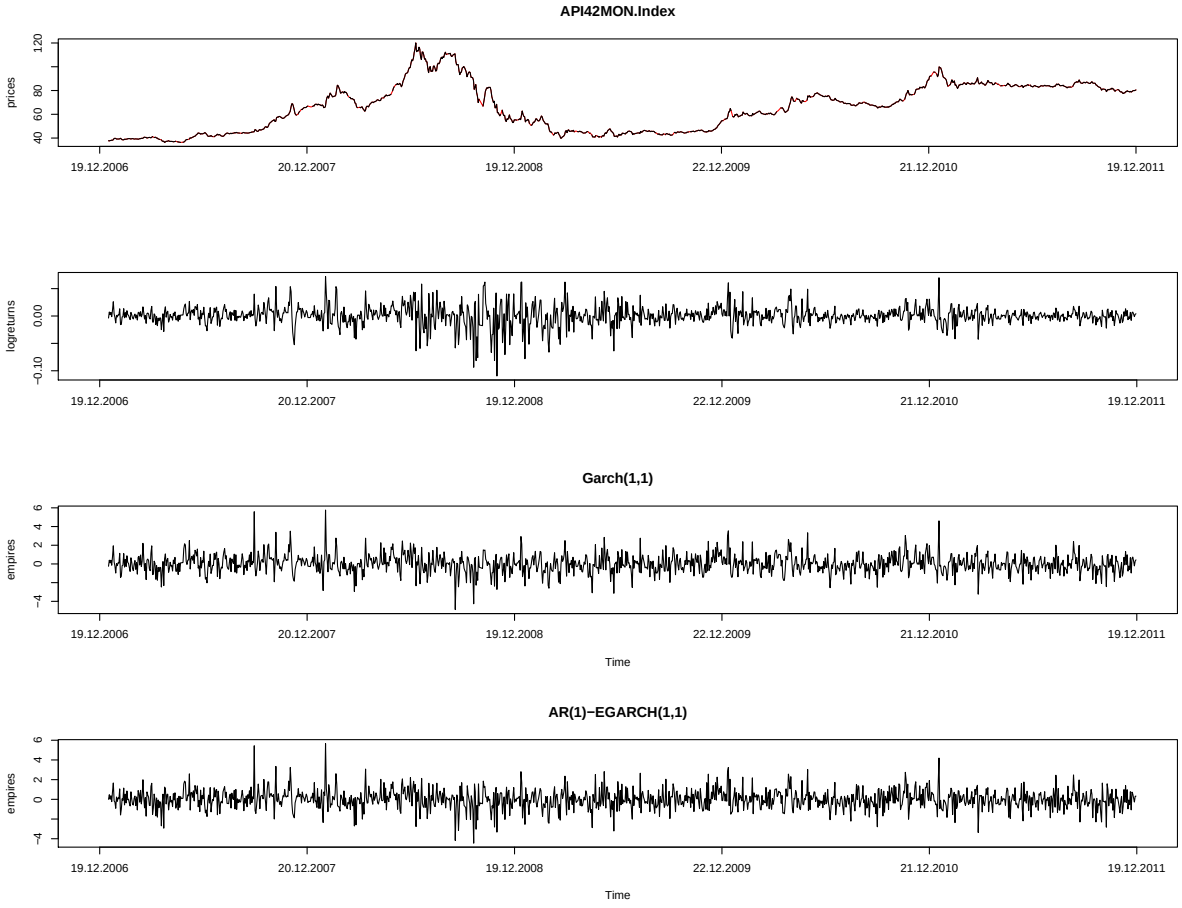
Figure 5: RSS and BIC with respect to number of possible breakpoints for API22MON.Index

```
 Optimal (m+1)-segment partition:

Call:
breakpoints.formula(formula = x ~ 1, h = 0.1)

Breakpoints at observation number:

m = 1           384
m = 2           421 565
m = 3           421 565            1086
m = 4           421 565     753 885
m = 5   143     384 565     753 885
m = 6   143     384 565     753 885 1086
m = 7   143 286 421 565     753 885 1086
m = 8   143 286 421 565     753 885 1023 1152
m = 9   128 256 384 512 640 768 896 1026 1154

Corresponding to breakdates:

m = 1                                          0.299065420560748
m = 2                                          0.327881619937695
m = 3                                          0.327881619937695
m = 4                                          0.327881619937695
m = 5   0.111370716510903                      0.299065420560748
m = 6   0.111370716510903                      0.299065420560748
m = 7   0.111370716510903  0.222741433021807 0.327881619937695
m = 8   0.111370716510903  0.222741433021807 0.327881619937695
m = 9   0.0996884735202492 0.199376947040498 0.299065420560748

m = 1
m = 2   0.440031152647975
m = 3   0.440031152647975
m = 4   0.440031152647975                    0.586448598130841 0.689252336448598
m = 5   0.440031152647975                    0.586448598130841 0.689252336448598
m = 6   0.440031152647975                    0.586448598130841 0.689252336448598
m = 7   0.440031152647975                    0.586448598130841 0.689252336448598
m = 8   0.440031152647975                    0.586448598130841 0.689252336448598
m = 9   0.398753894080997 0.498442367601246 0.598130841121495 0.697819314641745
```

```
m = 1
m = 2
m = 3   0.845794392523364
m = 4
m = 5
m = 6   0.845794392523364
m = 7   0.845794392523364
m = 8   0.796728971962617 0.897196261682243
m = 9   0.799065420560748 0.898753894080997

Fit:

m   0        1        2        3        4        5        6        7
RSS 1276.035 1269.899 1260.719 1256.108 1253.551 1251.244 1249.545 1249.383
BIC 3650.159 3658.286 3663.285 3672.897 3684.596 3696.546 3709.117 3723.266

m   8        9
RSS 1250.045 1255.791
BIC 3738.262 3758.465
```

2.2 API42MON.Index

2.2.1 Time Series Plots



2.2.2 GARCH(1,1)-Model Fit

```
Title:
 GARCH Modelling

Call:
 garchFit(formula = ~garch(1, 1), data = as.zoo(na.omit(x)), trace = F)

Mean and Variance Equation:
 data ~ garch(1, 1)
<environment: 0xa42b92c>
 [data = as.zoo(na.omit(x))]

Conditional Distribution:
 norm

Coefficient(s):
        mu       omega      alpha1       beta1
6.4709e-04  3.7992e-06  1.2606e-01  8.6859e-01

Std. Errors:
 based on Hessian
```

```
Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     6.471e-04   3.411e-04    1.897   0.0578 .
omega  3.799e-06   1.173e-06    3.239   0.0012 **
alpha1 1.261e-01   2.027e-02    6.218 5.03e-10 ***
beta1  8.686e-01   1.901e-02   45.702  < 2e-16 ***
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ âĂŹ 1

Log Likelihood:
 3624.167    normalized:  2.82256

Description:
 Wed Jan 11 13:50:13 2012 by user: maggie


Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  481.0073  0
 Shapiro-Wilk Test  R    W      0.9702862 1.389533e-15
 Ljung-Box Test     R    Q(10)  71.28312  2.50473e-11
 Ljung-Box Test     R    Q(15)  82.65032  2.280953e-11
 Ljung-Box Test     R    Q(20)  86.09437  3.542506e-10
 Ljung-Box Test     R^2  Q(10)  5.55537   0.8511347
 Ljung-Box Test     R^2  Q(15)  11.15916  0.7412322
 Ljung-Box Test     R^2  Q(20)  14.13935  0.823361
 LM Arch Test       R    TR^2   8.96778   0.70568

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-5.638889 -5.622821 -5.638908 -5.632856
```

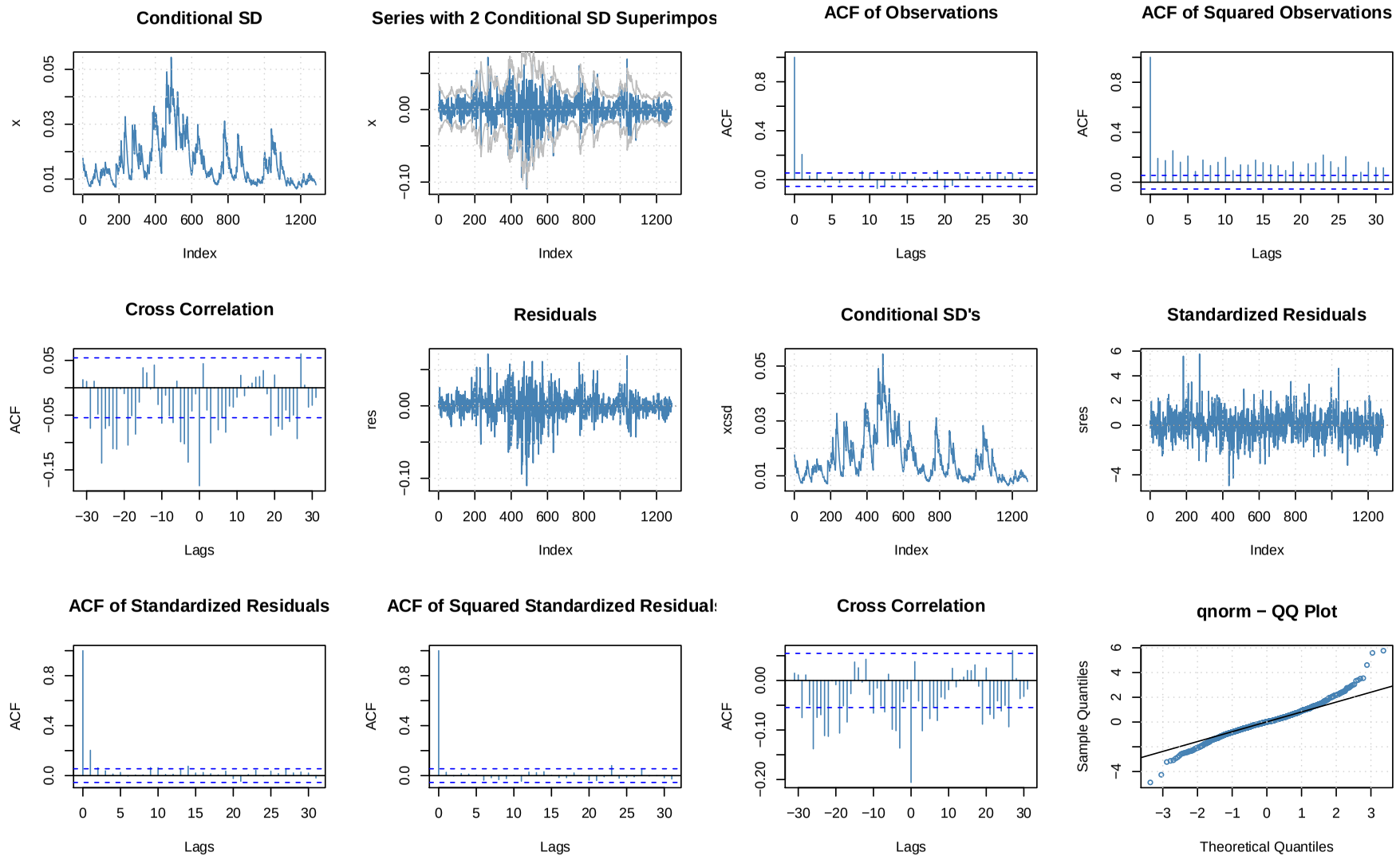


Figure 6: GARCH(1,1) diagnostic plots for API42MON.Index

2.2.3 AR(1)-EGARCH(1,1)-Model Fit

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model : eGARCH(1,1)
Mean Model : ARFIMA(1,0,0)
Distribution : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.000817    0.000417   1.9587 0.050152
ar1     0.214130    0.029283   7.3125 0.000000
omega  -0.071573    0.037673  -1.8998 0.057453
alpha1  0.036220    0.012356   2.9314 0.003375
beta1   0.990290    0.004542 218.0122 0.000000
gamma1  0.183325    0.028654   6.3978 0.000000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.000817    0.000440   1.8576 0.063220
ar1     0.214130    0.034183   6.2642 0.000000
omega  -0.071573    0.068223  -1.0491 0.294133
alpha1  0.036220    0.019618   1.8463 0.064855
beta1   0.990290    0.008222 120.4398 0.000000
gamma1  0.183325    0.062359   2.9398 0.003284

LogLikelihood : 3646.343

Information Criteria
------------------------------------

Akaike       -5.6703
Bayes        -5.6462
Shibata      -5.6704
Hannan-Quinn -5.6613

Q-Statistics on Standardized Residuals
------------------------------------
      statistic p-value
Lag10     11.45  0.2459
Lag15     19.47  0.1478
Lag20     23.89  0.2004

H0 : No serial correlation

Q-Statistics on Standardized Squared Residuals
------------------------------------
      statistic p-value
Lag10     7.413  0.5942
Lag15    10.987  0.6871
Lag20    13.564  0.8085

ARCH LM Tests
------------------------------------
             Statistic DoF P-Value
ARCH Lag[2]     0.7582   2  0.6845
ARCH Lag[5]     4.2494   5  0.5141
ARCH Lag[10]    7.2834  10  0.6984

Nyblom stability test
------------------------------------
Joint Statistic:  1.4188
Individual Statistics:
mu     0.09963
ar1    0.64929
omega  0.33862
alpha1 0.37548
beta1  0.33545
gamma1 0.10569

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:       1.49 1.68 2.12
Individual Statistic:  0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           0.6639 0.5069
Negative Sign Bias  0.1542 0.8775
Positive Sign Bias  1.0471 0.2953
Joint Effect        3.3686 0.3382


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     66.09    4.051e-07
2    30     77.68    2.517e-06
3    40     84.72    3.132e-05
4    50     92.25    1.833e-04


Elapsed time : 0.8593154
```

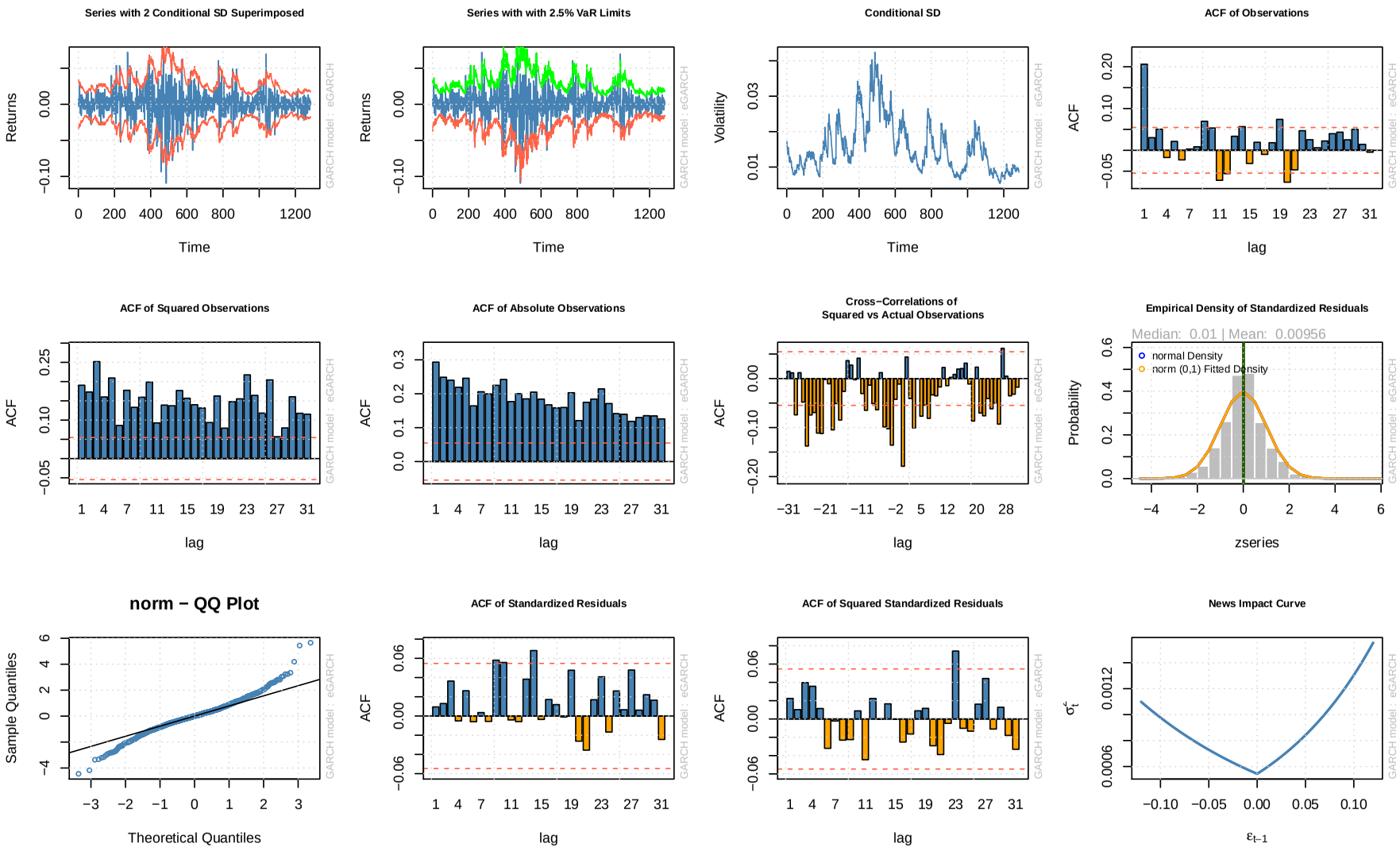


Figure 7: AR(1)-EGARCH(1,1) diagnostic plots for API42MON.Index

2.2.4 Empirical Fluctuation Process

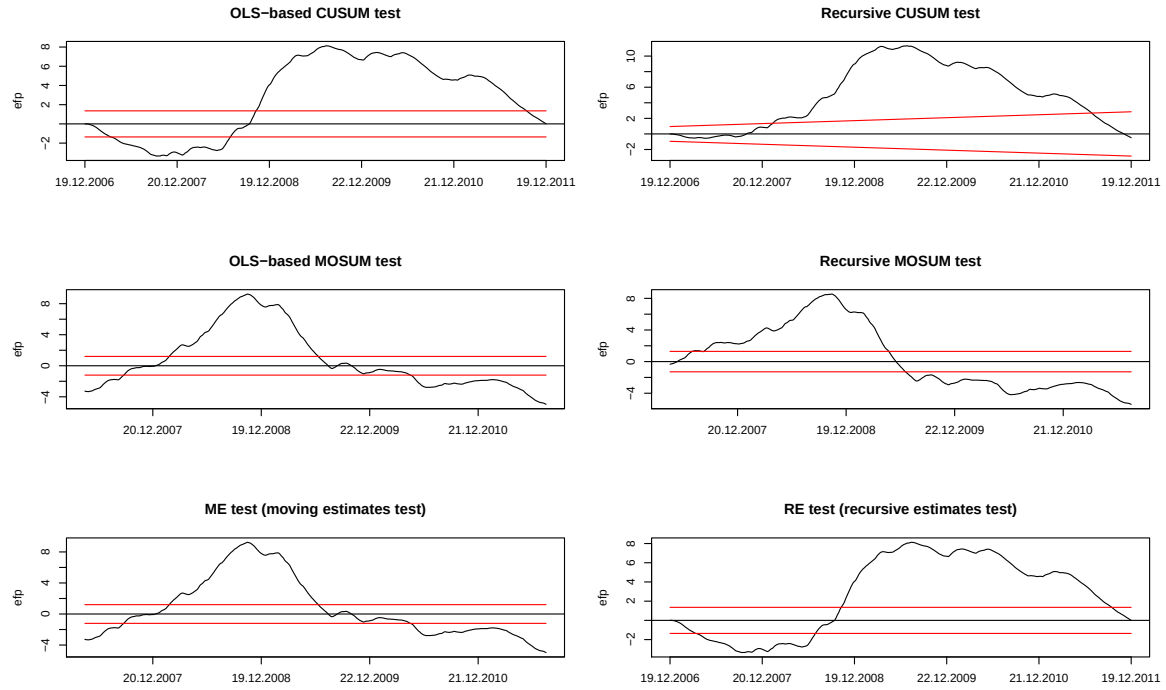


Figure 8: Empirical fluctuation processes for API42MON.Index

	statistic	p-value
OLS-CUSUM	8.12593	0.00000
Rec-CUSUM	5.86814	0.00000
OLS-MOSUM	9.23703	0.01000
Rec-MOSUM	8.53978	0.01000
ME	9.23703	0.01000
RE	8.12593	0.00000

Table 3: General Test for all models for API42MON.Index

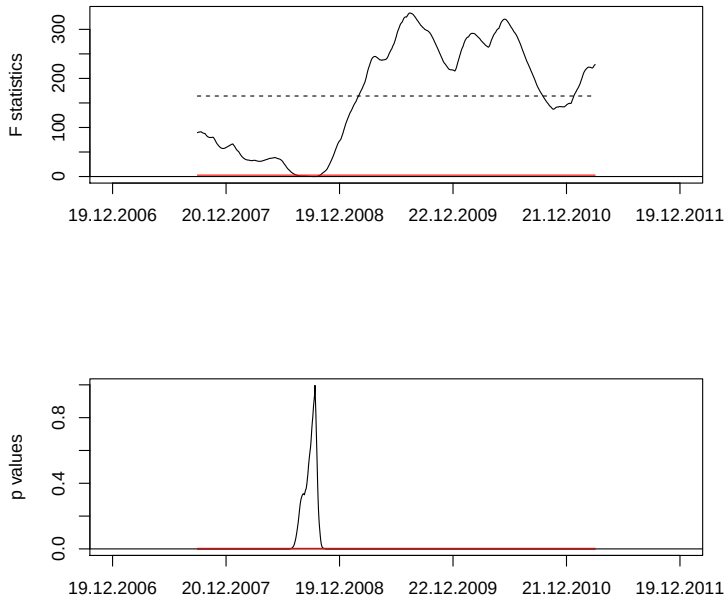


Figure 9: F-test statistic and p-values for API42MON.Index

	statistic	p-value
sup	333.31121	0.00000
ave	164.12427	0.00000
exp	161.93154	0.00000

Table 4: F-Test with different functionals for API42MON.Index

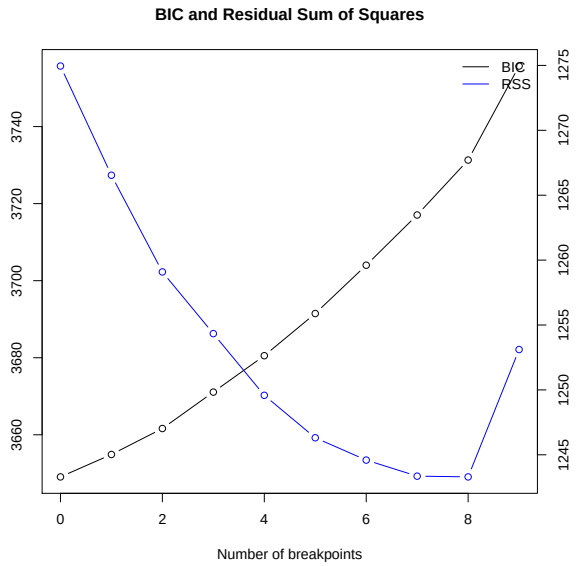


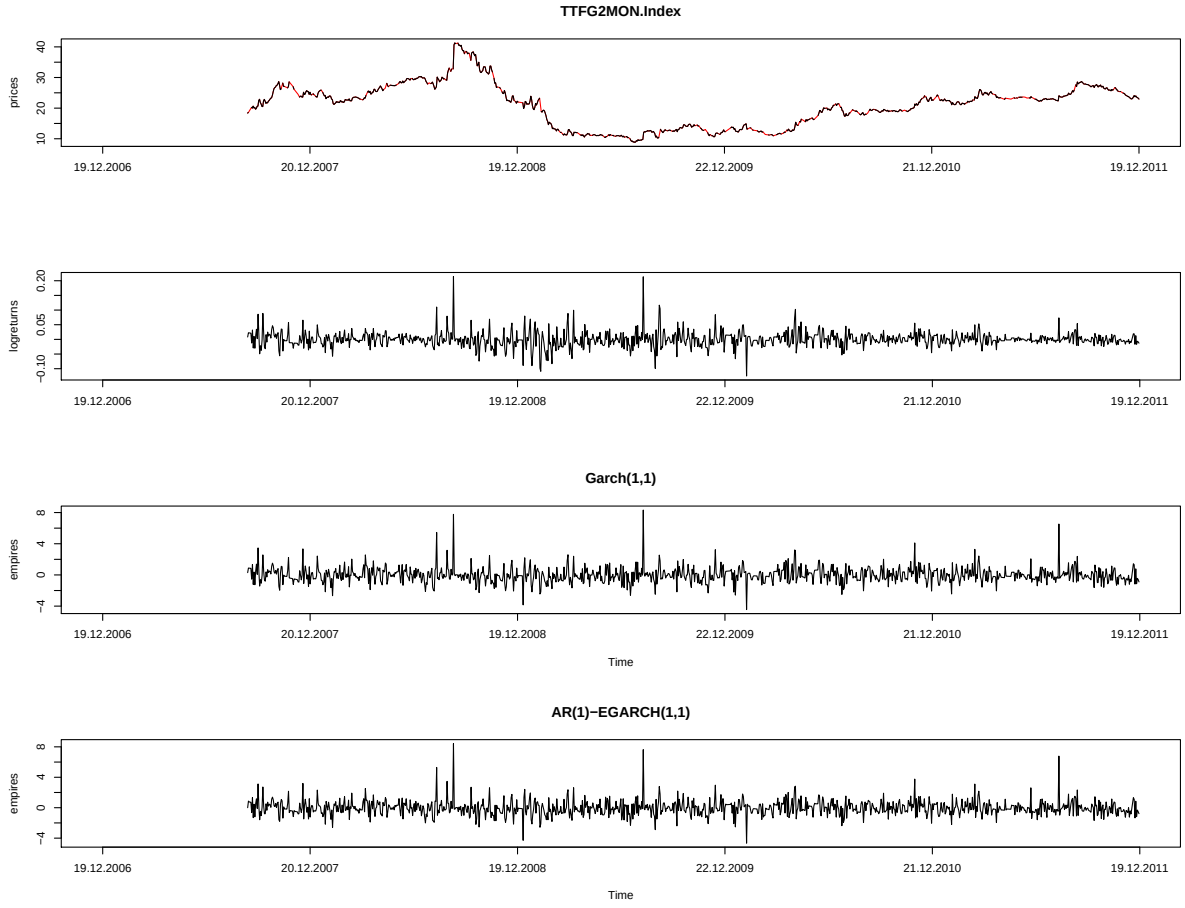
Figure 10: RSS and BIC with respect to number of possible breakpoints for API42MON.Index

```
 Optimal (m+1)-segment partition:

Call:
breakpoints.formula(formula = x ~ 1, h = 0.1)

Breakpoints at observation number:

m = 1           384
m = 2           384 565
m = 3   174     384 565
m = 4   174     384     605         1038
m = 5   174     384 565     747 883
m = 6   174     384 565     747 883 1038
m = 7   174     384 565 702     853 981  1124
m = 8   143 286 433 565 702     853 981  1124
m = 9   128 256 384 512 641 769 899 1027 1156

Corresponding to breakdates:

m = 1                                          0.299065420560748
m = 2                                          0.299065420560748
m = 3   0.135514018691589                      0.299065420560748
m = 4   0.135514018691589                      0.299065420560748
m = 5   0.135514018691589                      0.299065420560748
m = 6   0.135514018691589                      0.299065420560748
m = 7   0.135514018691589                      0.299065420560748
m = 8   0.111370716510903  0.222741433021807 0.337227414330218
m = 9   0.0996884735202492 0.199376947040498 0.299065420560748

m = 1
m = 2   0.440031152647975
m = 3   0.440031152647975
m = 4                   0.471183800623053
m = 5   0.440031152647975                   0.581775700934579 0.687694704049844
m = 6   0.440031152647975                   0.581775700934579 0.687694704049844
m = 7   0.440031152647975 0.546728971962617                   0.664330218068536
m = 8   0.440031152647975 0.546728971962617                   0.664330218068536
m = 9   0.398753894080997 0.499221183800623 0.598909657320872 0.700155763239875
```

```
m = 1
m = 2
m = 3
m = 4   0.808411214953271
m = 5
m = 6   0.808411214953271
m = 7   0.764018691588785 0.875389408099688
m = 8   0.764018691588785 0.875389408099688
m = 9   0.799844236760125 0.900311526479751

Fit:

m   0        1        2        3        4        5        6        7
RSS 1274.953 1266.540 1259.090 1254.336 1249.584 1246.313 1244.594 1243.357
BIC 3649.071 3654.885 3661.626 3671.084 3680.526 3691.475 3704.019 3717.057

m   8        9
RSS 1243.299 1253.110
BIC 3731.313 3755.722
```

2.3 TTFG2MON.Index

2.3.1 Time Series Plots



2.3.2 GARCH(1,1)-Model Fit

```
Title:
 GARCH Modelling

Call:
 garchFit(formula = ~garch(1, 1), data = as.zoo(na.omit(x)), trace = F)

Mean and Variance Equation:
 data ~ garch(1, 1)
<environment: 0xa547a20>
 [data = as.zoo(na.omit(x))]

Conditional Distribution:
 norm

Coefficient(s):
        mu       omega      alpha1       beta1
5.9736e-04  8.5050e-06  1.0490e-01  8.9565e-01

Std. Errors:
 based on Hessian
```

```
Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     5.974e-04   5.927e-04    1.008 0.313509
omega  8.505e-06   2.188e-06    3.888 0.000101 ***
alpha1 1.049e-01   1.604e-02    6.541 6.12e-11 ***
beta1  8.957e-01   1.256e-02   71.313  < 2e-16 ***
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ âĂŹ 1

Log Likelihood:
 2605.017    normalized:  2.338435

Description:
 Wed Jan 11 13:50:13 2012 by user: maggie


Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  5338.358  0
 Shapiro-Wilk Test  R    W      0.9095018 0
 Ljung-Box Test     R    Q(10)  24.50429  0.006368652
 Ljung-Box Test     R    Q(15)  25.60271  0.04241187
 Ljung-Box Test     R    Q(20)  26.39393  0.1531921
 Ljung-Box Test     R^2  Q(10)  2.368027  0.9926625
 Ljung-Box Test     R^2  Q(15)  3.50228   0.9989658
 Ljung-Box Test     R^2  Q(20)  5.897691  0.9990288
 LM Arch Test       R    TR^2   3.296033  0.9930686

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-4.669689 -4.651679 -4.669715 -4.662880
```

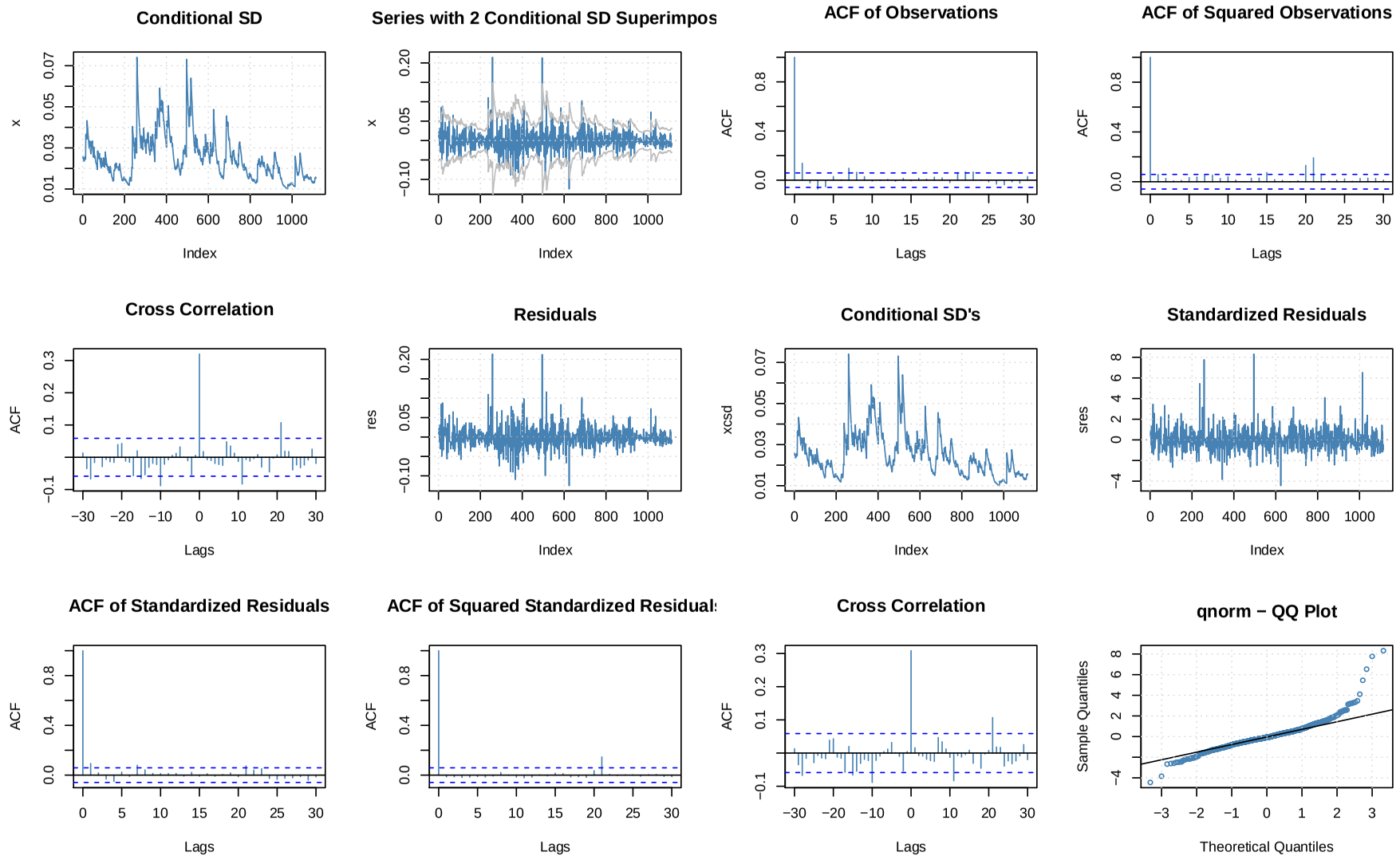


Figure 11: GARCH(1,1) diagnostic plots for TTFG2MON.Index

2.3.3 AR(1)-EGARCH(1,1)-Model Fit

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model : eGARCH(1,1)
Mean Model : ARFIMA(1,0,0)
Distribution : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error   t value Pr(>|t|)
mu      0.001097    0.000670   1.63671 0.101690
ar1     0.121496    0.033723   3.60281 0.000315
omega  -0.141102    0.041583  -3.39323 0.000691
alpha1  0.012962    0.021615   0.59969 0.548716
beta1   0.978745    0.005548 176.41325 0.000000
gamma1  0.198044    0.022525   8.79230 0.000000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001097    0.000942  1.16492 0.244051
ar1     0.121496    0.040041  3.03432 0.002411
omega  -0.141102    0.086662 -1.62818 0.103487
alpha1  0.012962    0.055454  0.23374 0.815184
beta1   0.978745    0.011329 86.39375 0.000000
gamma1  0.198044    0.031742  6.23927 0.000000

LogLikelihood : 2618.557

Information Criteria
------------------------------------

Akaike       -4.6904
Bayes        -4.6634
Shibata      -4.6905
Hannan-Quinn -4.6802

Q-Statistics on Standardized Residuals
------------------------------------
      statistic p-value
Lag10     13.64  0.1356
Lag15     14.85  0.3888
Lag20     16.33  0.6351

H0 : No serial correlation

Q-Statistics on Standardized Squared Residuals
------------------------------------
      statistic p-value
Lag10     3.506  0.9408
Lag15     4.711  0.9894
Lag20     6.985  0.9943

ARCH LM Tests
------------------------------------
             Statistic DoF P-Value
ARCH Lag[2]     0.2363   2  0.8886
ARCH Lag[5]     1.2079   5  0.9441
ARCH Lag[10]    3.4666  10  0.9682

Nyblom stability test
------------------------------------
Joint Statistic:  0.8581
Individual Statistics:
mu     0.04780
ar1    0.25445
omega  0.25232
alpha1 0.04763
beta1  0.22849
gamma1 0.13480

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:       1.49 1.68 2.12
Individual Statistic:  0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           0.7238 0.4693
Negative Sign Bias  1.1393 0.2548
Positive Sign Bias  0.2314 0.8170
Joint Effect        1.3565 0.7158


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     94.37    5.535e-12
2    30    112.88    7.722e-12
3    40    119.54    4.181e-10
4    50    134.56    6.558e-10


Elapsed time : 0.555038
```

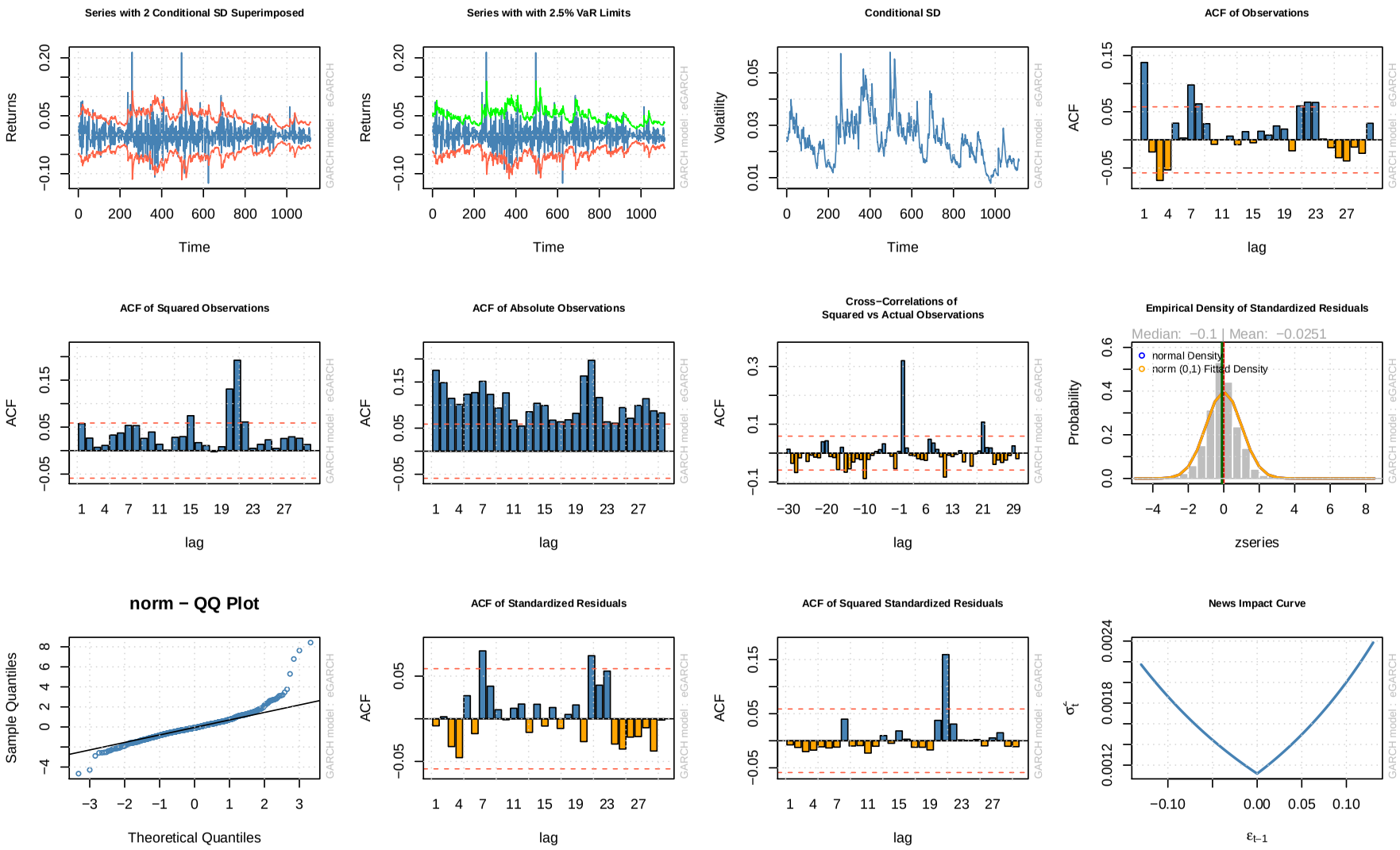


Figure 12: AR(1)-EGARCH(1,1) diagnostic plots for TTFG2MON.Index

2.3.4 Empirical Fluctuation Process

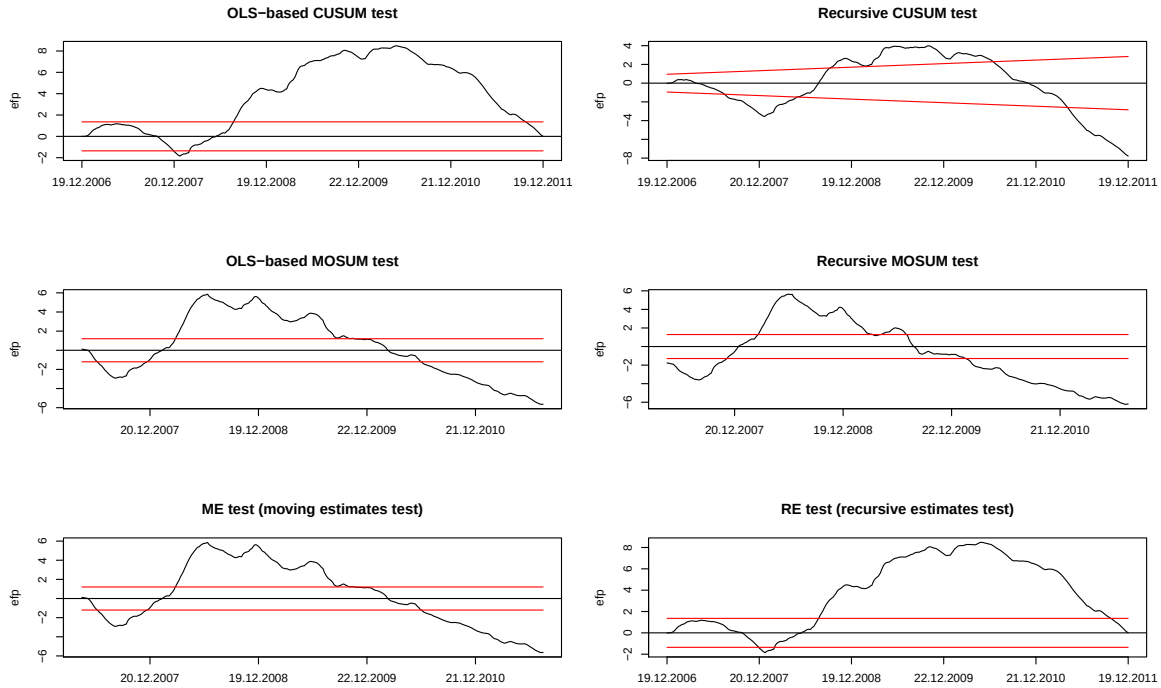


Figure 13: Empirical fluctuation processes for TTFG2MON.Index

	statistic	p-value
OLS-CUSUM	8.48408	0.00000
Rec-CUSUM	2.59454	0.00000
OLS-MOSUM	5.87228	0.01000
Rec-MOSUM	6.22325	0.01000
ME	5.87228	0.01000
RE	8.48408	0.00000

Table 5: General Test for all models for TTFG2MON.Index

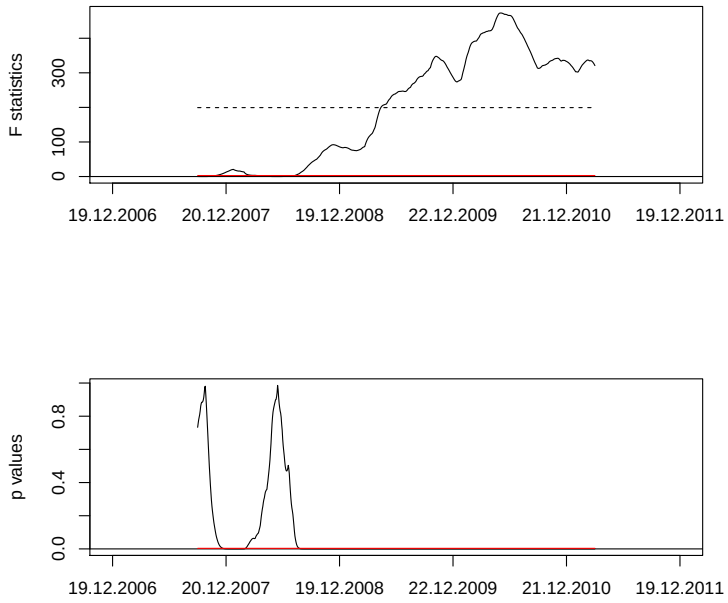


Figure 14: F-test statistic and p-values for TTFG2MON.Index

	statistic	p-value
sup	473.00540	0.00000
ave	199.37194	0.00000
exp	231.98550	0.00000

Table 6: F-Test with different functionals for TTFG2MON.Index

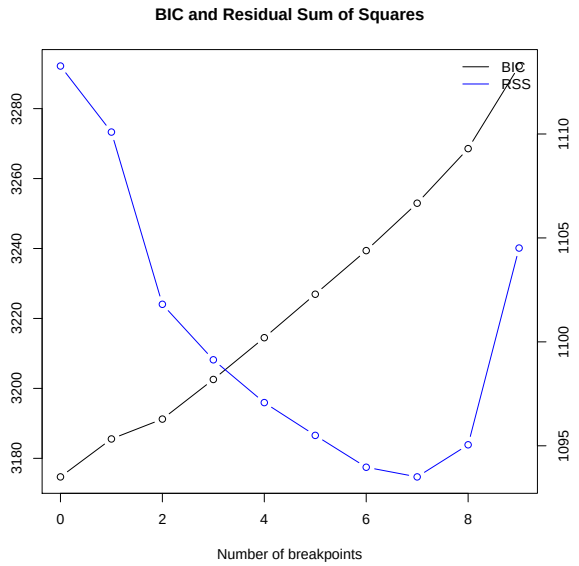


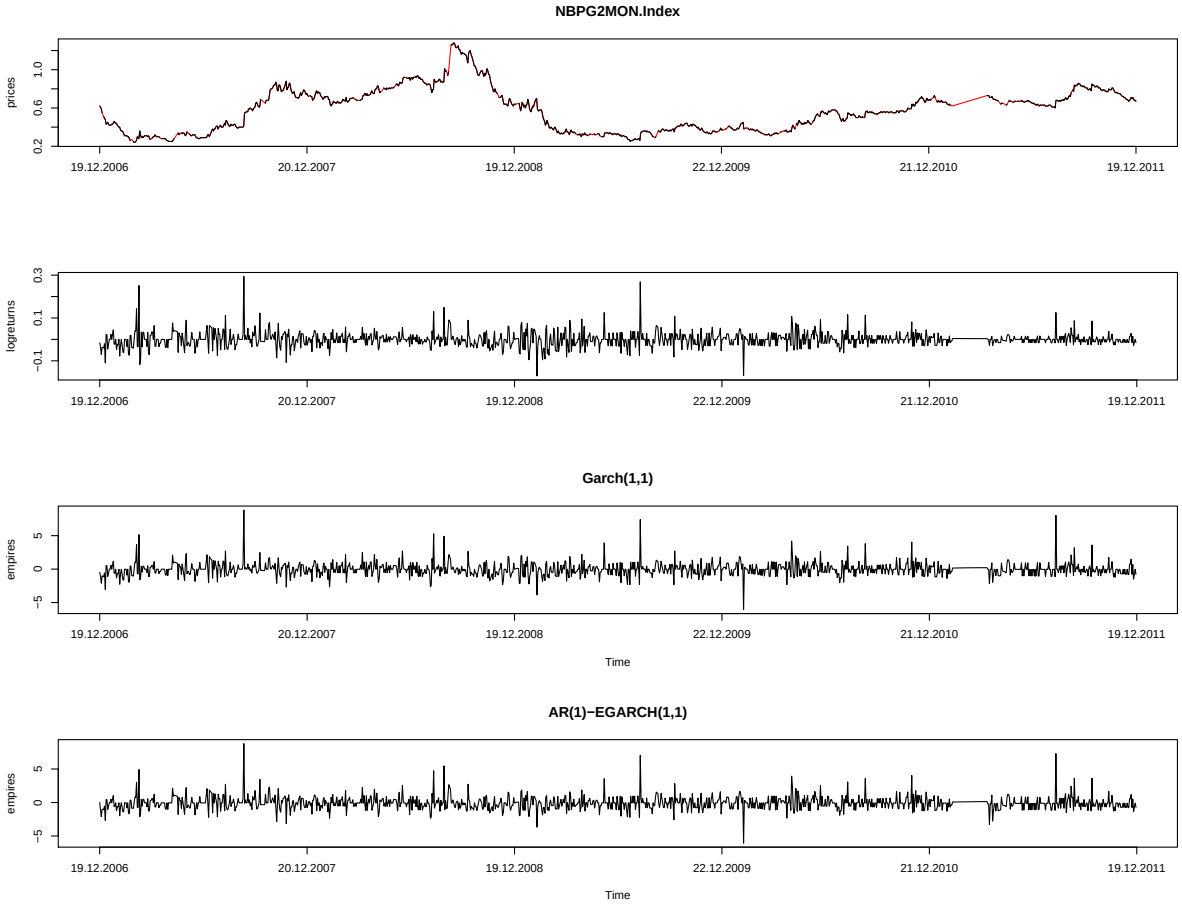
Figure 15: RSS and BIC with respect to number of possible breakpoints for TTFG2MON.Index

```
 Optimal (m+1)-segment partition:

Call:
breakpoints.formula(formula = x ~ 1, h = 0.1)

Breakpoints at observation number:

m = 1       258
m = 2       263     484
m = 3       263     484             915
m = 4       263     395     657     846
m = 5       263     395 545 657     846
m = 6   144 258     395 545 657     846
m = 7   144 258     395 545 657 778 915
m = 8   144 258     395 545 657 778 889 1003
m = 9   111 222 334 447 558 669 781 892 1003

Corresponding to breakdates:

m = 1                   0.231597845601436
m = 2                   0.236086175942549                   0.434470377019749
m = 3                   0.236086175942549                   0.434470377019749
m = 4                   0.236086175942549                   0.354578096947935
m = 5                   0.236086175942549                   0.354578096947935
m = 6   0.129263913824057 0.231597845601436                 0.354578096947935
m = 7   0.129263913824057 0.231597845601436                 0.354578096947935
m = 8   0.129263913824057 0.231597845601436                 0.354578096947935
m = 9   0.099640933572711 0.199281867145422 0.299820466786355 0.401256732495512

m = 1
m = 2
m = 3                                                       0.821364452423698
m = 4                   0.589766606822262                   0.759425493716337
m = 5   0.489228007181329 0.589766606822262                 0.759425493716337
m = 6   0.489228007181329 0.589766606822262                 0.759425493716337
m = 7   0.489228007181329 0.589766606822262 0.698384201077199 0.821364452423698
m = 8   0.489228007181329 0.589766606822262 0.698384201077199 0.79802513464991
m = 9   0.500897666068223 0.600538599640934 0.701077199281867 0.800718132854578
```

```
m = 1
m = 2
m = 3
m = 4
m = 5
m = 6
m = 7
m = 8   0.900359066427289
m = 9   0.900359066427289

Fit:

m   0        1        2        3        4        5        6        7
RSS 1113.270 1110.093 1101.808 1099.137 1097.079 1095.501 1093.969 1093.505
BIC 3174.696 3185.544 3191.230 3202.558 3214.501 3226.930 3239.402 3252.960

m   8        9
RSS 1095.049 1104.514
BIC 3268.564 3292.183
```

2.4 NBPG2MON.Index

2.4.1 Time Series Plots



2.4.2 GARCH(1,1)-Model Fit

```
Title:
 GARCH Modelling

Call:
 garchFit(formula = ~garch(1, 1), data = as.zoo(na.omit(x)), trace = F)

Mean and Variance Equation:
 data ~ garch(1, 1)
<environment: 0x9bce6d4>
 [data = as.zoo(na.omit(x))]

Conditional Distribution:
 norm

Coefficient(s):
        mu       omega      alpha1       beta1
5.9891e-04  7.6134e-06  5.1246e-02  9.4640e-01

Std. Errors:
 based on Hessian
```

```
Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     5.989e-04   7.588e-04    0.789 0.429923
omega  7.613e-06   2.092e-06    3.640 0.000273 ***
alpha1 5.125e-02   1.100e-02    4.658 3.19e-06 ***
beta1  9.464e-01   9.413e-03  100.545  < 2e-16 ***
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ âĂŹ 1

Log Likelihood:
 2635.743    normalized:  2.035323

Description:
 Wed Jan 11 13:50:14 2012 by user: maggie


Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  9552.145  0
 Shapiro-Wilk Test  R    W      0.8714641 0
 Ljung-Box Test     R    Q(10)  8.178035  0.6114515
 Ljung-Box Test     R    Q(15)  10.30774  0.799949
 Ljung-Box Test     R    Q(20)  11.74672  0.9245179
 Ljung-Box Test     R^2  Q(10)  2.326968  0.9931646
 Ljung-Box Test     R^2  Q(15)  5.571876  0.9860775
 Ljung-Box Test     R^2  Q(20)  7.515946  0.9946166
 LM Arch Test       R    TR^2   3.063269  0.9950806

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-4.064468 -4.048511 -4.064487 -4.058480
```

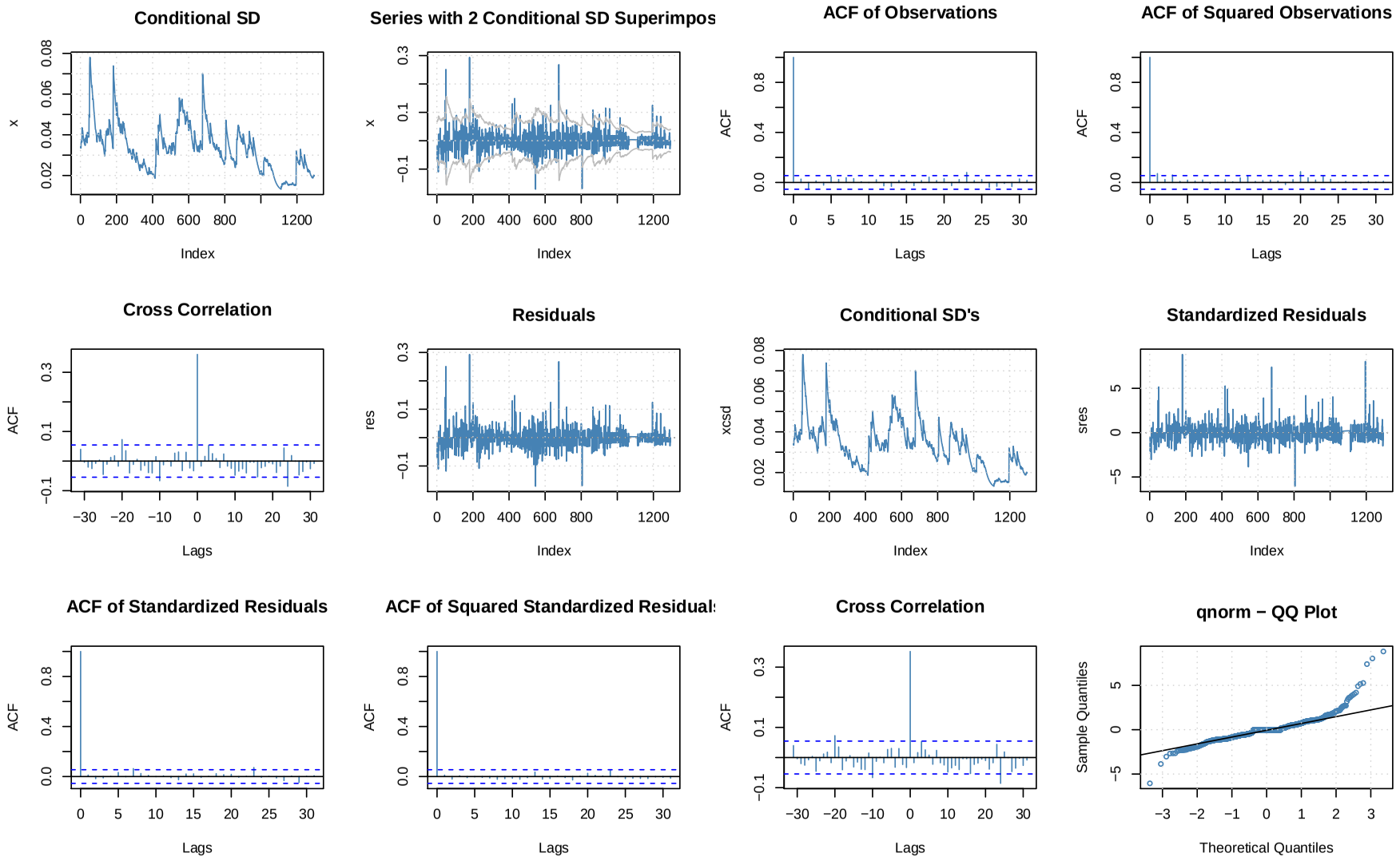


Figure 16: GARCH(1,1) diagnostic plots for NBPG2MON.Index

2.4.3 AR(1)-EGARCH(1,1)-Model Fit

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model : eGARCH(1,1)
Mean Model : ARFIMA(1,0,0)
Distribution : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error   t value Pr(>|t|)
mu      0.002116    0.000627   3.37550 0.000737
ar1     0.008392    0.044108   0.19026 0.849104
omega  -0.059149    0.022315  -2.65067 0.008033
alpha1 -0.033926    0.010754  -3.15491 0.001605
beta1   0.989853    0.003437 287.98269 0.000000
gamma1  0.101063    0.017853   5.66072 0.000000

Robust Standard Errors:
        Estimate  Std. Error   t value Pr(>|t|)
mu      0.002116    0.001176  1.799557 0.071931
ar1     0.008392    0.091411  0.091806 0.926852
omega  -0.059149    0.069438 -0.851831 0.394308
alpha1 -0.033926    0.024562 -1.381235 0.167207
beta1   0.989853    0.010560 93.733684 0.000000
gamma1  0.101063    0.056185  1.798764 0.072056

LogLikelihood : 2653.796

Information Criteria
------------------------------------

Akaike       -4.0893
Bayes        -4.0653
Shibata      -4.0893
Hannan-Quinn -4.0803

Q-Statistics on Standardized Residuals
------------------------------------
      statistic p-value
Lag10     11.53  0.2411
Lag15     13.03  0.5241
Lag20     15.29  0.7038

H0 : No serial correlation

Q-Statistics on Standardized Squared Residuals
------------------------------------
      statistic p-value
Lag10     2.220  0.9875
Lag15     5.264  0.9818
Lag20     9.861  0.9563

ARCH LM Tests
------------------------------------
             Statistic DoF P-Value
ARCH Lag[2]      1.007   2  0.6044
ARCH Lag[5]      1.115   5  0.9528
ARCH Lag[10]     2.187  10  0.9947

Nyblom stability test
------------------------------------
Joint Statistic:  1.1083
Individual Statistics:
mu     0.06028
ar1    0.30550
omega  0.19426
alpha1 0.03598
beta1  0.18717
gamma1 0.21464

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:       1.49 1.68 2.12
Individual Statistic:  0.35 0.47 0.75

Sign Bias Test
------------------------------------
                     t-value   prob sig
Sign Bias          0.007576 0.9940
Negative Sign Bias 1.376921 0.1688
Positive Sign Bias 0.836303 0.4031
Joint Effect       2.599235 0.4576


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20      1674            0
2    30      1664            0
3    40      1720            0
4    50      2716            0


Elapsed time : 0.8430073
```

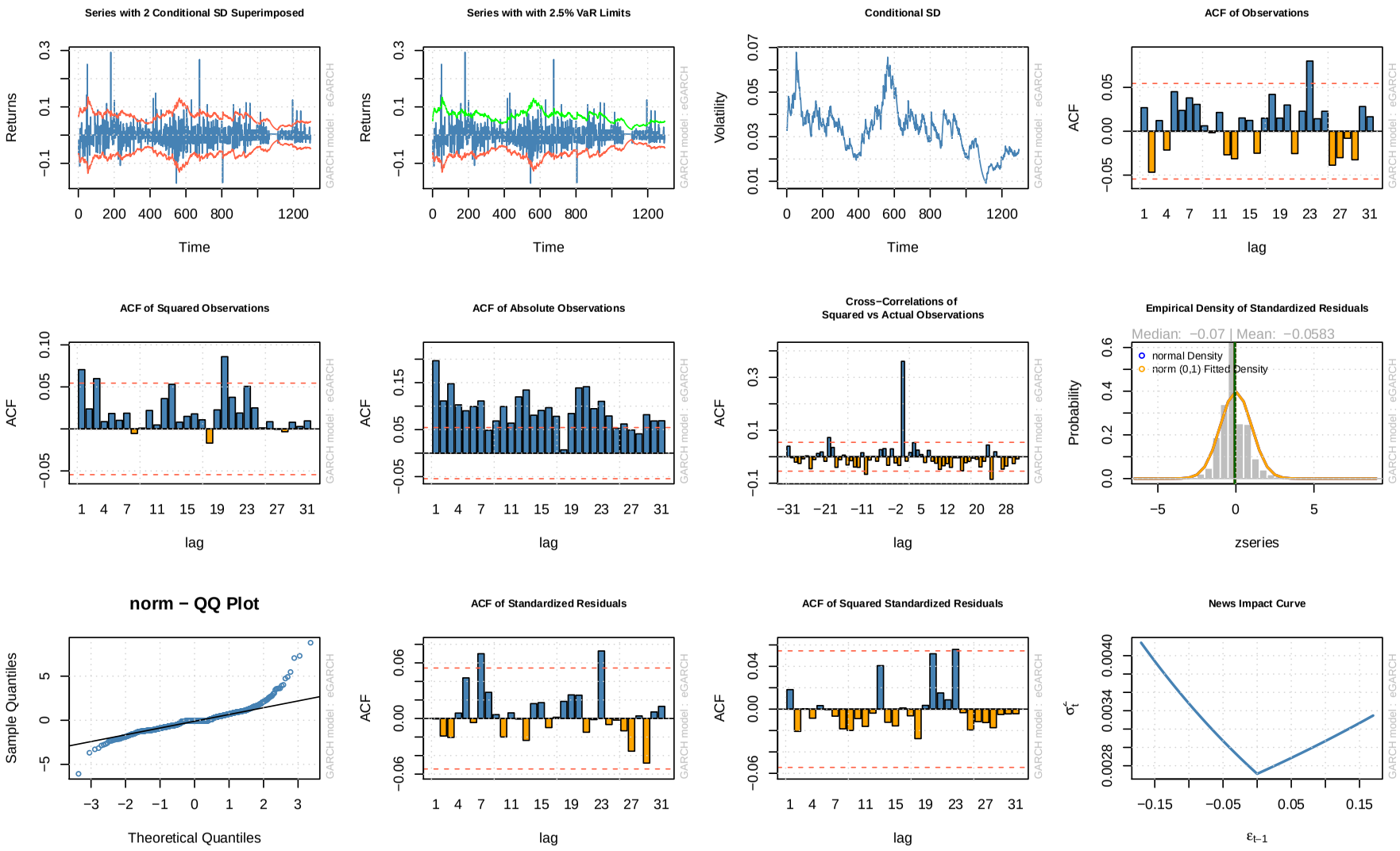


Figure 17: AR(1)-EGARCH(1,1) diagnostic plots for NBPG2MON.Index

2.4.4 Empirical Fluctuation Process

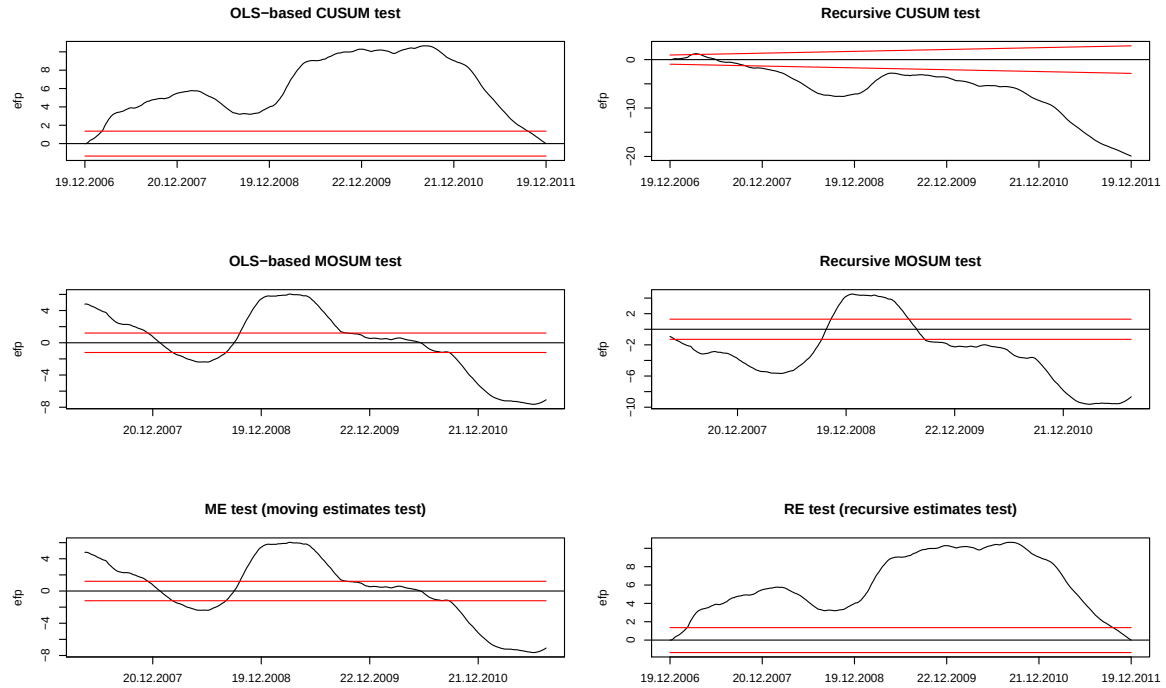


Figure 18: Empirical fluctuation processes for NBPG2MON.Index

	statistic	p-value
OLS-CUSUM	10.65314	0.00000
Rec-CUSUM	6.63929	0.00000
OLS-MOSUM	7.63504	0.01000
Rec-MOSUM	9.62738	0.01000
ME	7.63504	0.01000
RE	10.65314	0.00000

Table 7: General Test for all models for NBPG2MON.Index

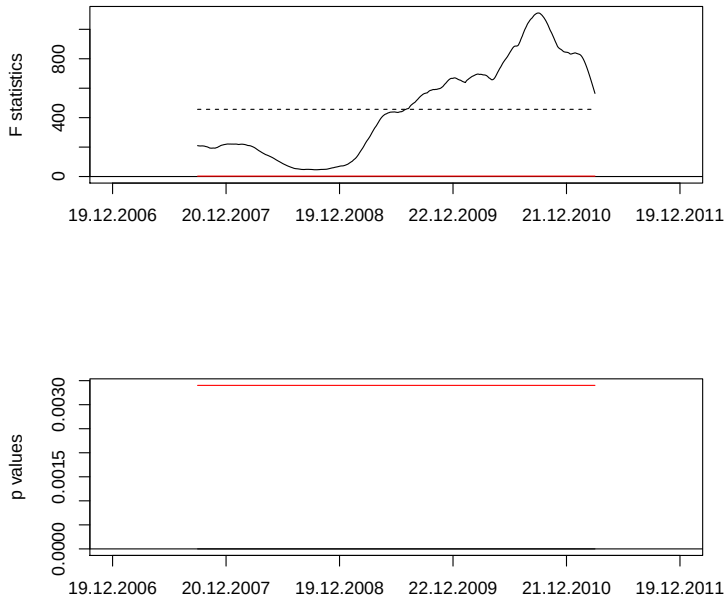


Figure 19: F-test statistic and p-values for NBPG2MON.Index

	statistic	p-value
sup	1110.78319	0.00000
ave	456.57301	0.00000
exp	550.34673	0.00000

Table 8: F-Test with different functionals for NBPG2MON.Index

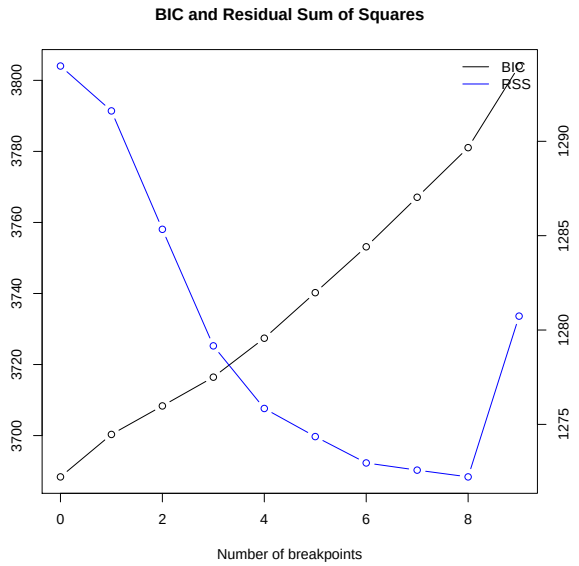


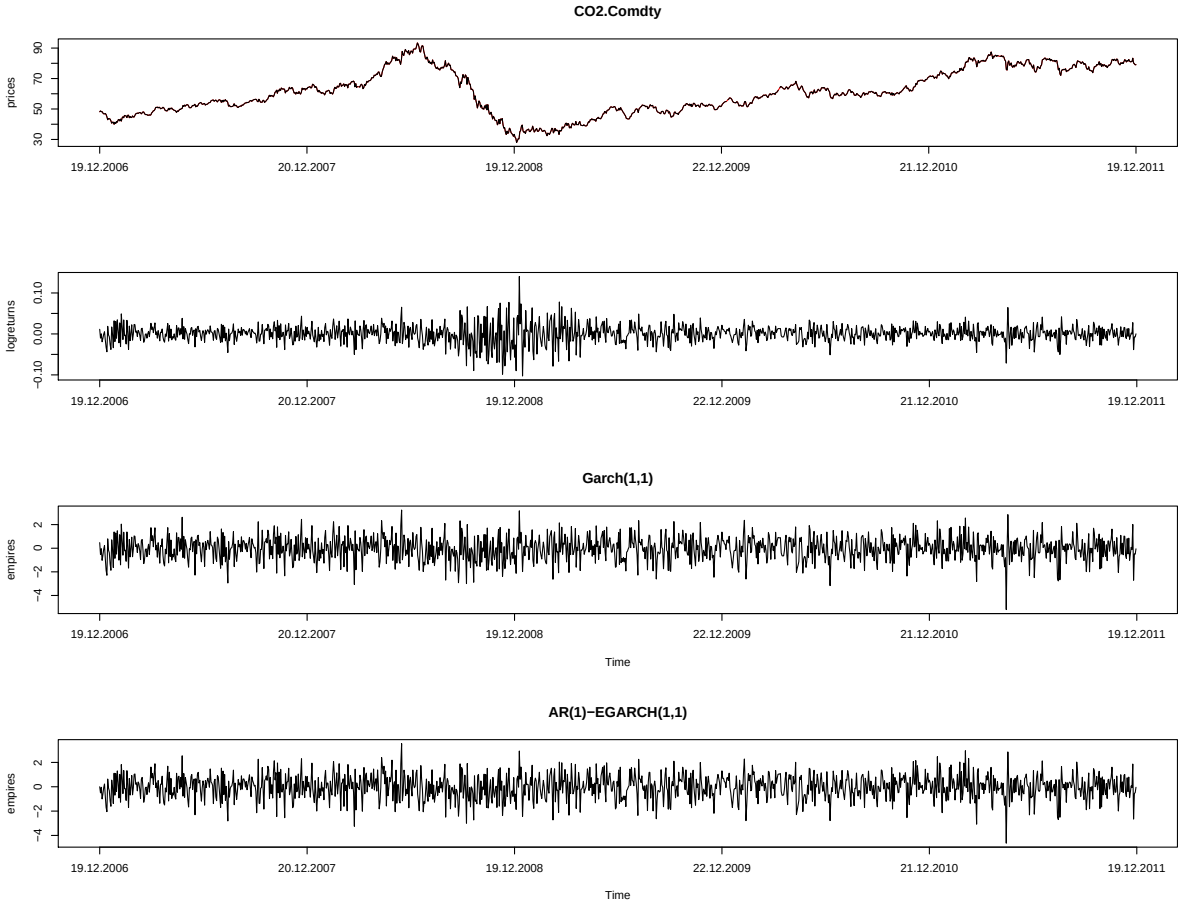
Figure 20: RSS and BIC with respect to number of possible breakpoints for NBPG2MON.Index

```
 Optimal (m+1)-segment partition:

Call:
breakpoints.formula(formula = x ~ 1, h = 0.1)

Breakpoints at observation number:

m = 1           442
m = 2           443 575
m = 3   133     443 575
m = 4   133     443     663             1109
m = 5   133     443 575     839     1027
m = 6   130 259 443 575     839     1027
m = 7   130 259 443 575 704 839     1027
m = 8   130 259 443 575 704 839     1027 1165
m = 9   130 259 390 521 650 779 908 1037 1166

Corresponding to breakdates:

m = 1                       0.341312741312741
m = 2                       0.342084942084942 0.444015444015444
m = 3   0.102702702702703   0.342084942084942 0.444015444015444
m = 4   0.102702702702703   0.342084942084942
m = 5   0.102702702702703   0.342084942084942 0.444015444015444
m = 6   0.1003861003861   0.2 0.342084942084942 0.444015444015444
m = 7   0.1003861003861   0.2 0.342084942084942 0.444015444015444
m = 8   0.1003861003861   0.2 0.342084942084942 0.444015444015444
m = 9   0.1003861003861   0.2 0.301158301158301 0.402316602316602

m = 1
m = 2
m = 3
m = 4   0.511969111969112
m = 5                   0.647876447876448                   0.793050193050193
m = 6                   0.647876447876448                   0.793050193050193
m = 7   0.543629343629344 0.647876447876448                   0.793050193050193
m = 8   0.543629343629344 0.647876447876448                   0.793050193050193
m = 9   0.501930501930502 0.601544401544401 0.701158301158301 0.800772200772201
```

```
m = 1
m = 2
m = 3
m = 4   0.856370656370656
m = 5
m = 6
m = 7
m = 8   0.8996138996139
m = 9   0.9003861003861

Fit:

m   0        1        2        3        4        5        6        7
RSS 1293.989 1291.611 1285.337 1279.162 1275.844 1274.354 1272.957 1272.577
BIC 3688.372 3700.323 3708.349 3716.445 3727.414 3740.233 3753.146 3767.092

m   8        9
RSS 1272.218 1280.737
BIC 3781.059 3804.034
```

2.5 CO2.Comdty

2.5.1 Time Series Plots



2.5.2 GARCH(1,1)-Model Fit

```
Title:
 GARCH Modelling

Call:
 garchFit(formula = ~garch(1, 1), data = as.zoo(na.omit(x)), trace = F)

Mean and Variance Equation:
 data ~ garch(1, 1)
<environment: 0xa437bec>
 [data = as.zoo(na.omit(x))]

Conditional Distribution:
 norm

Coefficient(s):
        mu       omega      alpha1       beta1
1.0791e-03  6.6401e-06  6.6487e-02  9.1586e-01

Std. Errors:
 based on Hessian
```

```
Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     1.079e-03   4.723e-04    2.285   0.0223 *
omega  6.640e-06   2.928e-06    2.267   0.0234 *
alpha1 6.649e-02   1.700e-02    3.911 9.19e-05 ***
beta1  9.159e-01   2.196e-02   41.714  < 2e-16 ***
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ âĂŹ 1

Log Likelihood:
 3312.888    normalized:  2.558214

Description:
 Wed Jan 11 13:50:14 2012 by user: maggie


Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  31.76962  1.262743e-07
 Shapiro-Wilk Test  R    W      0.9948208 0.0001916409
 Ljung-Box Test     R    Q(10)  6.329981  0.7868197
 Ljung-Box Test     R    Q(15)  11.43723  0.7210294
 Ljung-Box Test     R    Q(20)  12.74947  0.8878727
 Ljung-Box Test     R^2  Q(10)  15.48481  0.1153608
 Ljung-Box Test     R^2  Q(15)  21.93677  0.1094727
 Ljung-Box Test     R^2  Q(20)  25.8483   0.1708806
 LM Arch Test       R    TR^2   17.85932  0.1200294

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-5.110251 -5.094294 -5.110270 -5.104263
```

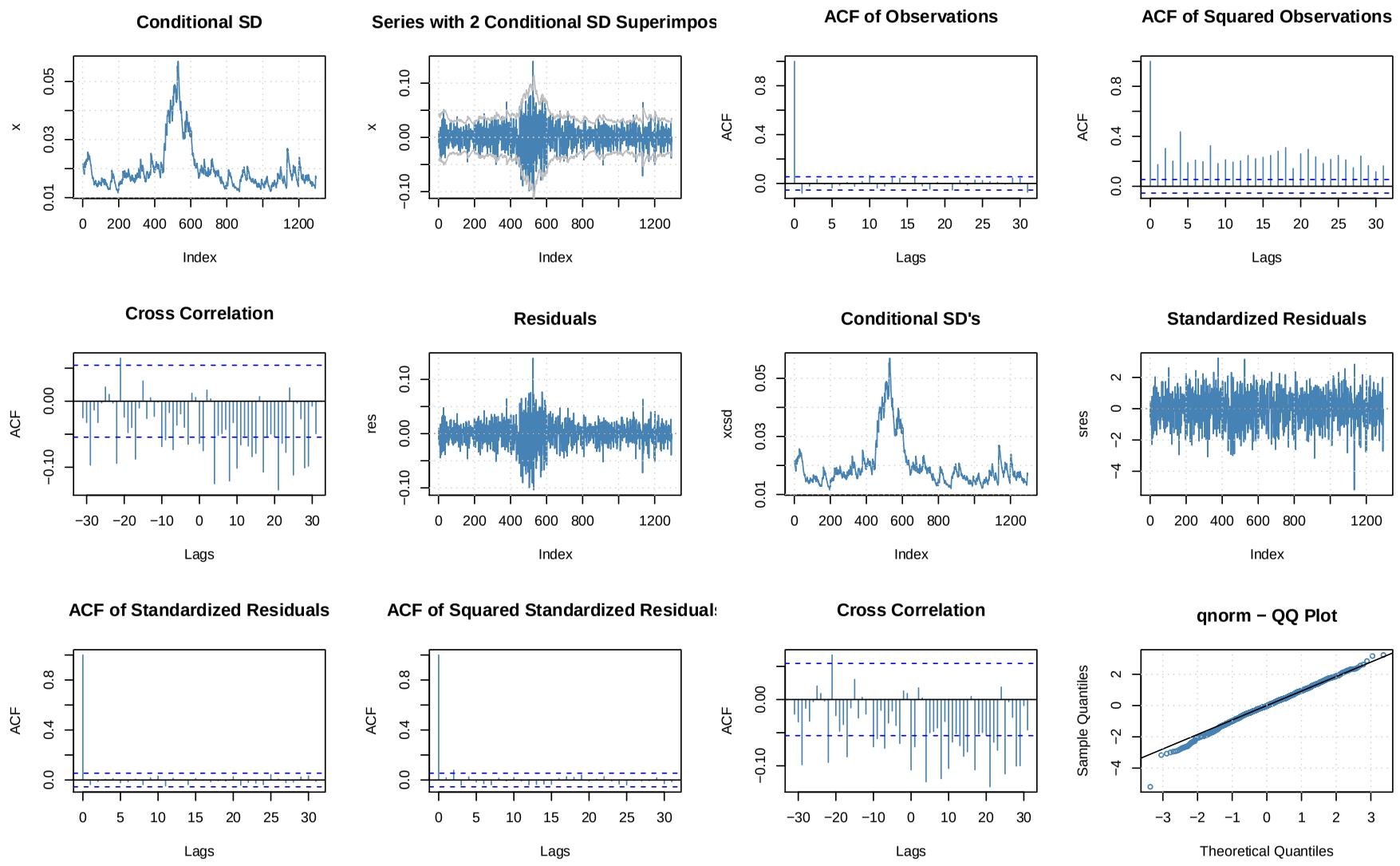


Figure 21: GARCH(1,1) diagnostic plots for CO2.Comdty

2.5.3 AR(1)-EGARCH(1,1)-Model Fit

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model : eGARCH(1,1)
Mean Model : ARFIMA(1,0,0)
Distribution : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.000671    0.000453   1.4824 0.138224
ar1    -0.038424    0.027321  -1.4064 0.159608
omega  -0.120813    0.038978  -3.0995 0.001939
alpha1 -0.070228    0.016442  -4.2714 0.000019
beta1   0.984697    0.004934 199.5723 0.000000
gamma1  0.106258    0.023117   4.5965 0.000004

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.000671    0.000441   1.5215 0.128147
ar1    -0.038424    0.027001  -1.4230 0.154724
omega  -0.120813    0.049275  -2.4518 0.014213
alpha1 -0.070228    0.022226  -3.1597 0.001580
beta1   0.984697    0.006306 156.1519 0.000000
gamma1  0.106258    0.024769   4.2900 0.000018

LogLikelihood : 3320.936

Information Criteria
------------------------------------

Akaike       -5.1196
Bayes        -5.0957
Shibata      -5.1196
Hannan-Quinn -5.1106

Q-Statistics on Standardized Residuals
------------------------------------
      statistic p-value
Lag10     4.321  0.8890
Lag15     9.550  0.7943
Lag20    10.865  0.9283

H0 : No serial correlation

Q-Statistics on Standardized Squared Residuals
------------------------------------
      statistic p-value
Lag10     15.15 0.08691
Lag15     19.73 0.13895
Lag20     22.66 0.25283

ARCH LM Tests
------------------------------------
             Statistic DoF P-Value
ARCH Lag[2]      7.561   2 0.02281
ARCH Lag[5]      9.686   5 0.08465
ARCH Lag[10]    14.562  10 0.14886

Nyblom stability test
------------------------------------
Joint Statistic:  1.2062
Individual Statistics:
mu     0.05867
ar1    0.40506
omega  0.17887
alpha1 0.08790
beta1  0.15607
gamma1 0.11103

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:       1.49 1.68 2.12
Individual Statistic:  0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias          1.50428 0.1328
Negative Sign Bias 0.43307 0.6650
Positive Sign Bias 0.07514 0.9401
Joint Effect       3.90345 0.2721


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     30.70      0.04355
2    30     37.67      0.12981
3    40     43.10      0.30018
4    50     53.76      0.29688


Elapsed time : 0.6272235
```

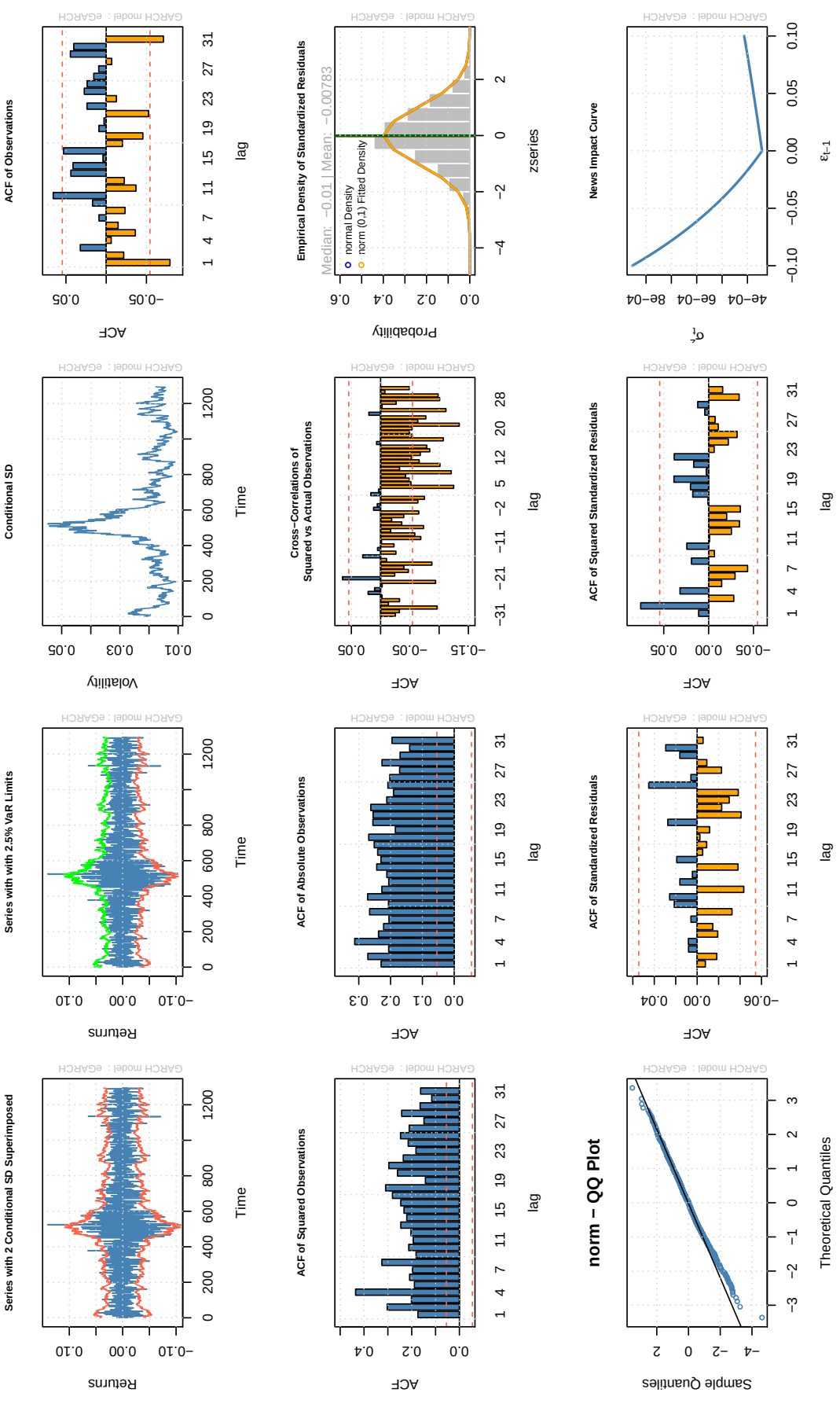


Figure 22: AR(1)-EGARCH(1,1) diagnostic plots for CO2.Comdty

2.5.4 Empirical Fluctuation Process

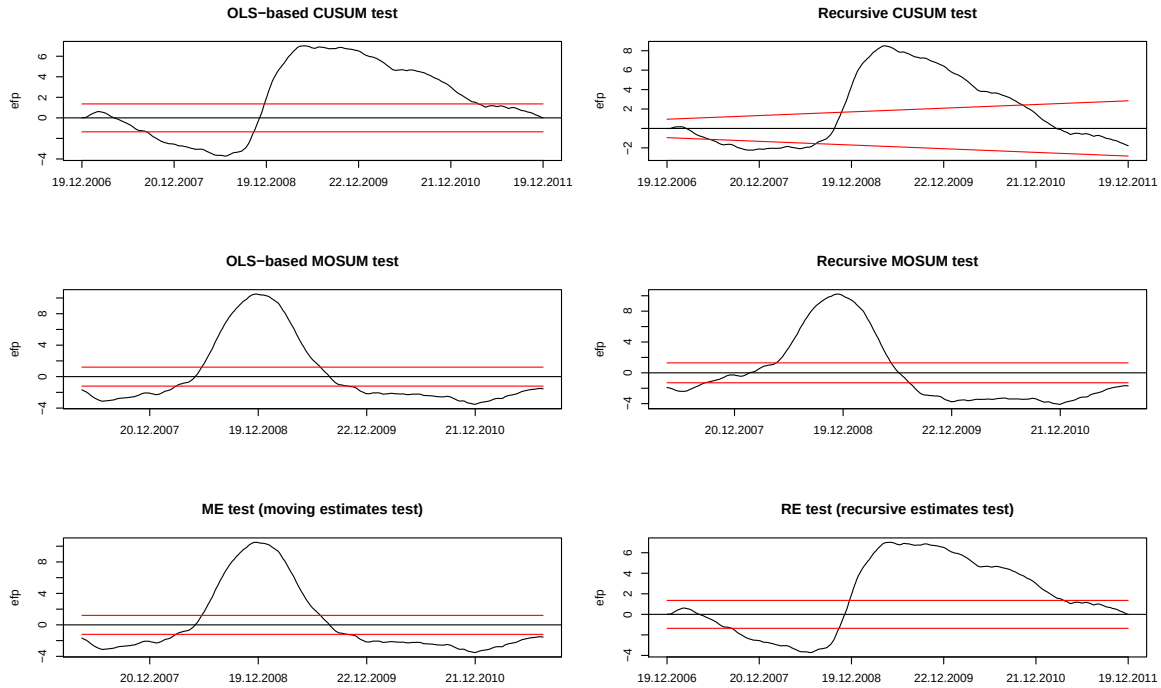


Figure 23: Empirical fluctuation processes for CO2.Comdty

	statistic	p-value
OLS-CUSUM	7.01269	0.00000
Rec-CUSUM	4.38107	0.00000
OLS-MOSUM	10.49481	0.01000
Rec-MOSUM	10.21681	0.01000
ME	10.49481	0.01000
RE	7.01269	0.00000

Table 9: General Test for all models for CO2.Comdty

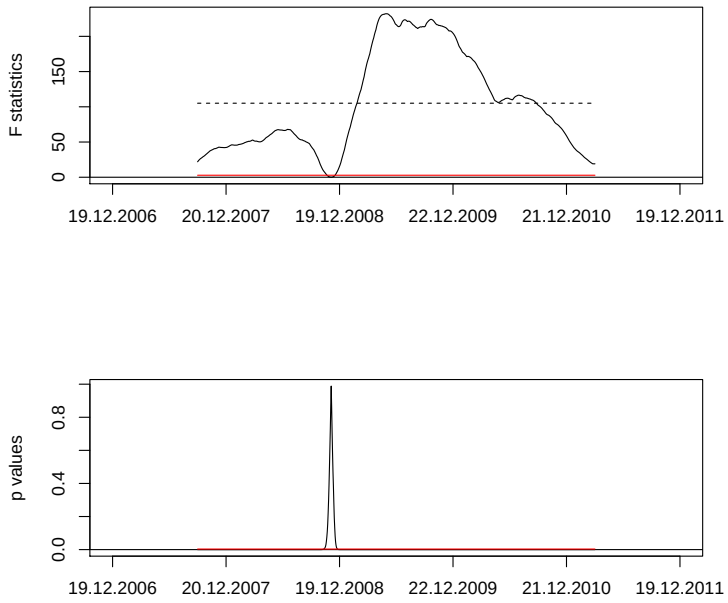


Figure 24: F-test statistic and p-values for CO2.Comdty

	statistic	p-value
sup	232.12711	0.00000
ave	105.09386	0.00000
exp	111.98964	0.00000

Table 10: F-Test with different functionals for CO2.Comdty

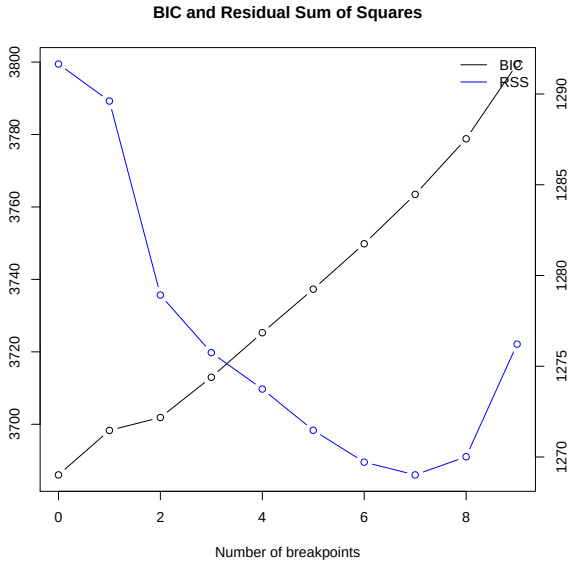


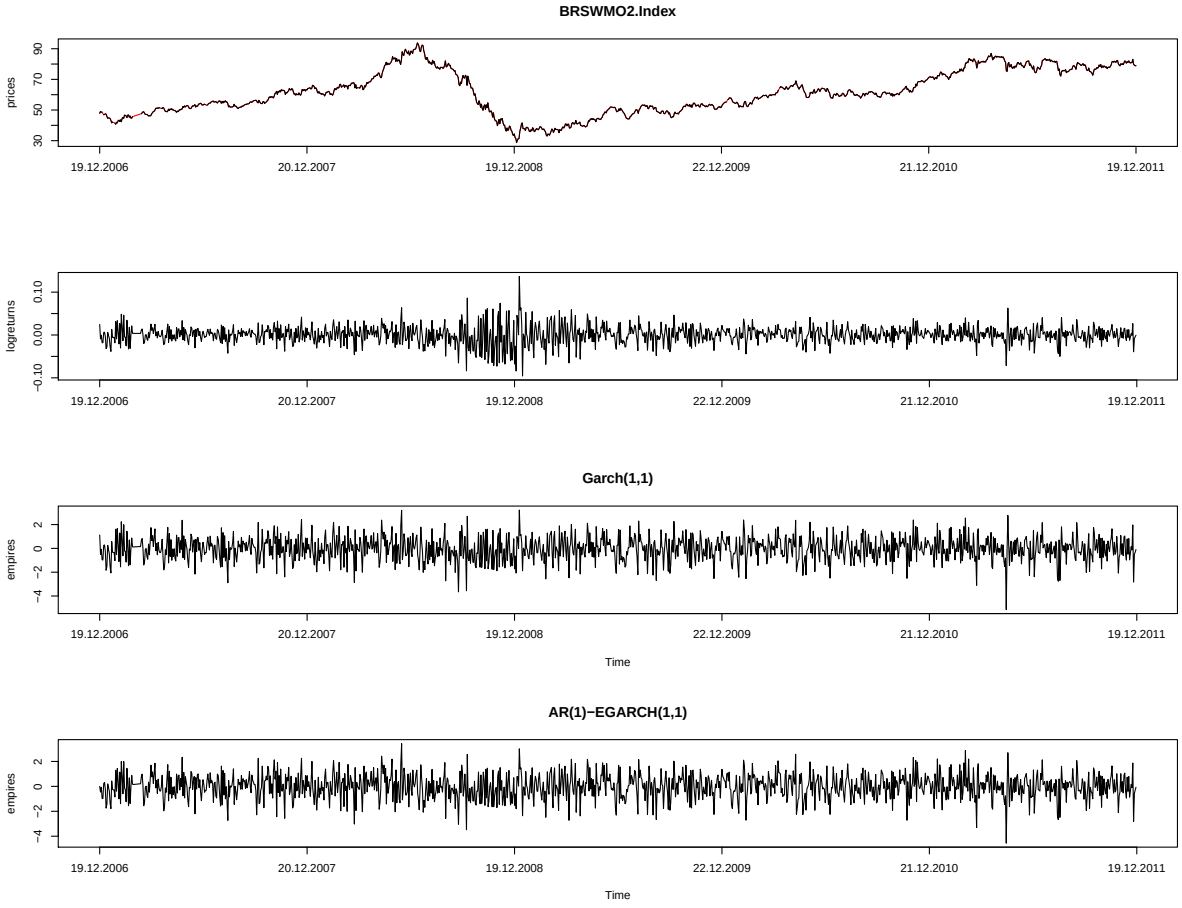
Figure 25: RSS and BIC with respect to number of possible breakpoints for CO2.Comdty

```
 Optimal (m+1)-segment partition:

Call:
breakpoints.formula(formula = x ~ 1, h = 0.1)

Breakpoints at observation number:

m = 1           397
m = 2           392 521
m = 3           392 521                1114
m = 4           392 521             976  1114
m = 5           392 521         870 999  1129
m = 6   173     392 521         870 999  1129
m = 7   173     392 521     715 870 999  1129
m = 8   132 261 392 521     715 870 999  1129
m = 9   132 261 392 521 650 779 908 1037 1166

Corresponding to breakdates:

m = 1                                       0.306563706563707
m = 2                                       0.302702702702703 0.402316602316602
m = 3                                       0.302702702702703 0.402316602316602
m = 4                                       0.302702702702703 0.402316602316602
m = 5                                       0.302702702702703 0.402316602316602
m = 6   0.133590733590734                   0.302702702702703 0.402316602316602
m = 7   0.133590733590734                   0.302702702702703 0.402316602316602
m = 8   0.101930501930502 0.201544401544402 0.302702702702703 0.402316602316602
m = 9   0.101930501930502 0.201544401544402 0.302702702702703 0.402316602316602

m = 1
m = 2
m = 3
m = 4                                                         0.753667953667954
m = 5                                       0.671814671814672 0.771428571428571
m = 6                                       0.671814671814672 0.771428571428571
m = 7                   0.552123552123552 0.671814671814672 0.771428571428571
m = 8                   0.552123552123552 0.671814671814672 0.771428571428571
m = 9   0.501930501930502 0.601544401544401 0.701158301158301 0.800772200772201
```

```
m = 1
m = 2
m = 3   0.86023166023166
m = 4   0.86023166023166
m = 5   0.871814671814672
m = 6   0.871814671814672
m = 7   0.871814671814672
m = 8   0.871814671814672
m = 9   0.9003861003861

Fit:

m   0        1        2        3        4        5        6        7
RSS 1291.653 1289.614 1278.924 1275.749 1273.745 1271.475 1269.715 1269.014
BIC 3686.032 3698.319 3701.871 3712.986 3725.282 3737.304 3749.844 3763.460

m   8        9
RSS 1270.014 1276.218
BIC 3778.813 3799.456
```

2.6 BRSWMO2.Index

2.6.1 Time Series Plots



2.6.2 GARCH(1,1)-Model Fit

```
Title:
 GARCH Modelling

Call:
 garchFit(formula = ~garch(1, 1), data = as.zoo(na.omit(x)), trace = F)

Mean and Variance Equation:
 data ~ garch(1, 1)
<environment: 0xa4403a8>
 [data = as.zoo(na.omit(x))]

Conditional Distribution:
 norm

Coefficient(s):
        mu       omega      alpha1       beta1
1.0557e-03  6.8945e-06  6.5394e-02  9.1581e-01

Std. Errors:
 based on Hessian
```

```
Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     1.056e-03   4.686e-04    2.253 0.024285 *
omega  6.895e-06   3.161e-06    2.181 0.029160 *
alpha1 6.539e-02   1.789e-02    3.655 0.000258 ***
beta1  9.158e-01   2.369e-02   38.659  < 2e-16 ***
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ âĂŹ 1

Log Likelihood:
 3334.199    normalized:  2.574671

Description:
 Wed Jan 11 13:50:14 2012 by user: maggie


Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  37.56763  6.954918e-09
 Shapiro-Wilk Test  R    W      0.9943592 8.328607e-05
 Ljung-Box Test     R    Q(10)  7.204263  0.7060308
 Ljung-Box Test     R    Q(15)  10.07687  0.8148766
 Ljung-Box Test     R    Q(20)  12.32499  0.9044359
 Ljung-Box Test     R^2  Q(10)  12.06722  0.280583
 Ljung-Box Test     R^2  Q(15)  20.27892  0.161492
 Ljung-Box Test     R^2  Q(20)  22.24703  0.3272578
 LM Arch Test       R    TR^2   15.16681  0.2324406

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-5.143164 -5.127207 -5.143183 -5.137176
```

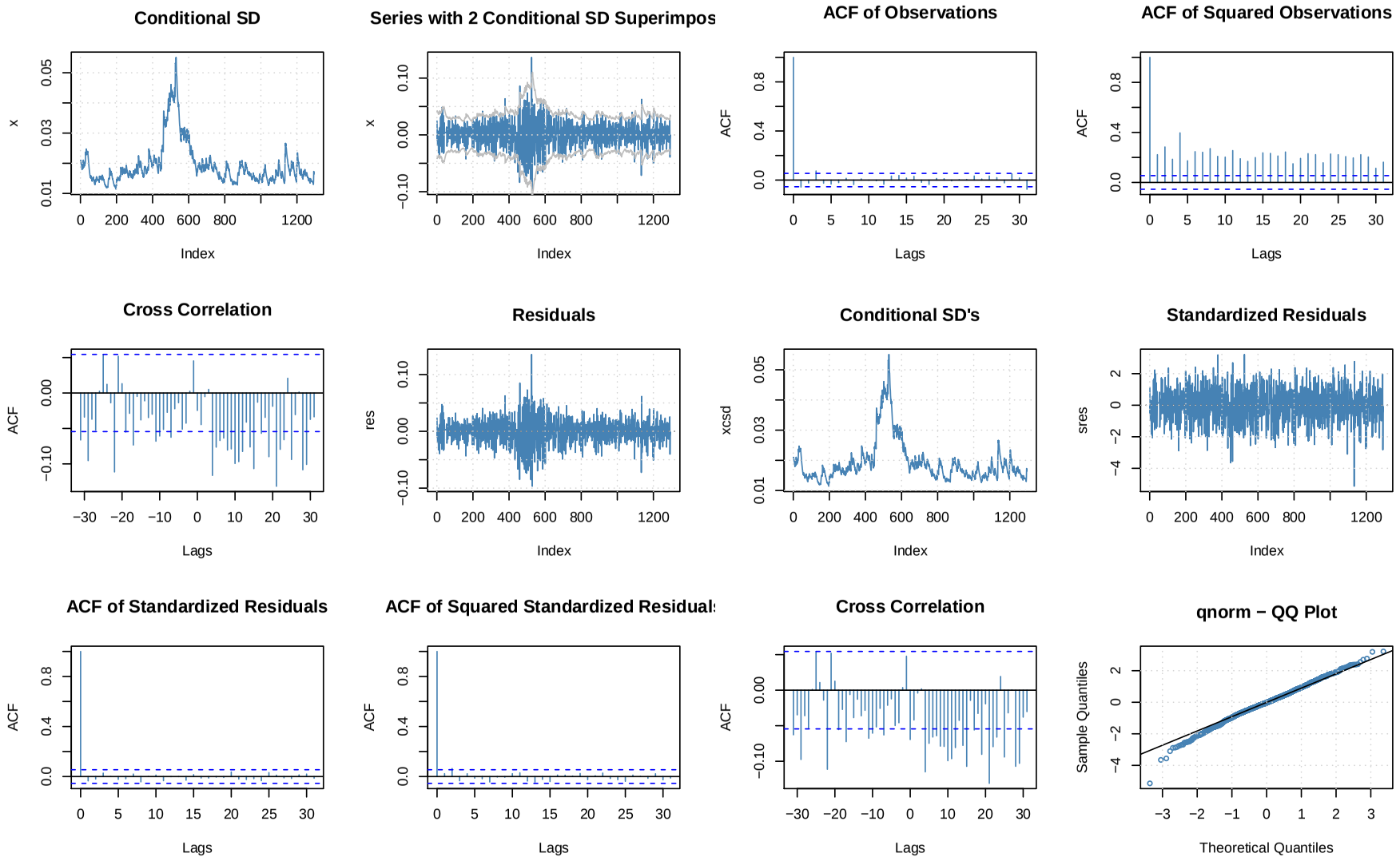


Figure 26: GARCH(1,1) diagnostic plots for BRSWMO2.Index

2.6.3 AR(1)-EGARCH(1,1)-Model Fit

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model : eGARCH(1,1)
Mean Model : ARFIMA(1,0,0)
Distribution : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.000595    0.000453   1.3155 0.188350
ar1    -0.038086    0.030290  -1.2574 0.208620
omega  -0.155002    0.053472  -2.8987 0.003747
alpha1 -0.072745    0.018306  -3.9739 0.000071
beta1   0.980386    0.006733 145.6198 0.000000
gamma1  0.116095    0.026899   4.3160 0.000016

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.000595    0.000436   1.3643 0.172479
ar1    -0.038086    0.032605  -1.1681 0.242759
omega  -0.155002    0.075731  -2.0467 0.040683
alpha1 -0.072745    0.026676  -2.7270 0.006391
beta1   0.980386    0.009602 102.1056 0.000000
gamma1  0.116095    0.032386   3.5847 0.000337

LogLikelihood : 3344.694

Information Criteria
------------------------------------

Akaike       -5.1563
Bayes        -5.1323
Shibata      -5.1563
Hannan-Quinn -5.1473

Q-Statistics on Standardized Residuals
------------------------------------
      statistic p-value
Lag10     5.123  0.8235
Lag15     8.420  0.8663
Lag20    10.810  0.9301

H0 : No serial correlation

Q-Statistics on Standardized Squared Residuals
------------------------------------
      statistic p-value
Lag10     11.82  0.2234
Lag15     18.59  0.1811
Lag20     20.08  0.3897

ARCH LM Tests
------------------------------------
             Statistic DoF P-Value
ARCH Lag[2]      5.059   2 0.07972
ARCH Lag[5]      6.136   5 0.29321
ARCH Lag[10]    11.384  10 0.32842

Nyblom stability test
------------------------------------
Joint Statistic:  1.0961
Individual Statistics:
mu     0.06359
ar1    0.22712
omega  0.17295
alpha1 0.09914
beta1  0.15144
gamma1 0.19784

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:       1.49 1.68 2.12
Individual Statistic:  0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           1.6514 0.0989   *
Negative Sign Bias  0.4746 0.6352
Positive Sign Bias  0.5555 0.5786
Joint Effect        3.1878 0.3636


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     21.34       0.3183
2    30     27.02       0.5709
3    40     43.04       0.3025
4    50     52.61       0.3363


Elapsed time : 0.7066424
```

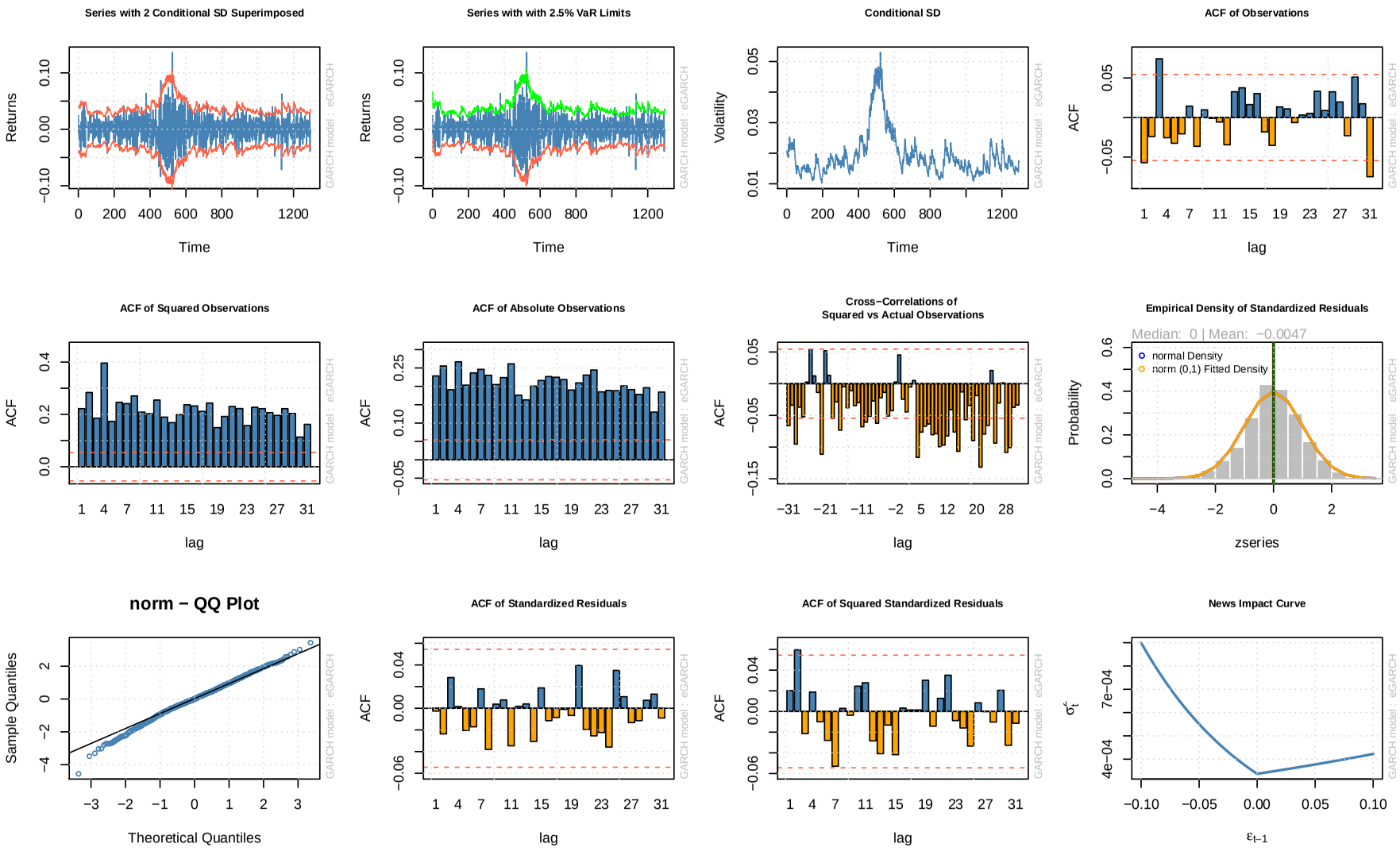


Figure 27: AR(1)-EGARCH(1,1) diagnostic plots for BRSWMO2.Index

2.6.4 Empirical Fluctuation Process

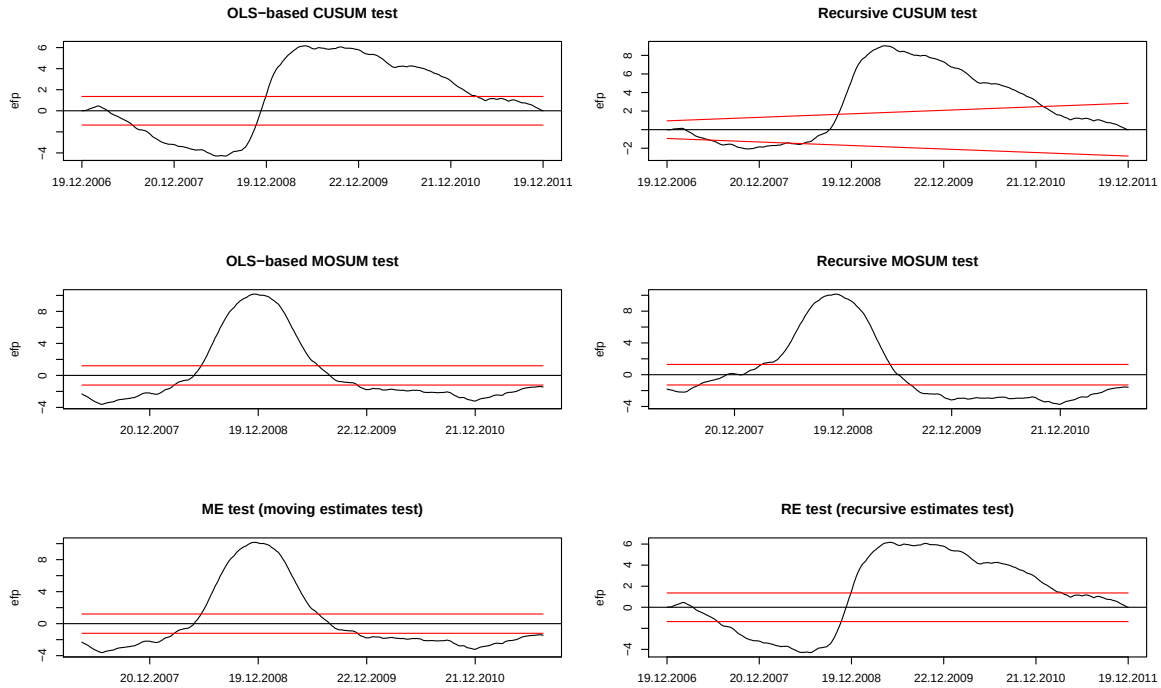


Figure 28: Empirical fluctuation processes for BRSWMO2.Index

	statistic	p-value
OLS-CUSUM	6.16646	0.00000
Rec-CUSUM	4.65191	0.00000
OLS-MOSUM	10.16339	0.01000
Rec-MOSUM	10.15038	0.01000
ME	10.16339	0.01000
RE	6.16646	0.00000

Table 11: General Test for all models for BRSWMO2.Index

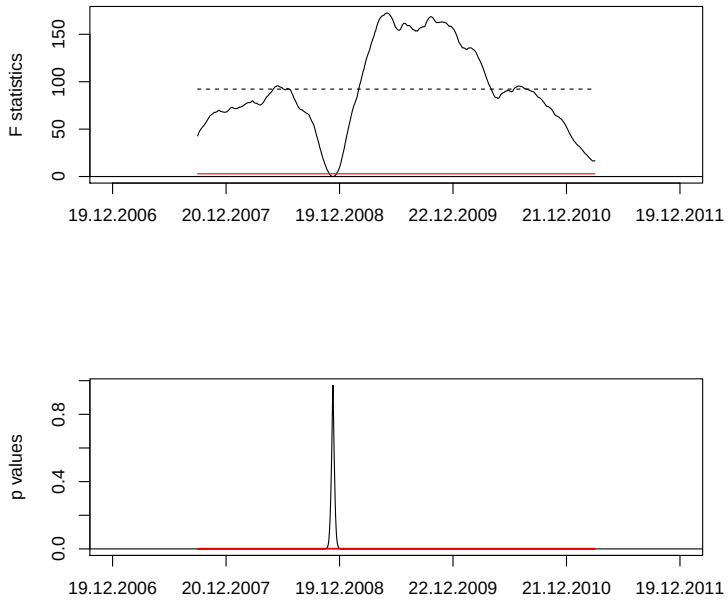


Figure 29: F-test statistic and p-values for BRSWMO2.Index

	statistic	p-value
sup	172.44240	0.00000
ave	92.20582	0.00000
exp	82.06458	0.00000

Table 12: F-Test with different functionals for BRSWMO2.Index

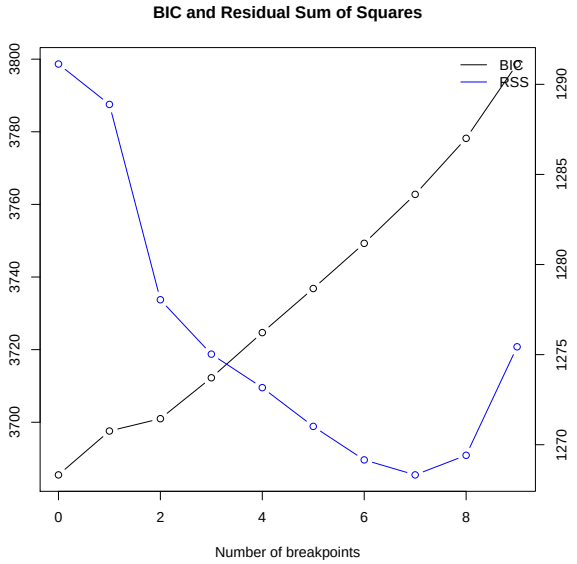


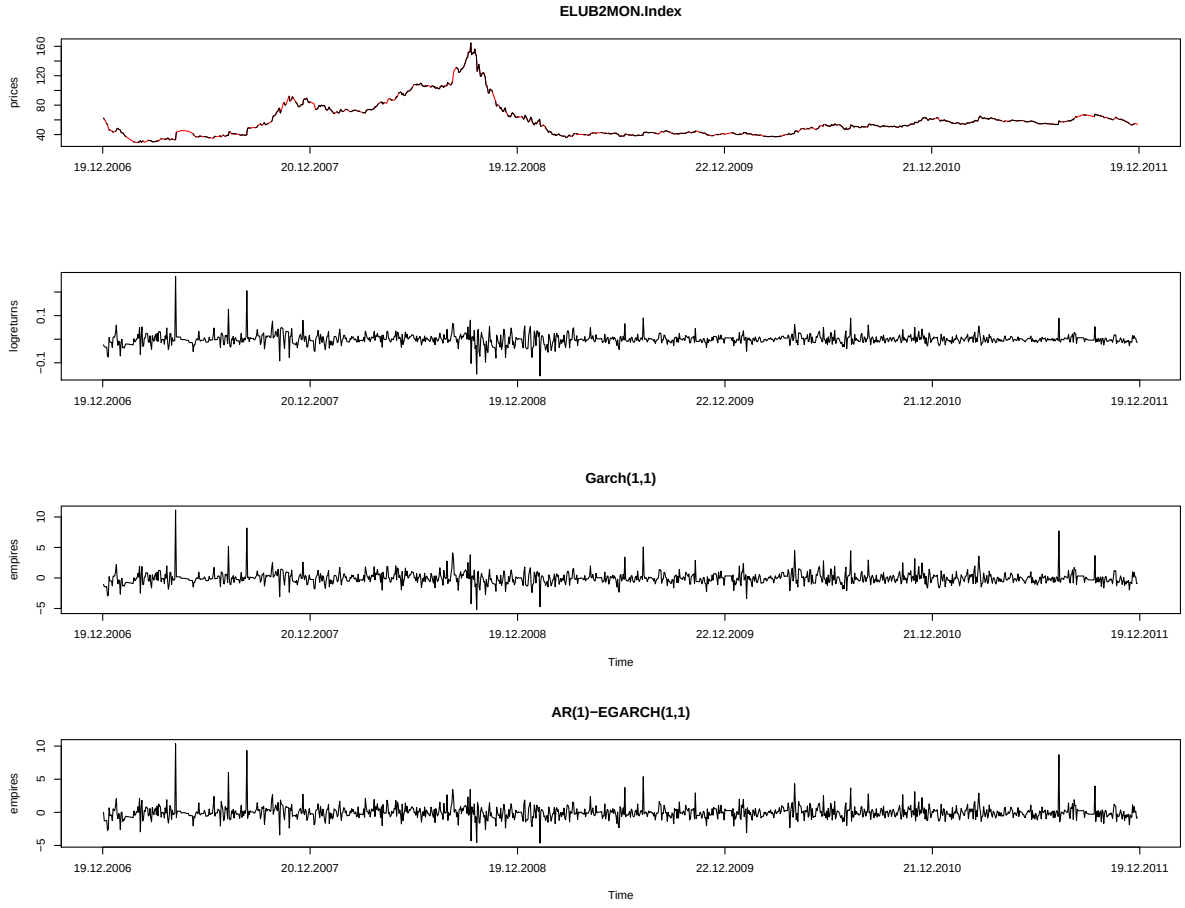
Figure 30: RSS and BIC with respect to number of possible breakpoints for BRSWMO2.Index

```
 Optimal (m+1)-segment partition:

Call:
breakpoints.formula(formula = x ~ 1, h = 0.1)

Breakpoints at observation number:

m = 1           397
m = 2           392 521
m = 3           392 521                 1114
m = 4   173     392 521                 1114
m = 5           392 521         870 999 1129
m = 6   173     392 521         870 999 1129
m = 7   173     392 521     715 870 999 1129
m = 8   129 259 392 521     715 870 999 1129
m = 9   129 259 392 521 650 779 908 1037 1166

Corresponding to breakdates:

m = 1                          0.306563706563707
m = 2                          0.302702702702703 0.402316602316602
m = 3                          0.302702702702703 0.402316602316602
m = 4   0.133590733590734      0.302702702702703 0.402316602316602
m = 5                          0.302702702702703 0.402316602316602
m = 6   0.133590733590734      0.302702702702703 0.402316602316602
m = 7   0.133590733590734      0.302702702702703 0.402316602316602
m = 8   0.0996138996138996 0.2 0.302702702702703 0.402316602316602
m = 9   0.0996138996138996 0.2 0.302702702702703 0.402316602316602

m = 1
m = 2
m = 3
m = 4
m = 5                                      0.671814671814672 0.771428571428571
m = 6                                      0.671814671814672 0.771428571428571
m = 7                     0.552123552123552 0.671814671814672 0.771428571428571
m = 8                     0.552123552123552 0.671814671814672 0.771428571428571
m = 9   0.501930501930502 0.601544401544401 0.701158301158301 0.800772200772201
```

```
m = 1
m = 2
m = 3   0.86023166023166
m = 4   0.86023166023166
m = 5   0.871814671814672
m = 6   0.871814671814672
m = 7   0.871814671814672
m = 8   0.871814671814672
m = 9   0.9003861003861

Fit:

m   0        1        2        3        4        5        6        7
RSS 1291.128 1288.886 1278.046 1275.028 1273.169 1271.018 1269.159 1268.327
BIC 3685.506 3697.587 3700.983 3712.253 3724.696 3736.839 3749.276 3762.759

m   8        9
RSS 1269.414 1275.436
BIC 3778.202 3798.662
```

2.7 ELUB2MON.Index

2.7.1 Time Series Plots



2.7.2 GARCH(1,1)-Model Fit

```
Title:
 GARCH Modelling

Call:
 garchFit(formula = ~garch(1, 1), data = as.zoo(na.omit(x)), trace = F)

Mean and Variance Equation:
 data ~ garch(1, 1)
<environment: 0xa31d330>
 [data = as.zoo(na.omit(x))]

Conditional Distribution:
 norm

Coefficient(s):
        mu       omega      alpha1       beta1
7.6426e-04  2.5886e-06  2.5708e-02  9.6968e-01

Std. Errors:
 based on Hessian
```

```
Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     7.643e-04   5.325e-04    1.435 0.151212
omega  2.589e-06   6.873e-07    3.766 0.000166 ***
alpha1 2.571e-02   4.257e-03    6.039 1.55e-09 ***
beta1  9.697e-01   4.328e-03  224.034  < 2e-16 ***
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ âĂŹ 1

Log Likelihood:
 3156.525    normalized:  2.443131

Description:
 Wed Jan 11 13:50:15 2012 by user: maggie


Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  22696.55  0
 Shapiro-Wilk Test  R    W      0.8477768 0
 Ljung-Box Test     R    Q(10)  29.43059  0.001060884
 Ljung-Box Test     R    Q(15)  34.56785  0.002832128
 Ljung-Box Test     R    Q(20)  43.52018  0.001744088
 Ljung-Box Test     R^2  Q(10)  1.083316  0.9997518
 Ljung-Box Test     R^2  Q(15)  1.976483  0.9999726
 Ljung-Box Test     R^2  Q(20)  2.58498   0.9999989
 LM Arch Test       R    TR^2   1.440412  0.999895

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-4.880069 -4.864082 -4.880089 -4.874069
```

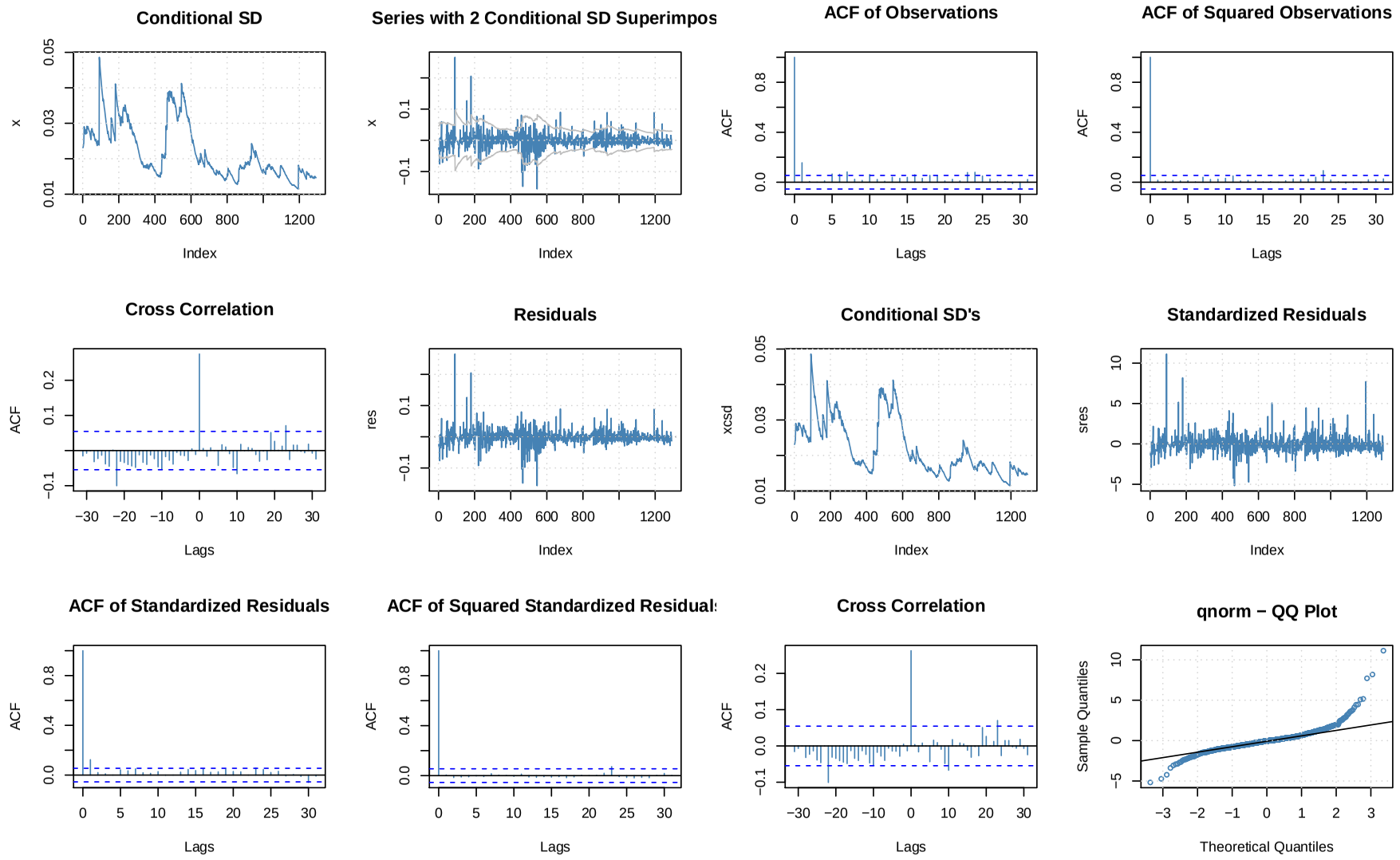


Figure 31: GARCH(1,1) diagnostic plots for ELUB2MON.Index

2.7.3 AR(1)-EGARCH(1,1)-Model Fit

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model : eGARCH(1,1)
Mean Model : ARFIMA(1,0,0)
Distribution : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.000872    0.000621   1.4038 0.160385
ar1     0.164143    0.031453   5.2187 0.000000
omega  -0.088920    0.032119  -2.7684 0.005633
alpha1  0.012231    0.010930   1.1190 0.263142
beta1   0.986765    0.004394 224.5743 0.000000
gamma1  0.095785    0.018398   5.2062 0.000000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.000872    0.000776  1.12430 0.260887
ar1     0.164143    0.047063  3.48773 0.000487
omega  -0.088920    0.118155 -0.75257 0.451709
alpha1  0.012231    0.025575  0.47822 0.632492
beta1   0.986765    0.015615 63.19433 0.000000
gamma1  0.095785    0.047781  2.00467 0.044999

LogLikelihood : 3186.322

Information Criteria
------------------------------------

Akaike       -4.9231
Bayes        -4.8991
Shibata      -4.9231
Hannan-Quinn -4.9141

Q-Statistics on Standardized Residuals
------------------------------------
      statistic p-value
Lag10     5.863  0.7536
Lag15     8.166  0.8805
Lag20    14.608  0.7472

H0 : No serial correlation

Q-Statistics on Standardized Squared Residuals
------------------------------------
      statistic p-value
Lag10     1.385  0.9979
Lag15     2.235  0.9998
Lag20     2.694  1.0000

ARCH LM Tests
------------------------------------
             Statistic DoF P-Value
ARCH Lag[2]     0.4569   2  0.7958
ARCH Lag[5]     1.0148   5  0.9614
ARCH Lag[10]    1.3522  10  0.9993

Nyblom stability test
------------------------------------
Joint Statistic:  1.4164
Individual Statistics:
mu     0.06077
ar1    0.29274
omega  0.29854
alpha1 0.13129
beta1  0.26642
gamma1 0.11027

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:       1.49 1.68 2.12
Individual Statistic:  0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           0.6812 0.4959
Negative Sign Bias  0.2137 0.8309
Positive Sign Bias  1.1081 0.2680
Joint Effect        1.2993 0.7293


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     187.3    1.145e-29
2    30     209.1    3.536e-29
3    40     223.8    8.759e-28
4    50     233.5    7.568e-26


Elapsed time : 0.8092835
```

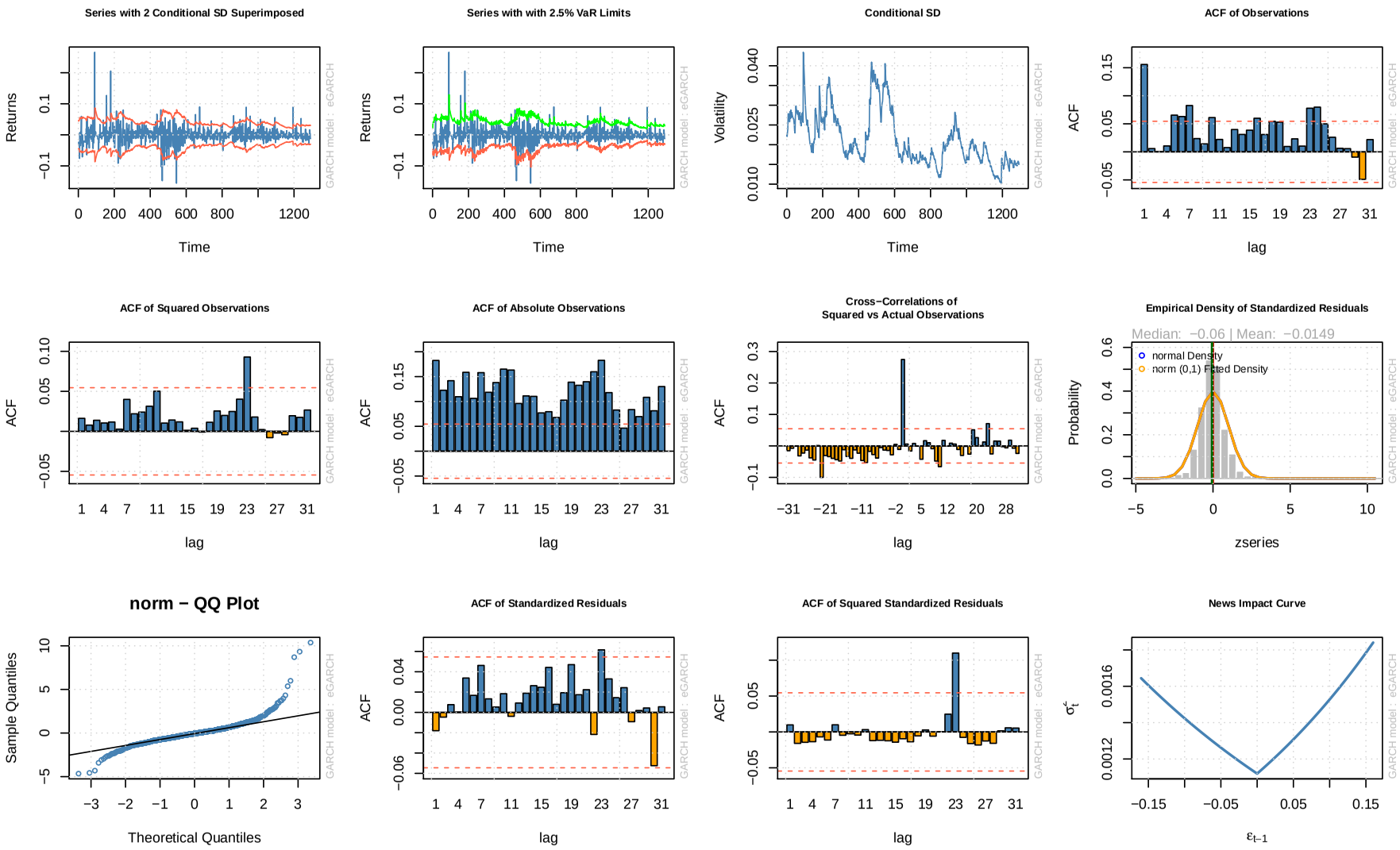


Figure 32: AR(1)-EGARCH(1,1) diagnostic plots for ELUB2MON.Index

2.7.4 Empirical Fluctuation Process

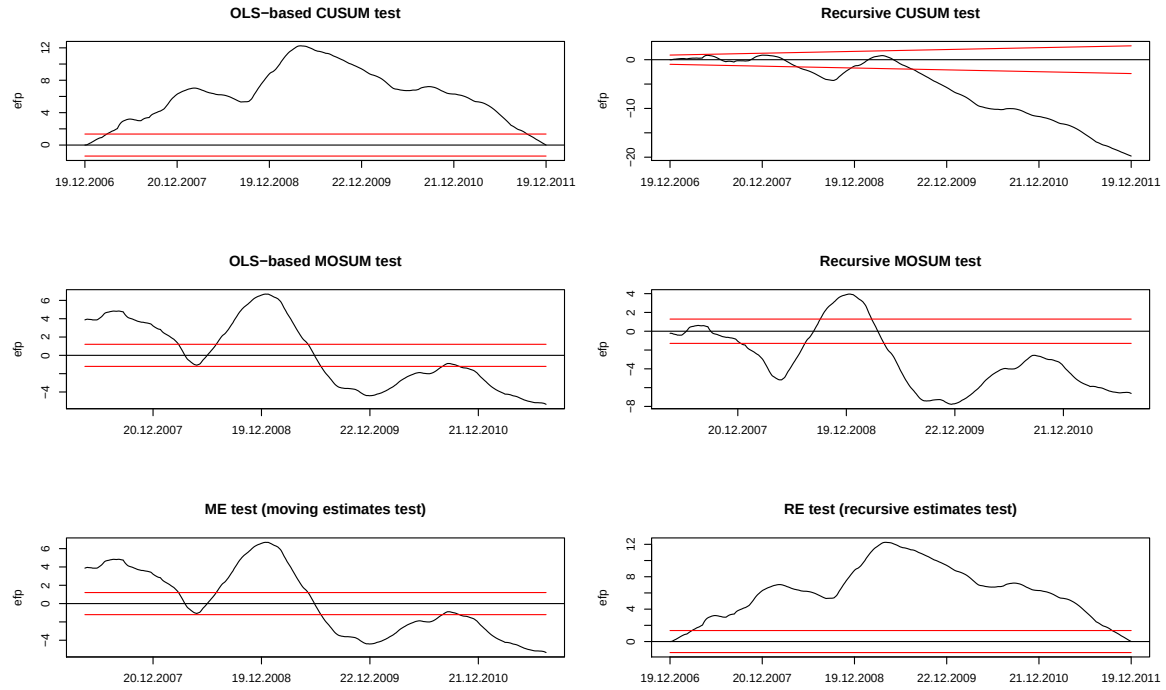


Figure 33: Empirical fluctuation processes for ELUB2MON.Index

	statistic	p-value
OLS-CUSUM	12.24786	0.00000
Rec-CUSUM	6.58991	0.00000
OLS-MOSUM	6.68879	0.01000
Rec-MOSUM	7.77380	0.01000
ME	6.68879	0.01000
RE	12.24786	0.00000

Table 13: General Test for all models for ELUB2MON.Index

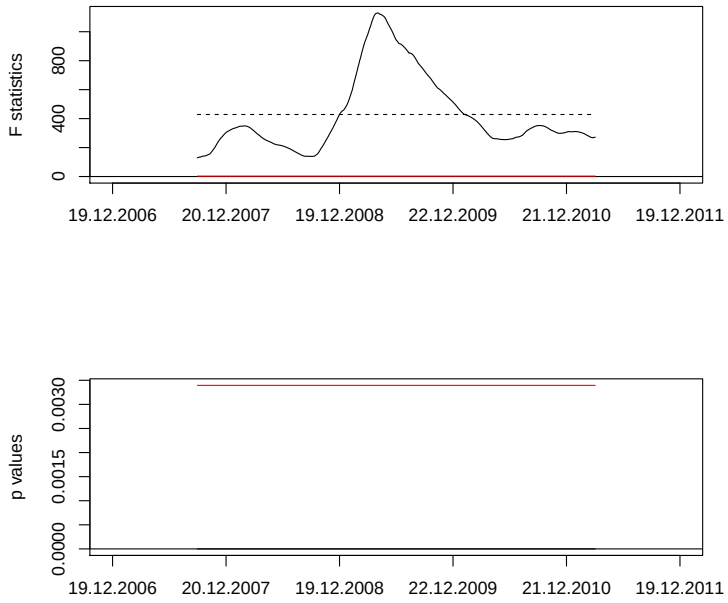


Figure 34: F-test statistic and p-values for ELUB2MON.Index

	statistic	p-value
sup	1129.17562	0.00000
ave	428.43308	0.00000
exp	559.30990	0.00000

Table 14: F-Test with different functionals for ELUB2MON.Index

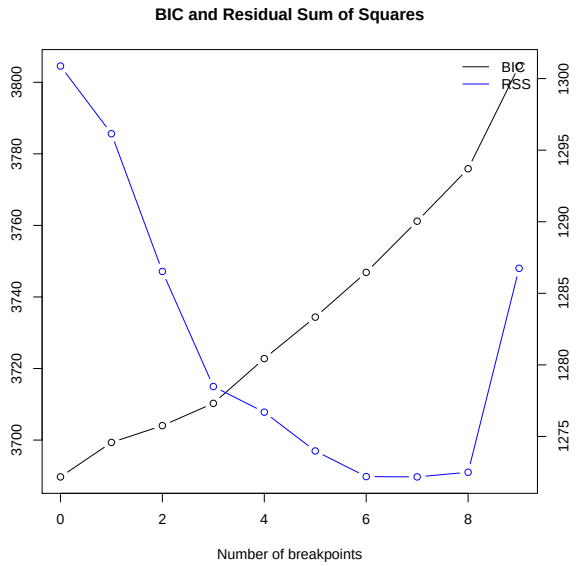


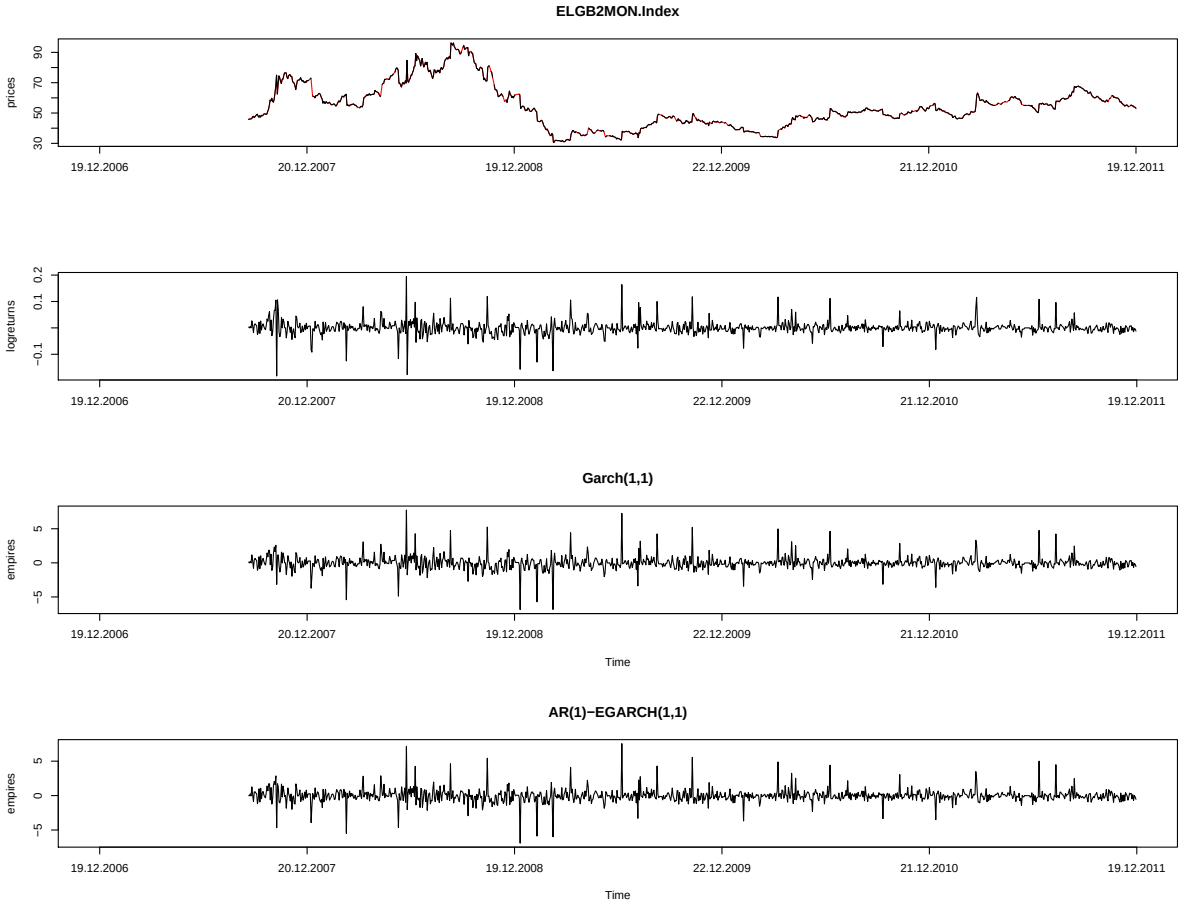
Figure 35: RSS and BIC with respect to number of possible breakpoints for ELUB2MON.Index

```
 Optimal (m+1)-segment partition:

Call:
breakpoints.formula(formula = x ~ 1, h = 0.1)

Breakpoints at observation number:

m = 1              459
m = 2   137        459
m = 3   137        459 593
m = 4   137     327 459 593
m = 5   137        459 593 838     1026
m = 6   137     327 459 593 838     1026
m = 7   137     327 459 593 838     1026 1159
m = 8   137     327 459 589 718 847 1026 1159
m = 9   131 260 389 518 647 776 905 1034 1163

Corresponding to breakdates:

m = 1                                                          0.355263157894737
m = 2   0.106037151702786                                      0.355263157894737
m = 3   0.106037151702786                                      0.355263157894737
m = 4   0.106037151702786                   0.253095975232198  0.355263157894737
m = 5   0.106037151702786                                      0.355263157894737
m = 6   0.106037151702786                   0.253095975232198  0.355263157894737
m = 7   0.106037151702786                   0.253095975232198  0.355263157894737
m = 8   0.106037151702786                   0.253095975232198  0.355263157894737
m = 9   0.101393188854489 0.201238390092879 0.301083591331269  0.400928792569659

m = 1
m = 2
m = 3   0.458978328173375
m = 4   0.458978328173375
m = 5   0.458978328173375 0.648606811145511                    0.794117647058823
m = 6   0.458978328173375 0.648606811145511                    0.794117647058823
m = 7   0.458978328173375 0.648606811145511                    0.794117647058823
m = 8   0.455882352941176 0.555727554179567 0.655572755417957  0.794117647058823
m = 9   0.50077399380805  0.60061919504644  0.70046439628483   0.80030959752322
```

```
m = 1
m = 2
m = 3
m = 4
m = 5
m = 6
m = 7   0.897058823529412
m = 8   0.897058823529412
m = 9   0.90015479876161

Fit:

m   0        1        2        3        4        5        6        7
RSS 1300.878 1296.152 1286.528 1278.489 1276.698 1273.987 1272.195 1272.175
BIC 3689.713 3699.338 3704.038 3710.266 3722.783 3734.364 3746.874 3761.182

m   8        9
RSS 1272.500 1286.743
BIC 3775.840 3804.549
```

2.8 ELGB2MON.Index

2.8.1 Time Series Plots



2.8.2 GARCH(1,1)-Model Fit

```
Title:
 GARCH Modelling

Call:
 garchFit(formula = ~garch(1, 1), data = as.zoo(na.omit(x)), trace = F)

Mean and Variance Equation:
 data ~ garch(1, 1)
<environment: 0xa41b798>
 [data = as.zoo(na.omit(x))]

Conditional Distribution:
 norm

Coefficient(s):
        mu       omega      alpha1       beta1
0.00034611  0.00049799  0.25604244  0.01921193

Std. Errors:
 based on Hessian
```

```
Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     3.461e-04   7.321e-04    0.473    0.636
omega  4.980e-04   4.052e-05   12.290  < 2e-16 ***
alpha1 2.560e-01   5.965e-02    4.293 1.77e-05 ***
beta1  1.921e-02   5.204e-02    0.369    0.712
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ âĂŹ 1

Log Likelihood:
 2539.75    normalized:  2.290126

Description:
 Wed Jan 11 13:50:15 2012 by user: maggie


Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  11237     0
 Shapiro-Wilk Test  R    W      0.7911191 0
 Ljung-Box Test     R    Q(10)  7.764792  0.6517994
 Ljung-Box Test     R    Q(15)  13.23485  0.5841639
 Ljung-Box Test     R    Q(20)  14.01493  0.8297382
 Ljung-Box Test     R^2  Q(10)  5.007974  0.8906447
 Ljung-Box Test     R^2  Q(15)  7.721311  0.934469
 Ljung-Box Test     R^2  Q(20)  20.28501  0.4402325
 LM Arch Test       R    TR^2   6.991164  0.8581969

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-4.573038 -4.554963 -4.573064 -4.566203
```

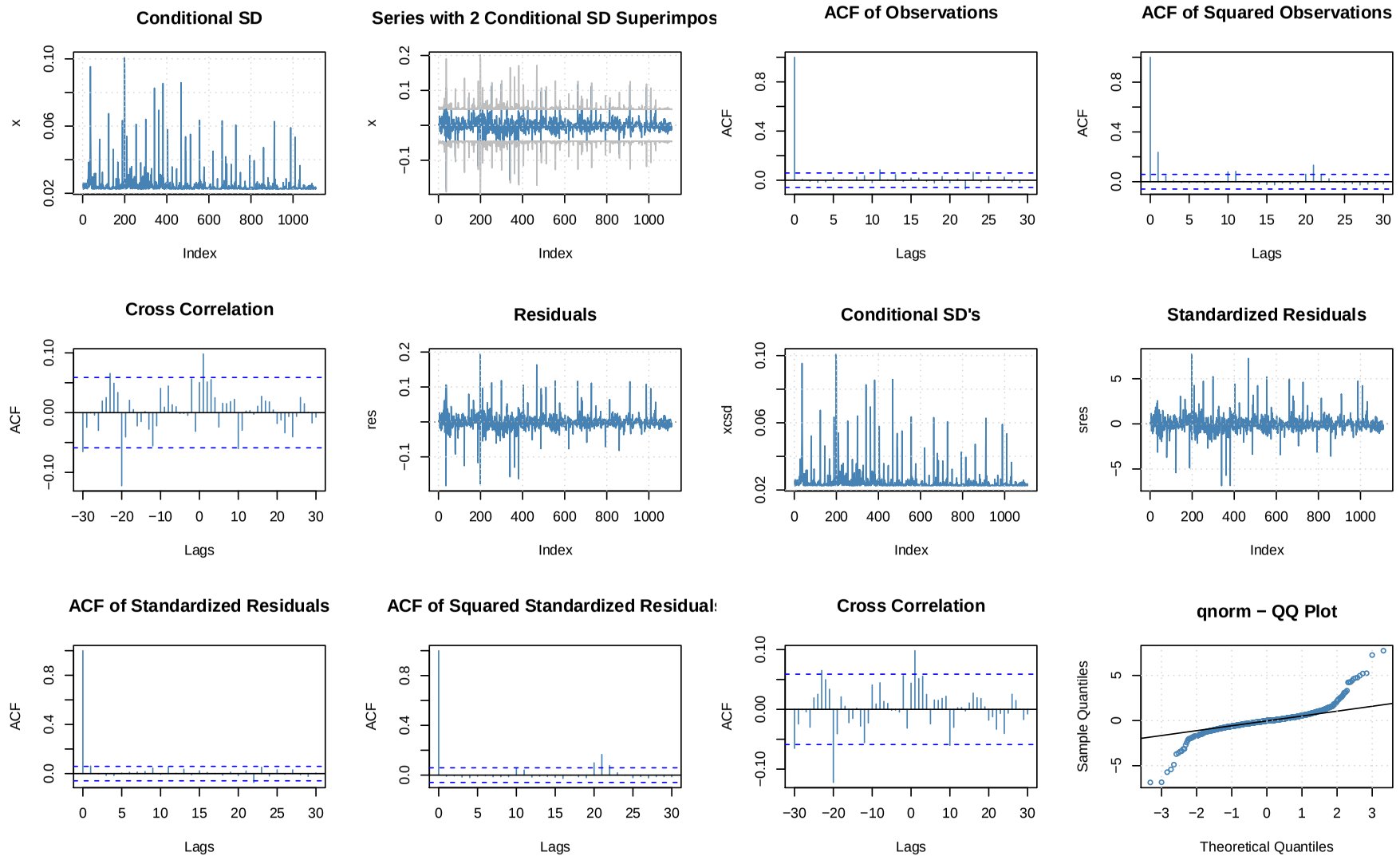


Figure 36: GARCH(1,1) diagnostic plots for ELGB2MON.Index

2.8.3 AR(1)-EGARCH(1,1)-Model Fit

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model : eGARCH(1,1)
Mean Model : ARFIMA(1,0,0)
Distribution : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.000319    0.000819  0.38931 0.697045
ar1     0.159865    0.043917  3.64012 0.000273
omega  -5.357840    0.938621 -5.70820 0.000000
alpha1 -0.032504    0.048219 -0.67409 0.500257
beta1   0.265693    0.126824  2.09497 0.036174
gamma1  0.461992    0.062962  7.33764 0.000000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.000319    0.000950  0.33592 0.736930
ar1     0.159865    0.055530  2.87888 0.003991
omega  -5.357840    2.081222 -2.57437 0.010042
alpha1 -0.032504    0.089399 -0.36358 0.716174
beta1   0.265693    0.282216  0.94145 0.346473
gamma1  0.461992    0.137495  3.36005 0.000779

LogLikelihood : 2546.296

Information Criteria
------------------------------------

Akaike       -4.5812
Bayes        -4.5541
Shibata      -4.5813
Hannan-Quinn -4.5710

Q-Statistics on Standardized Residuals
------------------------------------
      statistic p-value
Lag10     3.840  0.9216
Lag15     8.957  0.8338
Lag20    10.400  0.9424

H0 : No serial correlation

Q-Statistics on Standardized Squared Residuals
------------------------------------
      statistic p-value
Lag10     3.621  0.9345
Lag15     5.826  0.9707
Lag20    15.936  0.6615

ARCH LM Tests
------------------------------------
             Statistic DoF P-Value
ARCH Lag[2]     0.1376   2  0.9335
ARCH Lag[5]     1.0538   5  0.9581
ARCH Lag[10]    3.6009  10  0.9636

Nyblom stability test
------------------------------------
Joint Statistic:  1.6905
Individual Statistics:
mu     0.06754
ar1    0.08417
omega  0.97298
alpha1 0.11208
beta1  0.97716
gamma1 0.29516

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:       1.49 1.68 2.12
Individual Statistic:  0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias          0.36478 0.7153
Negative Sign Bias 0.01229 0.9902
Positive Sign Bias 0.60093 0.5480
Joint Effect       0.37933 0.9445


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     346.9    4.412e-62
2    30     365.3    7.636e-60
3    40     376.7    7.753e-57
4    50     385.3    9.682e-54


Elapsed time : 0.8313982
```

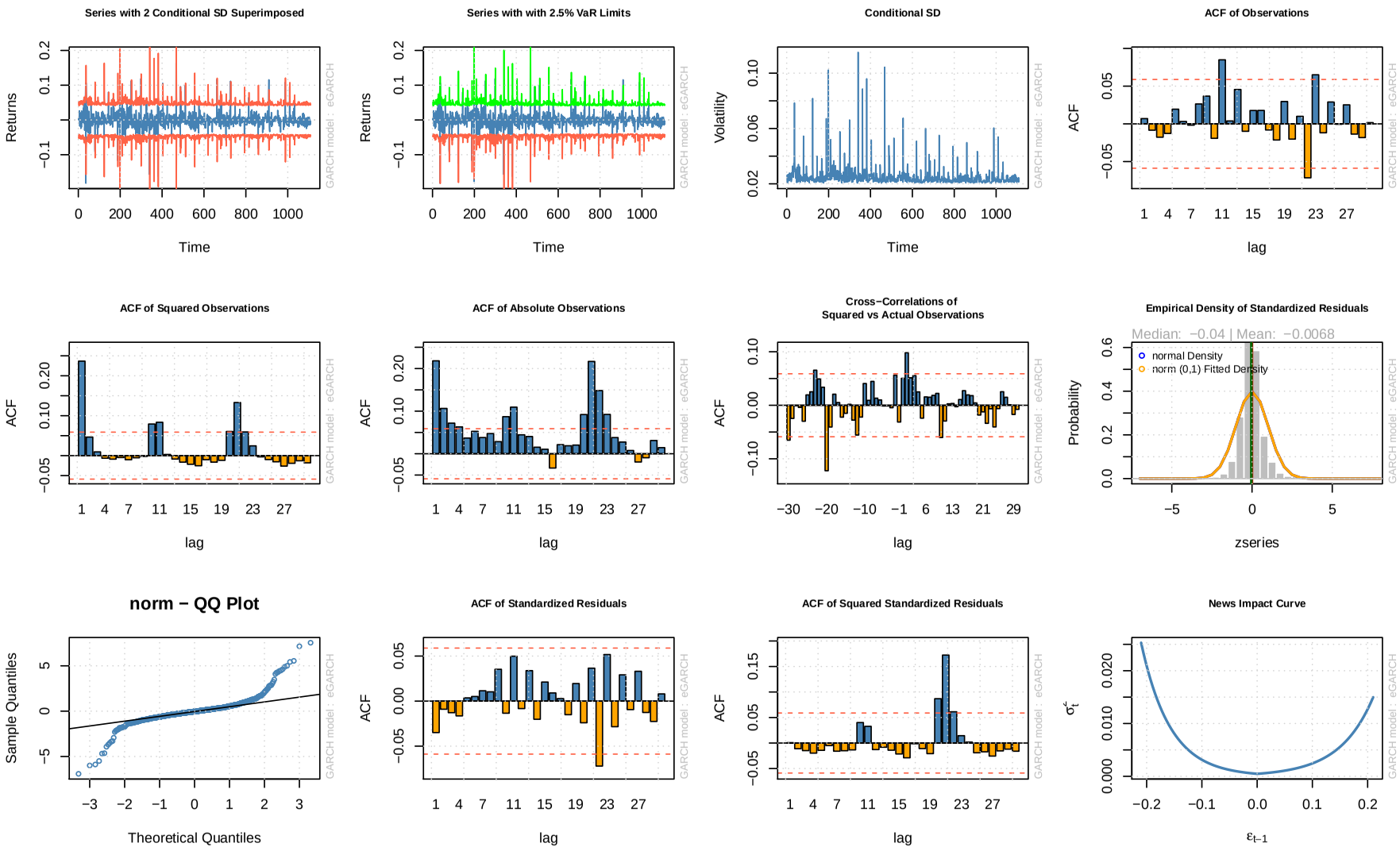


Figure 37: AR(1)-EGARCH(1,1) diagnostic plots for ELGB2MON.Index

2.8.4 Empirical Fluctuation Process

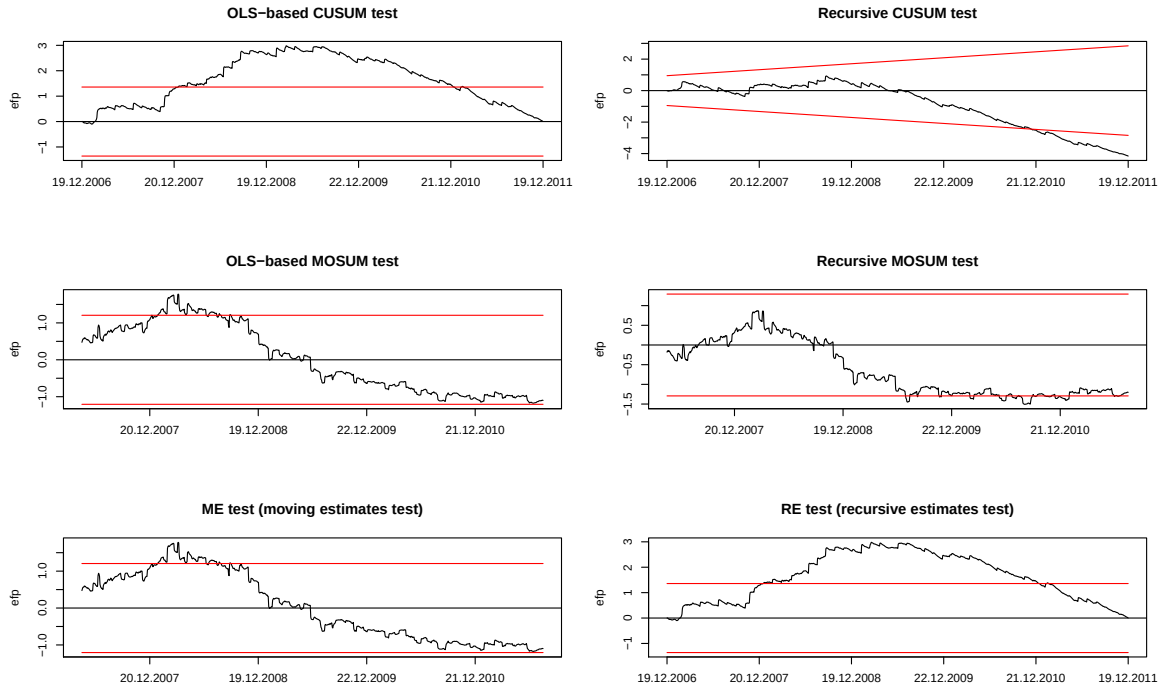


Figure 38: Empirical fluctuation processes for ELGB2MON.Index

	statistic	p-value
OLS-CUSUM	2.98001	0.00000
Rec-CUSUM	1.38586	0.00088
OLS-MOSUM	1.78058	0.01000
Rec-MOSUM	1.50642	0.01000
ME	1.78058	0.01000
RE	2.98001	0.00000

Table 15: General Test for all models for ELGB2MON.Index

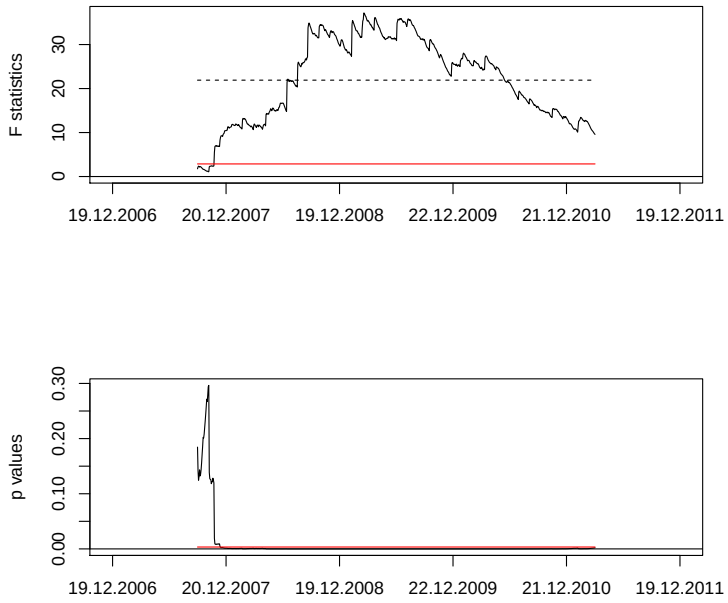


Figure 39: F-test statistic and p-values for ELGB2MON.Index

	statistic	p-value
sup	37.16893	0.00000
ave	21.90474	0.00000
exp	15.71931	0.00000

Table 16: F-Test with different functionals for ELGB2MON.Index

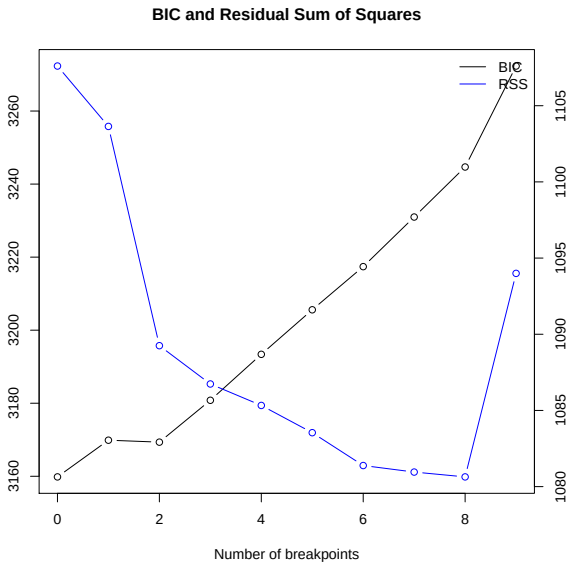


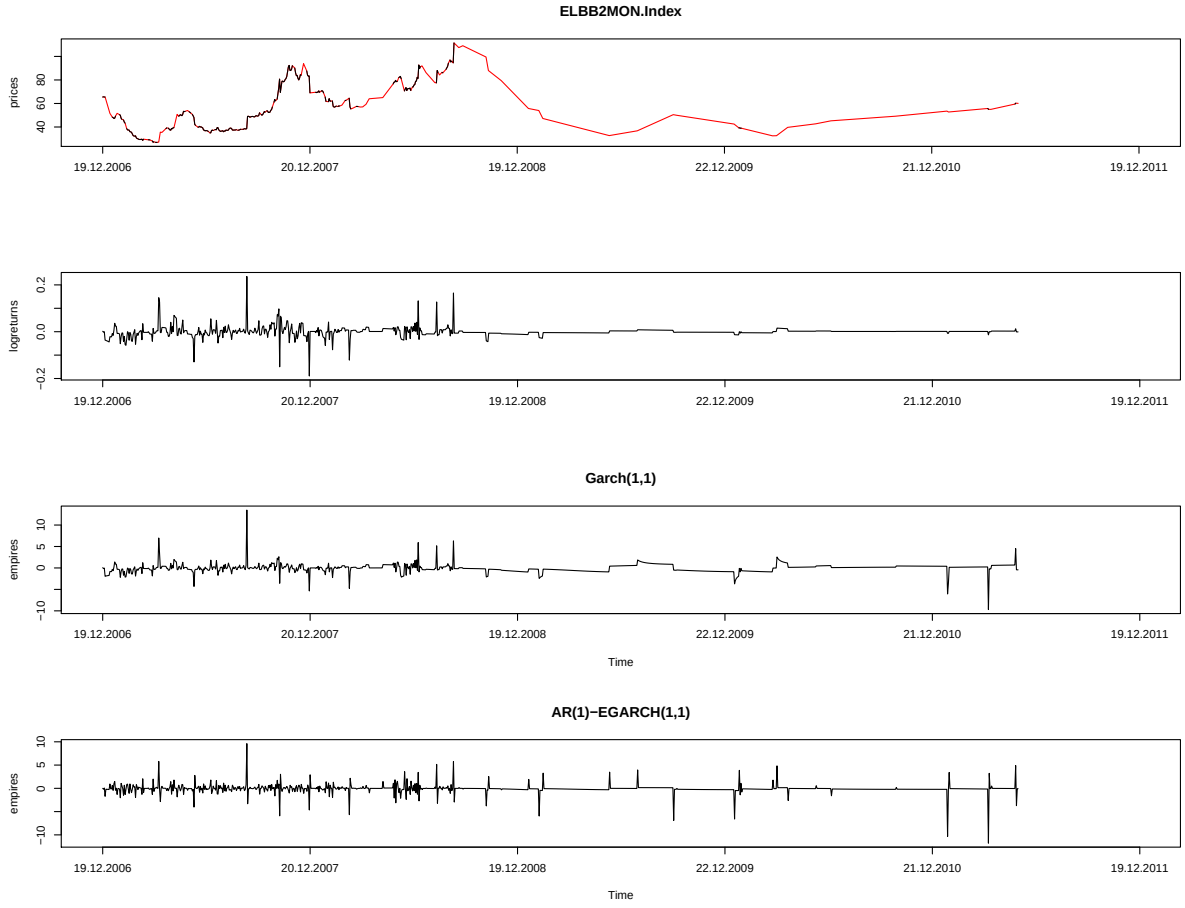
Figure 40: RSS and BIC with respect to number of possible breakpoints for ELGB2MON.Index

```
 Optimal (m+1)-segment partition:

Call:
breakpoints.formula(formula = x ~ 1, h = 0.1)

Breakpoints at observation number:

m = 1       253
m = 2       269 381
m = 3   139 269 381
m = 4   139 269 381     512
m = 5   139 269 381     512 659
m = 6   139 269 381     512 659 771
m = 7   139 269 381     512 659 772 885
m = 8   139 269 381     512 659 772 885 996
m = 9   110 221 336 446 556 666 776 886 996

Corresponding to breakdates:

m = 1                     0.228133453561767
m = 2                     0.242560865644725 0.343552750225428
m = 3   0.125338142470694 0.242560865644725 0.343552750225428
m = 4   0.125338142470694 0.242560865644725 0.343552750225428
m = 5   0.125338142470694 0.242560865644725 0.343552750225428
m = 6   0.125338142470694 0.242560865644725 0.343552750225428
m = 7   0.125338142470694 0.242560865644725 0.343552750225428
m = 8   0.125338142470694 0.242560865644725 0.343552750225428
m = 9   0.0991884580703336 0.199278629395852 0.30297565374211

m = 1
m = 2
m = 3
m = 4                     0.461677186654644
m = 5                     0.461677186654644 0.594229035166817
m = 6                     0.461677186654644 0.594229035166817 0.69522091974752
m = 7                     0.461677186654644 0.594229035166817 0.696122633002705
m = 8                     0.461677186654644 0.594229035166817 0.696122633002705
m = 9   0.402164111812444 0.501352569882777 0.600541027953111 0.699729486023445
```

```
m = 1
m = 2
m = 3
m = 4
m = 5
m = 6
m = 7   0.798016230838593
m = 8   0.798016230838593 0.898106402164112
m = 9   0.798917944093778 0.898106402164112

Fit:

m   0        1        2        3        4        5        6        7
RSS 1107.605 1103.641 1089.250 1086.738 1085.329 1083.540 1081.385 1080.956
BIC 3159.832 3169.879 3169.345 3180.807 3193.391 3205.584 3217.398 3230.981

m   8        9
RSS 1080.639 1093.996
BIC 3244.678 3272.323
```

2.9 ELBB2MON.Index

2.9.1 Time Series Plots



2.9.2 GARCH(1,1)-Model Fit

```
Title:
 GARCH Modelling

Call:
 garchFit(formula = ~garch(1, 1), data = as.zoo(na.omit(x)), trace = F)

Mean and Variance Equation:
 data ~ garch(1, 1)
<environment: 0xa30dea4>
 [data = as.zoo(na.omit(x))]

Conditional Distribution:
 norm

Coefficient(s):
        mu       omega      alpha1       beta1
7.5210e-04  5.6205e-08  4.9693e-02  9.5701e-01

Std. Errors:
 based on Hessian
```

```
Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     7.521e-04   9.144e-05    8.225 2.22e-16 ***
omega  5.620e-08          NA       NA       NA
alpha1 4.969e-02   3.393e-03   14.648  < 2e-16 ***
beta1  9.570e-01   1.238e-03  773.176  < 2e-16 ***
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ âĂŹ 1

Log Likelihood:
 3643.799    normalized:  3.185139

Description:
 Wed Jan 11 13:50:15 2012 by user: maggie


Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  89693.03  0
 Shapiro-Wilk Test  R    W      0.7427188 0
 Ljung-Box Test     R    Q(10)  774.6787  0
 Ljung-Box Test     R    Q(15)  929.6848  0
 Ljung-Box Test     R    Q(20)  1068.441  0
 Ljung-Box Test     R^2  Q(10)  1.280223  0.9994725
 Ljung-Box Test     R^2  Q(15)  1.951581  0.9999748
 Ljung-Box Test     R^2  Q(20)  2.374399  0.9999995
 LM Arch Test       R    TR^2   1.521713  0.999859

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-6.363286 -6.345655 -6.363310 -6.356629
```

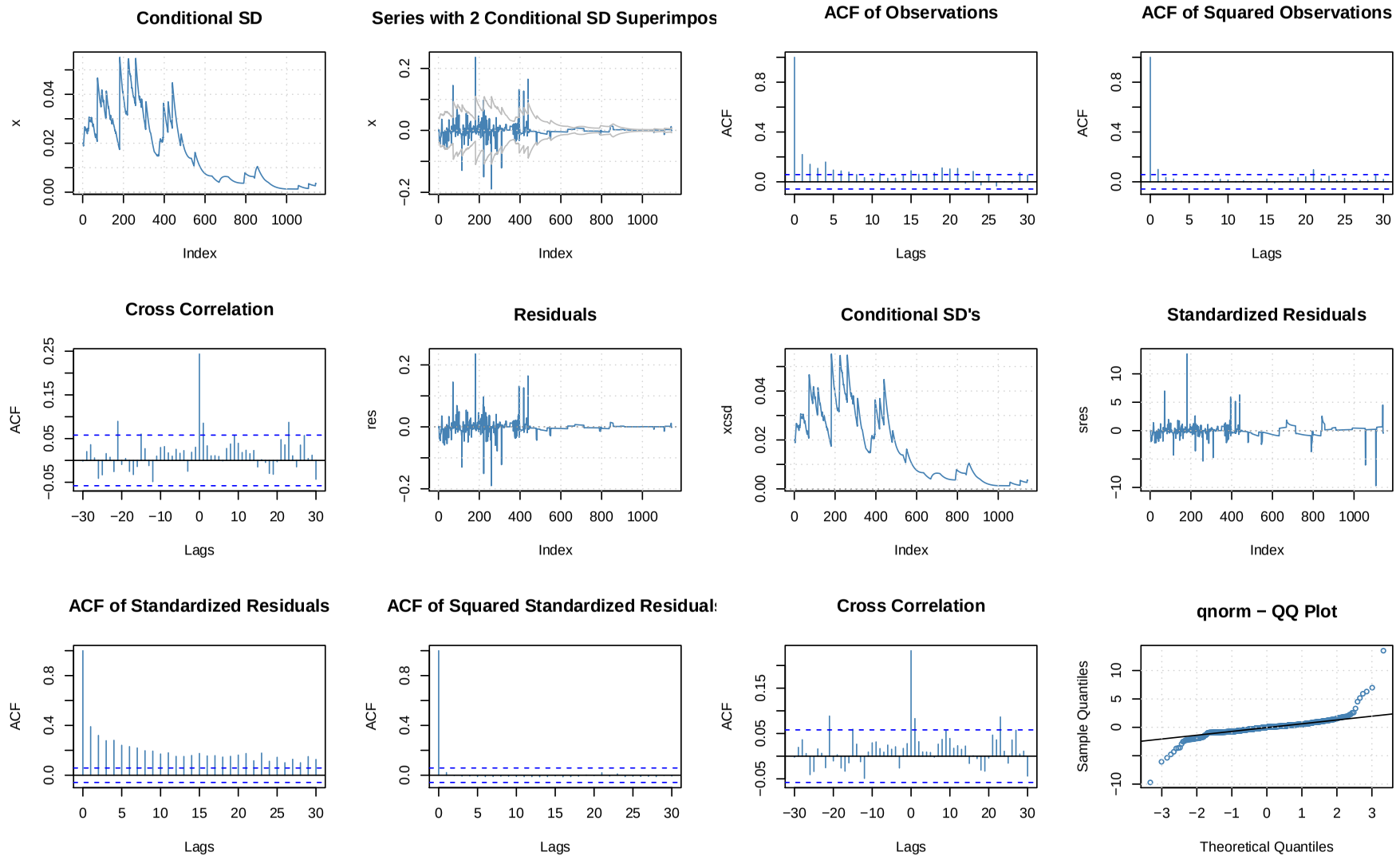


Figure 41: GARCH(1,1) diagnostic plots for ELBB2MON.Index

2.9.3 AR(1)-EGARCH(1,1)-Model Fit

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model : eGARCH(1,1)
Mean Model : ARFIMA(1,0,0)
Distribution : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.003541    0.000283  12.5160  0.00000
ar1     0.922711    0.010802  85.4223  0.00000
omega  -0.010239    0.008282  -1.2363  0.21633
alpha1  0.015919    0.011482   1.3865  0.16560
beta1   0.990603    0.001155 857.8283  0.00000
gamma1  0.207258    0.013550  15.2962  0.00000

Robust Standard Errors:
        Estimate  Std. Error   t value Pr(>|t|)
mu      0.003541    0.001141   3.10275 0.001917
ar1     0.922711    0.037768  24.43093 0.000000
omega  -0.010239    0.030264  -0.33831 0.735126
alpha1  0.015919    0.026197   0.60769 0.543391
beta1   0.990603    0.004224 234.51463 0.000000
gamma1  0.207258    0.032484   6.38034 0.000000

LogLikelihood : 4086.436

Information Criteria
------------------------------------

Akaike       -7.1336
Bayes        -7.1072
Shibata      -7.1337
Hannan-Quinn -7.1236

Q-Statistics on Standardized Residuals
------------------------------------
      statistic   p-value
Lag10     54.63 1.435e-08
Lag15     57.80 2.838e-07
Lag20     58.86 5.858e-06

H0 : No serial correlation

Q-Statistics on Standardized Squared Residuals
------------------------------------
      statistic p-value
Lag10     6.543  0.6846
Lag15     7.535  0.9121
Lag20     8.419  0.9819

ARCH LM Tests
------------------------------------
             Statistic DoF P-Value
ARCH Lag[2]      4.947   2  0.0843
ARCH Lag[5]      5.377   5  0.3716
ARCH Lag[10]     6.168  10  0.8010

Nyblom stability test
------------------------------------
Joint Statistic:  0.7617
Individual Statistics:
mu     0.10245
ar1    0.09643
omega  0.12065
alpha1 0.17016
beta1  0.11833
gamma1 0.11879

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:       1.49 1.68 2.12
Individual Statistic:  0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           0.4327 0.6653
Negative Sign Bias  1.0852 0.2781
Positive Sign Bias  1.5003 0.1338
Joint Effect        4.4264 0.2190


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20      3238            0
2    30      3668            0
3    40      3942            0
4    50      4131            0


Elapsed time : 0.8349519
```

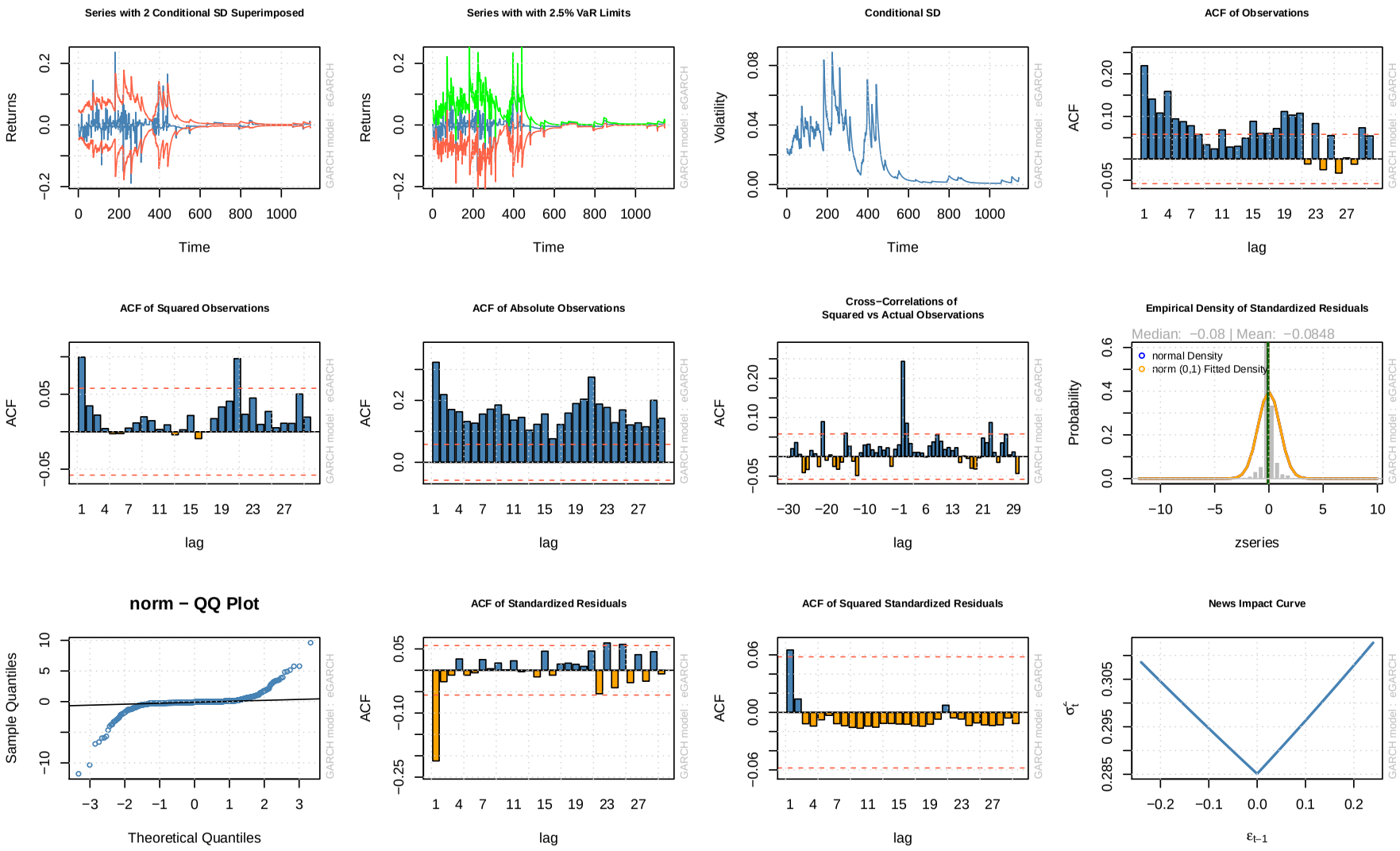


Figure 42: AR(1)-EGARCH(1,1) diagnostic plots for ELBB2MON.Index

2.9.4 Empirical Fluctuation Process

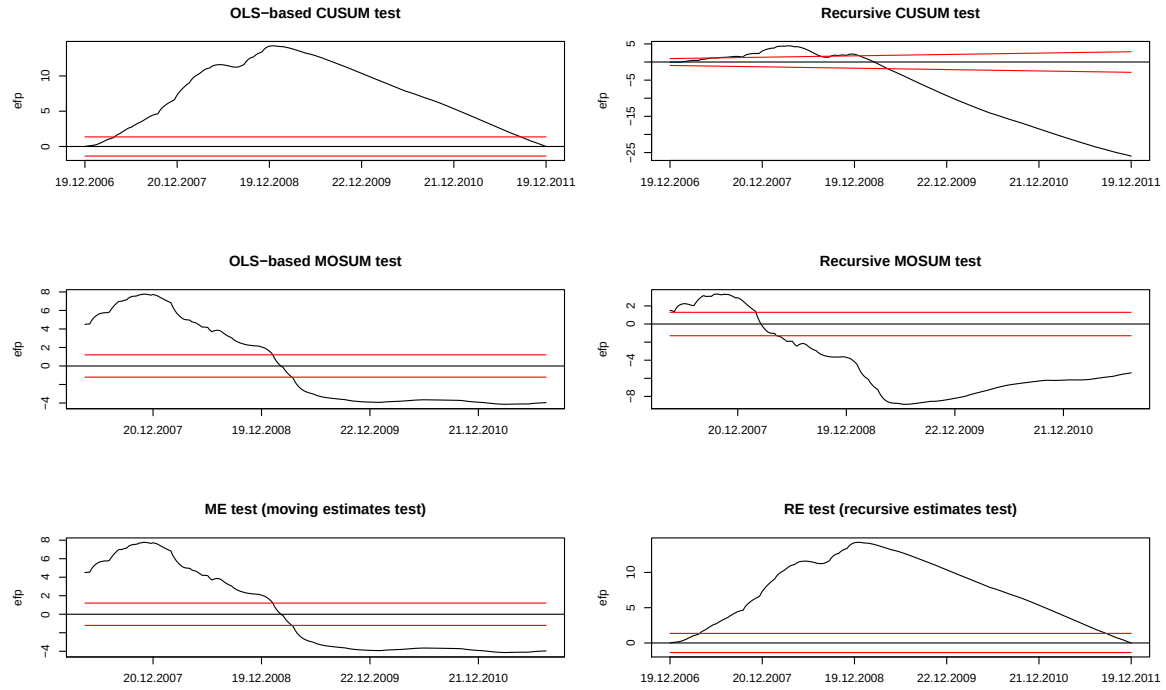


Figure 43: Empirical fluctuation processes for ELBB2MON.Index

	statistic	p-value
OLS-CUSUM	14.26548	0.00000
Rec-CUSUM	8.65681	0.00000
OLS-MOSUM	7.76834	0.01000
Rec-MOSUM	8.87289	0.01000
ME	7.76834	0.01000
RE	14.26548	0.00000

Table 17: General Test for all models for ELBB2MON.Index

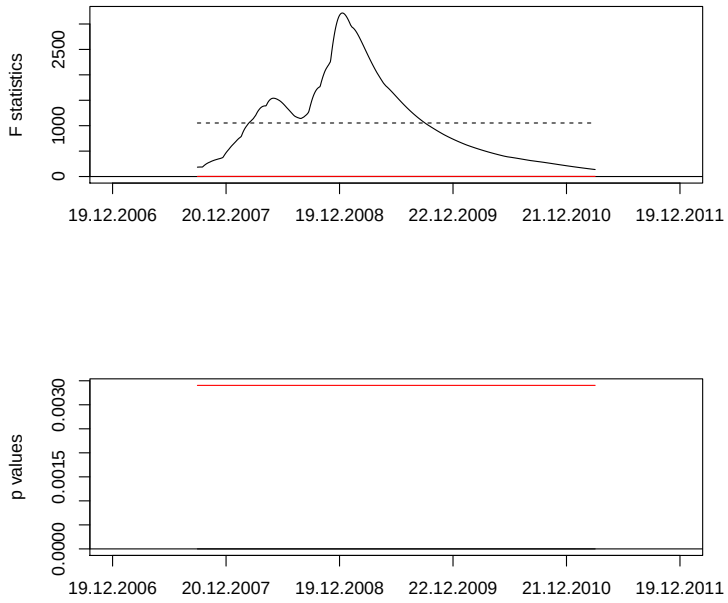


Figure 44: F-test statistic and p-values for ELBB2MON.Index

	statistic	p-value
sup	3216.08199	0.00000
ave	1053.04140	0.00000
exp	Inf	

Table 18: F-Test with different functionals for ELBB2MON.Index

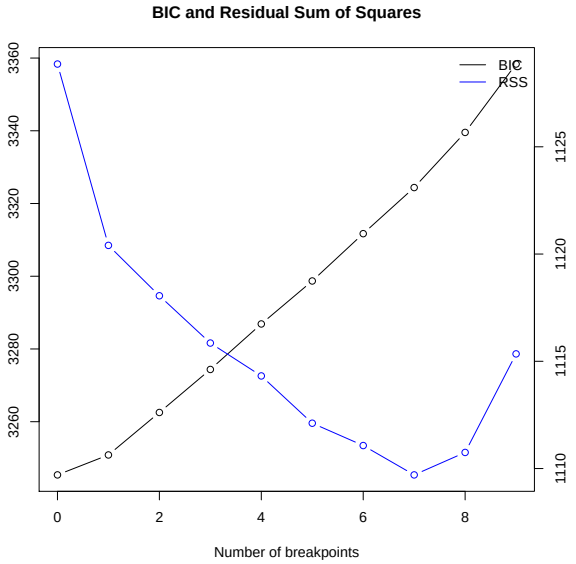


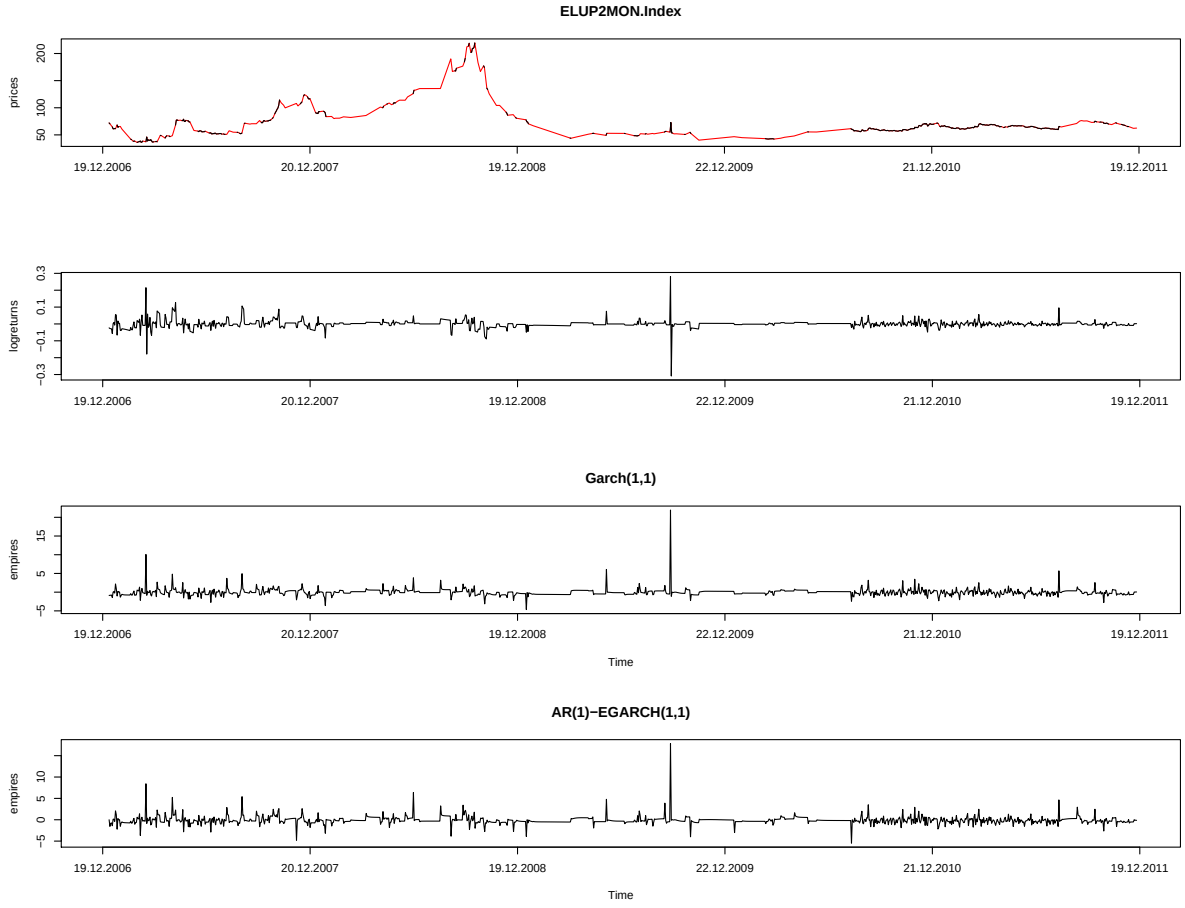
Figure 45: RSS and BIC with respect to number of possible breakpoints for ELBB2MON.Index

```
 Optimal (m+1)-segment partition:

Call:
breakpoints.formula(formula = x ~ 1, h = 0.1)

Breakpoints at observation number:

m = 1                       713
m = 2                       713 827
m = 3                       713 837 951
m = 4               439 599 713 827
m = 5               439 599 713 837 951
m = 6           309 439 599 713 837 951
m = 7       181 309 439 599 713 837 951
m = 8   115 232 350 479 599 713 837 951
m = 9   115 232 346 460 574 688 802 916 1030

Corresponding to breakdates:

m = 1
m = 2
m = 3
m = 4                                                       0.383741258741259
m = 5                                                       0.383741258741259
m = 6                                   0.270104895104895 0.383741258741259
m = 7                     0.158216783216783 0.270104895104895 0.383741258741259
m = 8   0.100524475524476 0.202797202797203 0.305944055944056 0.418706293706294
m = 9   0.100524475524476 0.202797202797203 0.302447552447552 0.402097902097902

m = 1                     0.623251748251748
m = 2                     0.623251748251748 0.722902097902098
m = 3                     0.623251748251748 0.731643356643357 0.831293706293706
m = 4   0.523601398601399 0.623251748251748 0.722902097902098
m = 5   0.523601398601399 0.623251748251748 0.731643356643357 0.831293706293706
m = 6   0.523601398601399 0.623251748251748 0.731643356643357 0.831293706293706
m = 7   0.523601398601399 0.623251748251748 0.731643356643357 0.831293706293706
m = 8   0.523601398601399 0.623251748251748 0.731643356643357 0.831293706293706
m = 9   0.501748251748252 0.601398601398601 0.701048951048951 0.800699300699301
```

```
m = 1
m = 2
m = 3
m = 4
m = 5
m = 6
m = 7
m = 8
m = 9   0.90034965034965

Fit:

m   0        1        2        3        4        5        6        7
RSS 1128.859 1120.404 1118.055 1115.851 1114.318 1112.114 1111.072 1109.704
BIC 3245.374 3250.858 3262.541 3274.369 3286.880 3298.700 3311.713 3324.387

m   8        9
RSS 1110.750 1115.346
BIC 3339.550 3358.358
```

2.10 ELUP2MON.Index

2.10.1 Time Series Plots



2.10.2 GARCH(1,1)-Model Fit

```
Title:
 GARCH Modelling

Call:
 garchFit(formula = ~garch(1, 1), data = as.zoo(na.omit(x)), trace = F)

Mean and Variance Equation:
 data ~ garch(1, 1)
<environment: 0xa47791c>
 [data = as.zoo(na.omit(x))]

Conditional Distribution:
 norm

Coefficient(s):
        mu       omega      alpha1       beta1
0.00110635  0.00004375  0.99999999  0.47299863

Std. Errors:
 based on Hessian
```

```
Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     1.106e-03   7.192e-05   15.382  < 2e-16 ***
omega  4.375e-05   7.198e-06    6.078 1.21e-09 ***
alpha1 1.000e+00   9.676e-02   10.335  < 2e-16 ***
beta1  4.730e-01   5.371e-02    8.806  < 2e-16 ***
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ âĂŹ 1

Log Likelihood:
 3336.727    normalized:  2.598697

Description:
 Wed Jan 11 13:50:16 2012 by user: maggie


Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  1763328   0
 Shapiro-Wilk Test  R    W      0.5899611 0
 Ljung-Box Test     R    Q(10)  71.39331  2.384715e-11
 Ljung-Box Test     R    Q(15)  78.53771  1.291431e-10
 Ljung-Box Test     R    Q(20)  102.947   3.725908e-13
 Ljung-Box Test     R^2  Q(10)  0.1574182 1
 Ljung-Box Test     R^2  Q(15)  0.208238  1
 Ljung-Box Test     R^2  Q(20)  0.250678  1
 LM Arch Test       R    TR^2   0.1906271 1

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-5.191164 -5.175096 -5.191184 -5.185132
```

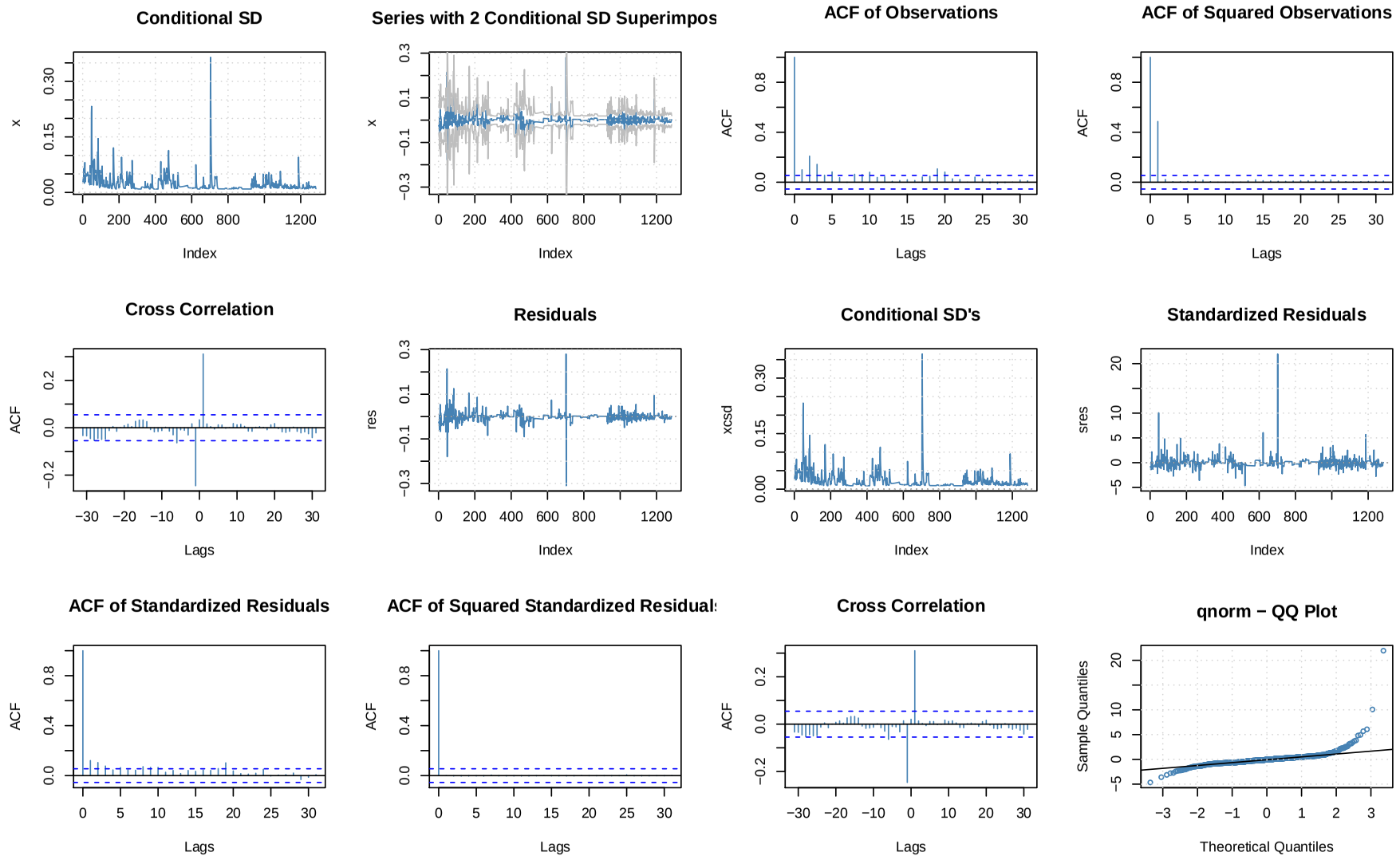


Figure 46: GARCH(1,1) diagnostic plots for ELUP2MON.Index

2.10.3 AR(1)-EGARCH(1,1)-Model Fit

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model : eGARCH(1,1)
Mean Model : ARFIMA(1,0,0)
Distribution : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.003303    0.000258  12.7962  0.00000
ar1     0.158197    0.122218   1.2944  0.19553
omega  -0.869927    0.105765  -8.2251  0.00000
alpha1 -0.282455    0.034613  -8.1605  0.00000
beta1   0.872830    0.013983  62.4211  0.00000
gamma1  0.903305    0.068358  13.2144  0.00000

Robust Standard Errors:
        Estimate  Std. Error   t value Pr(>|t|)
mu      0.003303    0.005008  0.659558  0.50954
ar1     0.158197    2.507523  0.063089  0.94970
omega  -0.869927    0.990737 -0.878060  0.37991
alpha1 -0.282455    0.205304 -1.375789  0.16889
beta1   0.872830    0.151192  5.773002  0.00000
gamma1  0.903305    1.087373  0.830722  0.40613

LogLikelihood : 3532.343

Information Criteria
------------------------------------

Akaike       -5.4927
Bayes        -5.4686
Shibata      -5.4928
Hannan-Quinn -5.4837

Q-Statistics on Standardized Residuals
------------------------------------
      statistic   p-value
Lag10     67.20 5.382e-11
Lag15     69.46 2.422e-09
Lag20     88.34 6.531e-11

H0 : No serial correlation

Q-Statistics on Standardized Squared Residuals
------------------------------------
      statistic p-value
Lag10     3.078  0.9611
Lag15     3.221  0.9986
Lag20     3.320  1.0000

ARCH LM Tests
------------------------------------
             Statistic DoF P-Value
ARCH Lag[2]    0.06759   2  0.9668
ARCH Lag[5]    0.19182   5  0.9992
ARCH Lag[10]   2.99806  10  0.9815

Nyblom stability test
------------------------------------
Joint Statistic:  2.0299
Individual Statistics:
mu     0.08347
ar1    0.06532
omega  0.14904
alpha1 0.13609
beta1  0.14263
gamma1 0.19415

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:       1.49 1.68 2.12
Individual Statistic:  0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           0.1418 0.8873
Negative Sign Bias  0.4061 0.6848
Positive Sign Bias  0.1651 0.8689
Joint Effect        0.1951 0.9784


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     691.5   2.256e-134
2    30     732.1   1.917e-135
3    40     807.1   3.381e-144
4    50     836.6   7.184e-144


Elapsed time : 0.9561591
```

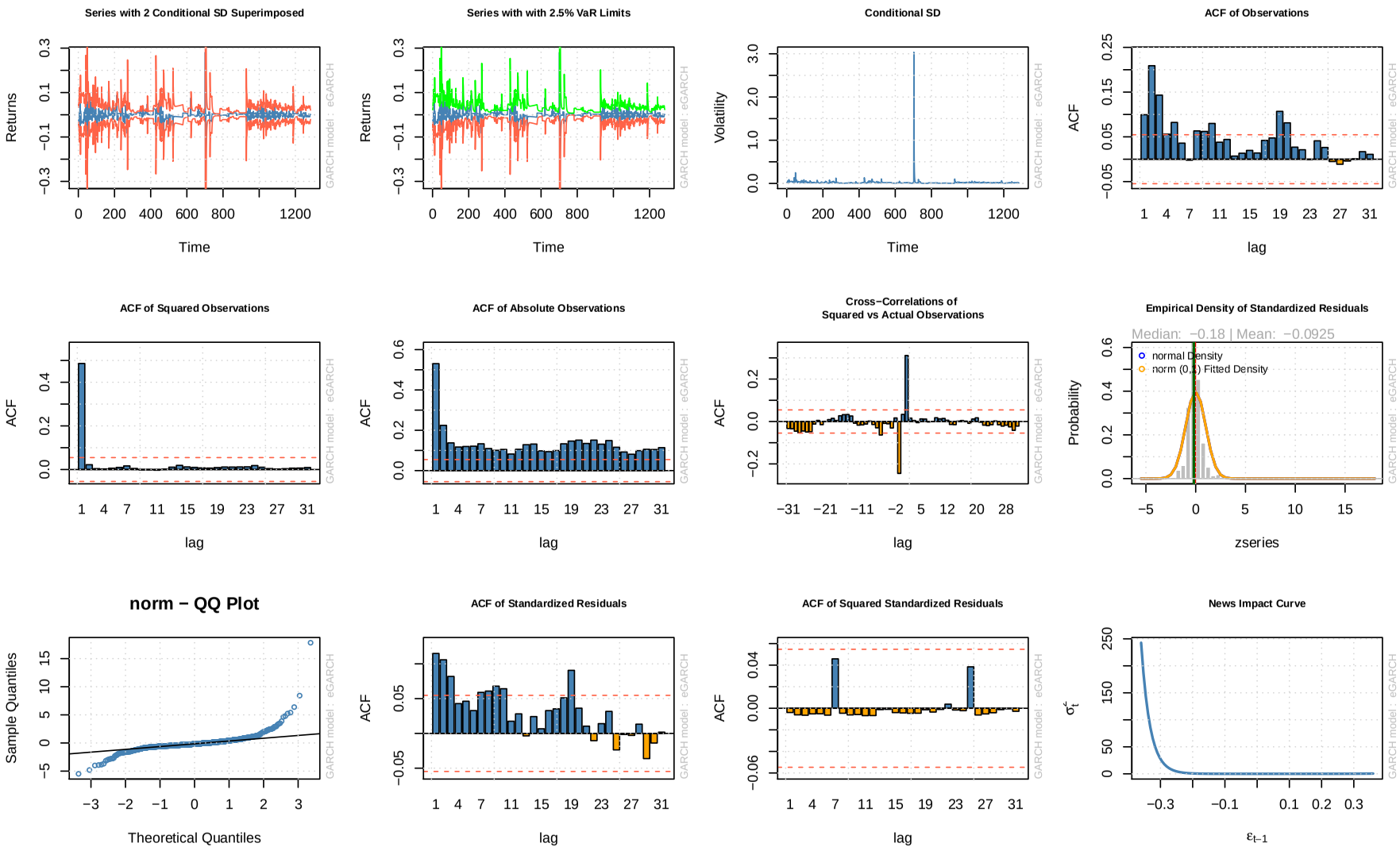


Figure 47: AR(1)-EGARCH(1,1) diagnostic plots for ELUP2MON.Index

2.10.4 Empirical Fluctuation Process

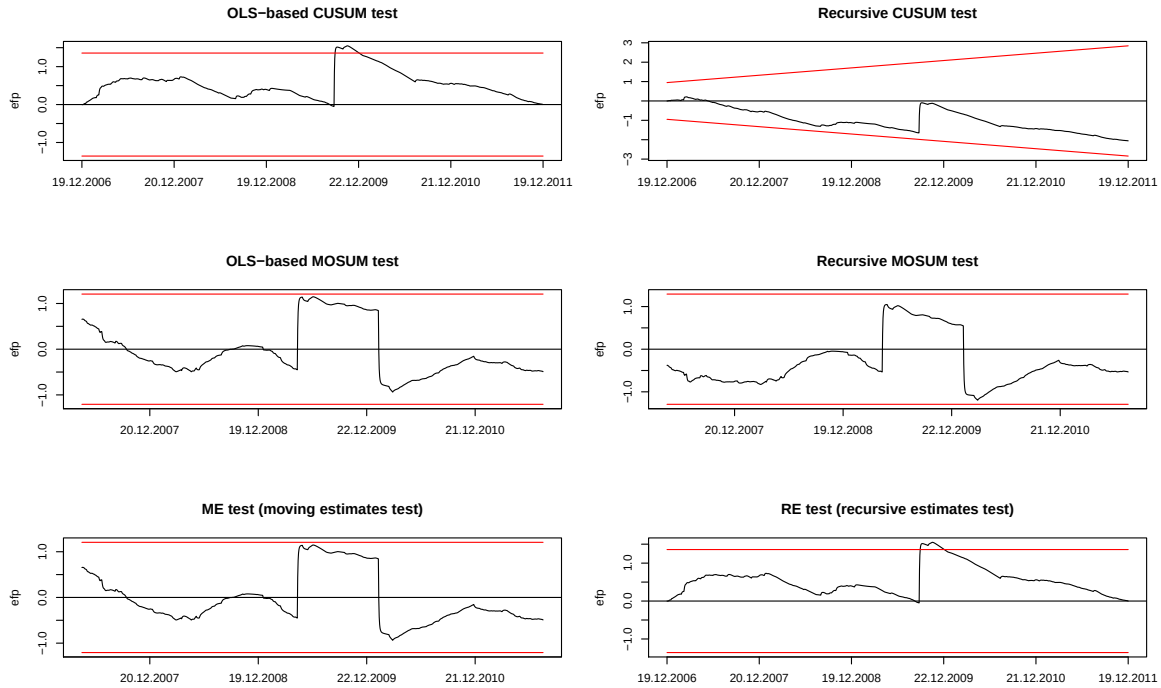


Figure 48: Empirical fluctuation processes for ELUP2MON.Index

	statistic	p-value
OLS-CUSUM	1.54864	0.01652
Rec-CUSUM	0.78748	0.14945
OLS-MOSUM	1.14631	0.08514
Rec-MOSUM	1.19723	0.10374
ME	1.14631	0.08514
RE	1.54864	0.01652

Table 19: General Test for all models for ELUP2MON.Index

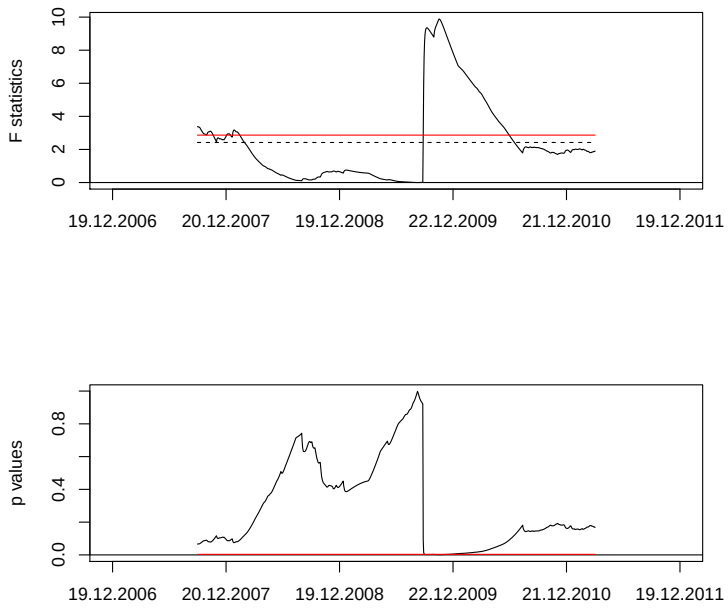


Figure 49: F-test statistic and p-values for ELUP2MON.Index

	statistic	p-value
sup	9.88524	0.02785
ave	2.42224	0.07475
exp	2.42046	0.03156

Table 20: F-Test with different functionals for ELUP2MON.Index

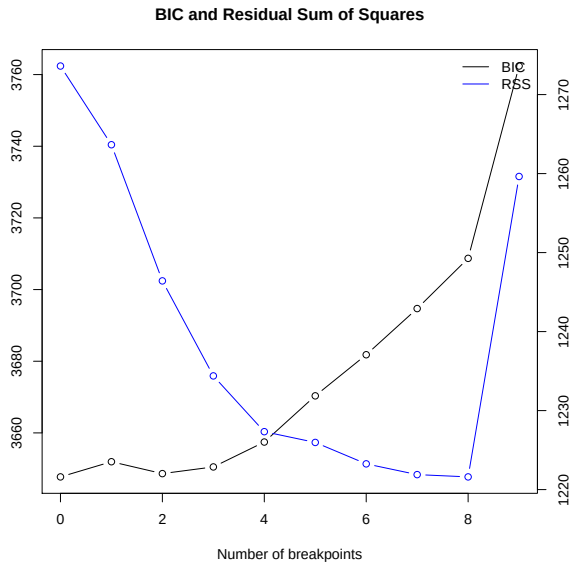


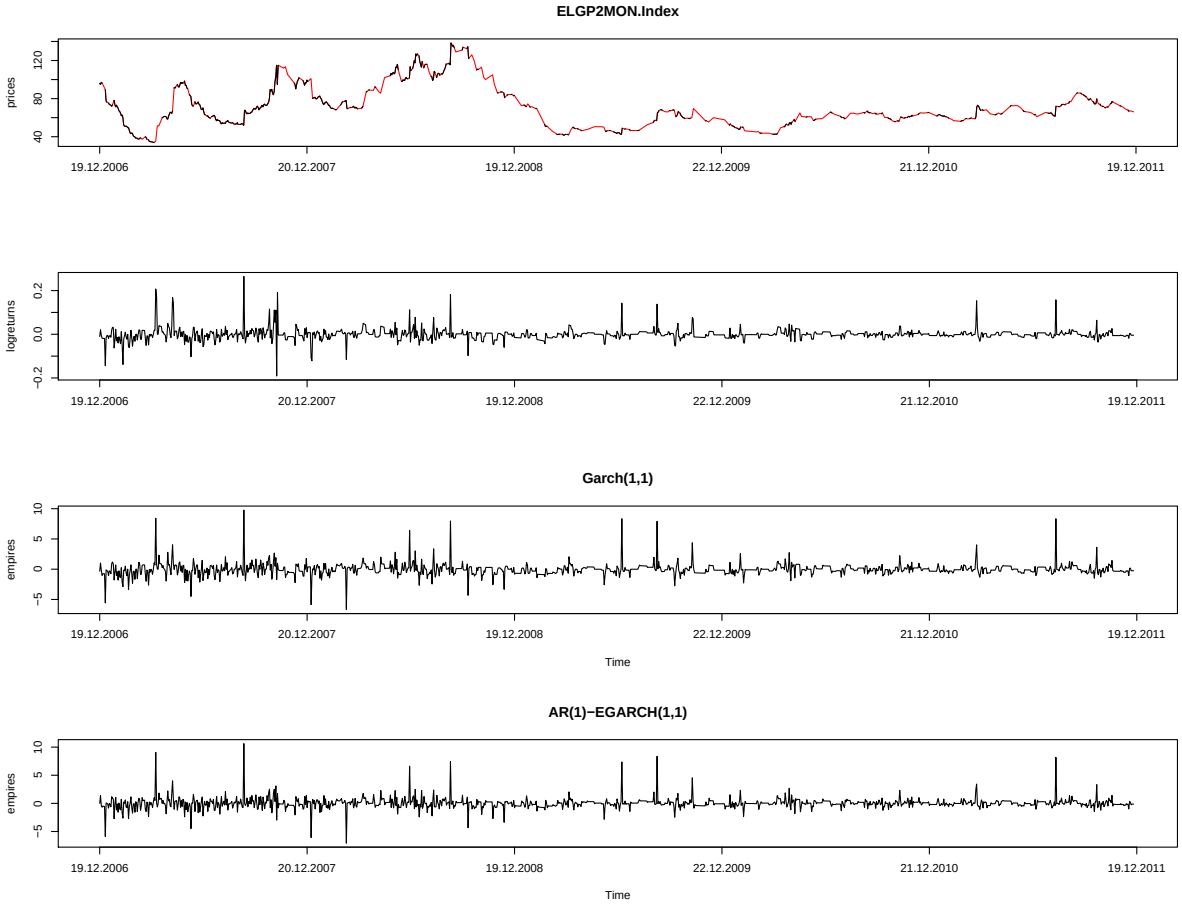
Figure 50: RSS and BIC with respect to number of possible breakpoints for ELUP2MON.Index

```
 Optimal (m+1)-segment partition:

Call:
breakpoints.formula(formula = x ~ 1, h = 0.1)

Breakpoints at observation number:

m = 1               457
m = 2           450 578
m = 3       321 450 578
m = 4       321 450 578     726
m = 5   164 321 450 578     726
m = 6       321 450 578 706 834     1019
m = 7   164 321 450 578 706 834     1019
m = 8   164 321 450 578 706 834     1019 1147
m = 9   131 259 388 516 644 772 900 1028 1156

Corresponding to breakdates:

m = 1                                                       0.355919003115265
m = 2                                       0.350467289719626 0.450155763239875
m = 3                     0.25              0.350467289719626 0.450155763239875
m = 4                     0.25              0.350467289719626 0.450155763239875
m = 5   0.127725856697819 0.25              0.350467289719626 0.450155763239875
m = 6                     0.25              0.350467289719626 0.450155763239875
m = 7   0.127725856697819 0.25              0.350467289719626 0.450155763239875
m = 8   0.127725856697819 0.25              0.350467289719626 0.450155763239875
m = 9   0.10202492211838  0.201713395638629 0.302180685358255 0.401869158878505

m = 1
m = 2
m = 3
m = 4                     0.565420560747664
m = 5                     0.565420560747664
m = 6   0.549844236760125 0.649532710280374                   0.793613707165109
m = 7   0.549844236760125 0.649532710280374                   0.793613707165109
m = 8   0.549844236760125 0.649532710280374                   0.793613707165109
m = 9   0.501557632398754 0.601246105919003 0.700934579439252 0.800623052959502
```

```
m = 1
m = 2
m = 3
m = 4
m = 5
m = 6
m = 7
m = 8   0.893302180685358
m = 9   0.900311526479751

Fit:

m   0        1        2        3        4        5        6        7
RSS 1273.617 1263.654 1246.421 1234.392 1227.331 1225.969 1223.258 1221.896
BIC 3647.724 3651.957 3648.641 3650.504 3657.454 3670.344 3681.817 3694.702

m   8        9
RSS 1221.609 1259.636
BIC 3708.715 3762.391
```

2.11 ELGP2MON.Index

2.11.1 Time Series Plots



2.11.2 GARCH(1,1)-Model Fit

```
Title:
 GARCH Modelling

Call:
 garchFit(formula = ~garch(1, 1), data = as.zoo(na.omit(x)), trace = F)

Mean and Variance Equation:
 data ~ garch(1, 1)
<environment: 0xa39b200>
 [data = as.zoo(na.omit(x))]

Conditional Distribution:
 norm

Coefficient(s):
        mu       omega      alpha1       beta1
9.6289e-05  2.8919e-04  1.0000e+00  2.1010e-03

Std. Errors:
 based on Hessian
```

```
Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     9.629e-05   5.250e-04    0.183    0.854
omega  2.892e-04   1.853e-05   15.608   <2e-16 ***
alpha1 1.000e+00   1.055e-01    9.482   <2e-16 ***
beta1  2.101e-03   3.577e-03    0.587    0.557
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ âĂŹ 1

Log Likelihood:
 3071.085    normalized:  2.377001

Description:
 Wed Jan 11 13:50:16 2012 by user: maggie


Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  40912.66  0
 Shapiro-Wilk Test  R    W      0.7372044 0
 Ljung-Box Test     R    Q(10)  55.79137  2.246143e-08
 Ljung-Box Test     R    Q(15)  66.34919  1.979873e-08
 Ljung-Box Test     R    Q(20)  69.08236  2.56984e-07
 Ljung-Box Test     R^2  Q(10)  0.9195524 0.999883
 Ljung-Box Test     R^2  Q(15)  1.569376  0.9999942
 Ljung-Box Test     R^2  Q(20)  1.974923  0.9999999
 LM Arch Test       R    TR^2   1.072839  0.9999791

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-4.747810 -4.731823 -4.747829 -4.741810
```

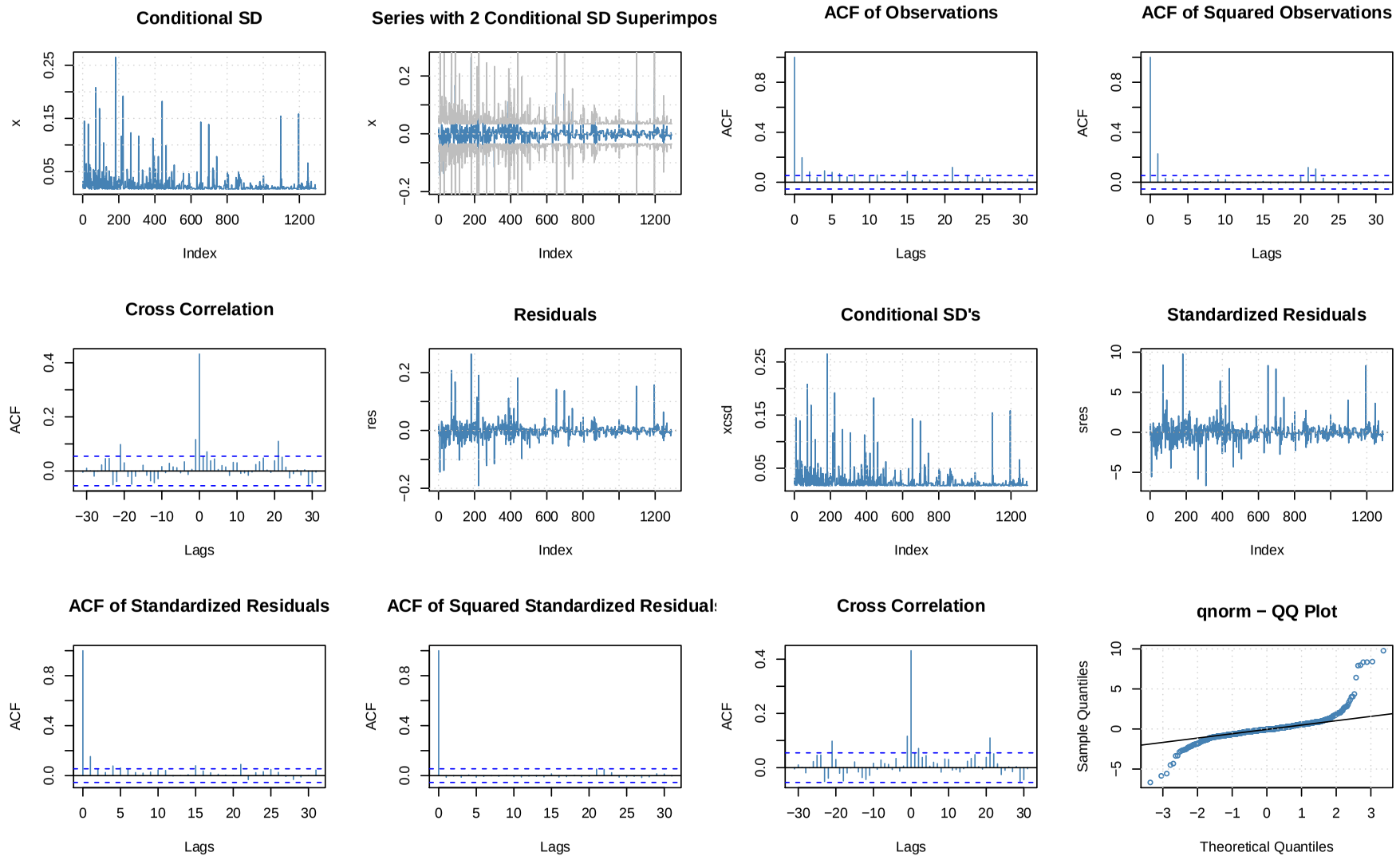


Figure 51: GARCH(1,1) diagnostic plots for ELGP2MON.Index

2.11.3 AR(1)-EGARCH(1,1)-Model Fit

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model : eGARCH(1,1)
Mean Model : ARFIMA(1,0,0)
Distribution : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu     -0.001108    0.000036 -30.7003 0.000000
ar1     0.313086    0.001153 271.5679 0.000000
omega  -4.278318    0.448961  -9.5294 0.000000
alpha1 -0.106178    0.045997  -2.3084 0.020979
beta1   0.405037    0.058577   6.9146 0.000000
gamma1  0.990317    0.074786  13.2419 0.000000

Robust Standard Errors:
        Estimate  Std. Error   t value Pr(>|t|)
mu     -0.001108    0.000015 -73.37086 0.000000
ar1     0.313086    0.000829 377.56718 0.000000
omega  -4.278318    1.168789  -3.66047 0.000252
alpha1 -0.106178    0.174991  -0.60676 0.544007
beta1   0.405037    0.159556   2.53853 0.011132
gamma1  0.990317    0.249209   3.97384 0.000071

LogLikelihood : 3096.207

Information Criteria
------------------------------------

Akaike       -4.7836
Bayes        -4.7596
Shibata      -4.7836
Hannan-Quinn -4.7746

Q-Statistics on Standardized Residuals
------------------------------------
      statistic p-value
Lag10     12.35 0.19418
Lag15     21.64 0.08622
Lag20     23.17 0.22986

H0 : No serial correlation

Q-Statistics on Standardized Squared Residuals
------------------------------------
      statistic p-value
Lag10    0.9898  0.9995
Lag15    1.5192  1.0000
Lag20    1.8881  1.0000

ARCH LM Tests
------------------------------------
             Statistic DoF P-Value
ARCH Lag[2]     0.4078   2  0.8155
ARCH Lag[5]     0.8104   5  0.9764
ARCH Lag[10]    0.9362  10  0.9999

Nyblom stability test
------------------------------------
Joint Statistic:  3.4079
Individual Statistics:
mu     0.05241
ar1    1.65267
omega  1.31910
alpha1 0.05568
beta1  1.37988
gamma1 1.07484

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:       1.49 1.68 2.12
Individual Statistic:  0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias          0.26053 0.7945
Negative Sign Bias 0.05026 0.9599
Positive Sign Bias 0.12166 0.9032
Joint Effect       0.09896 0.9920


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     605.3   3.772e-116
2    30     648.8   4.660e-118
3    40     682.9   1.423e-118
4    50     724.0   6.908e-121


Elapsed time : 0.8885119
```

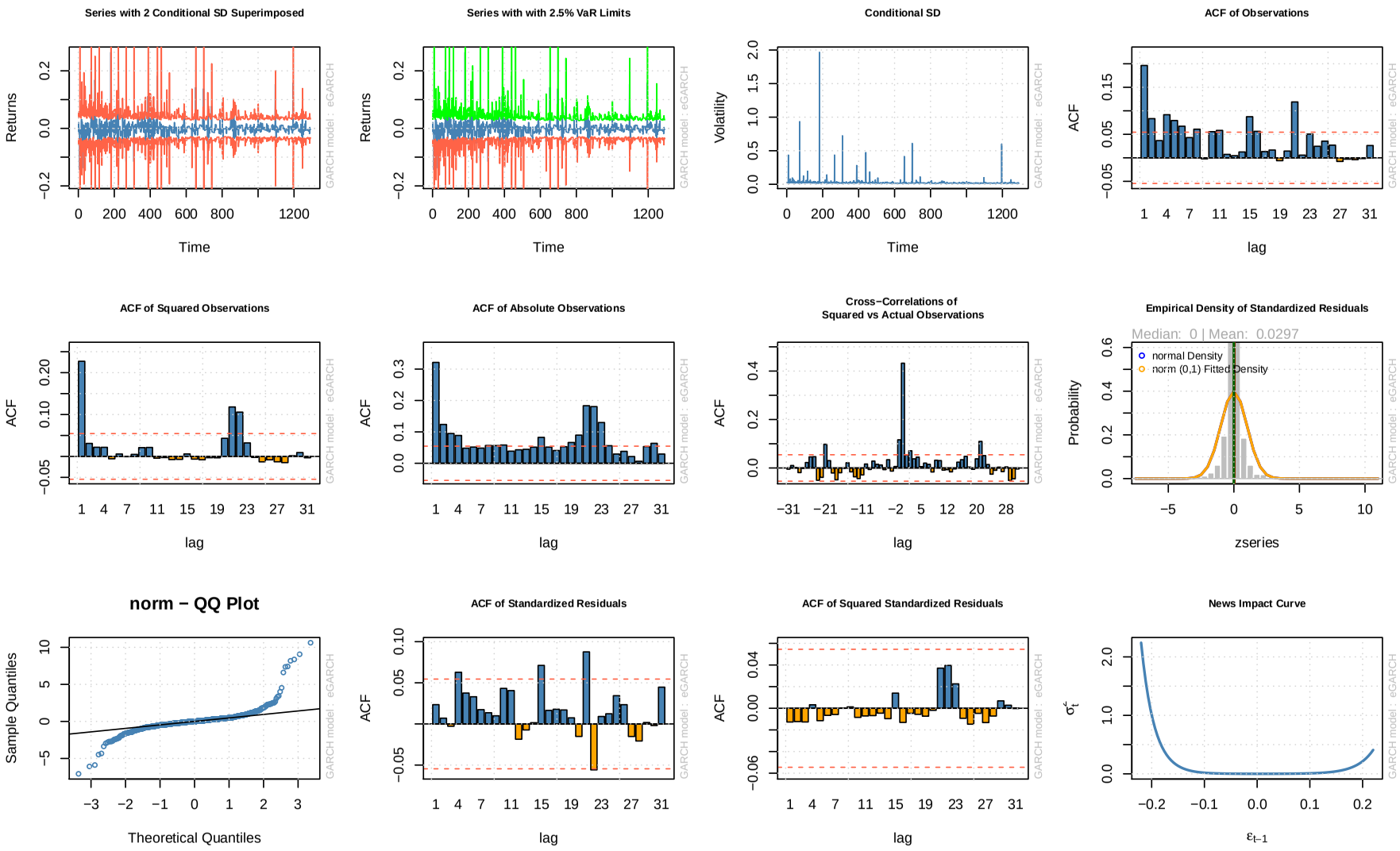


Figure 52: AR(1)-EGARCH(1,1) diagnostic plots for ELGP2MON.Index

2.11.4 Empirical Fluctuation Process

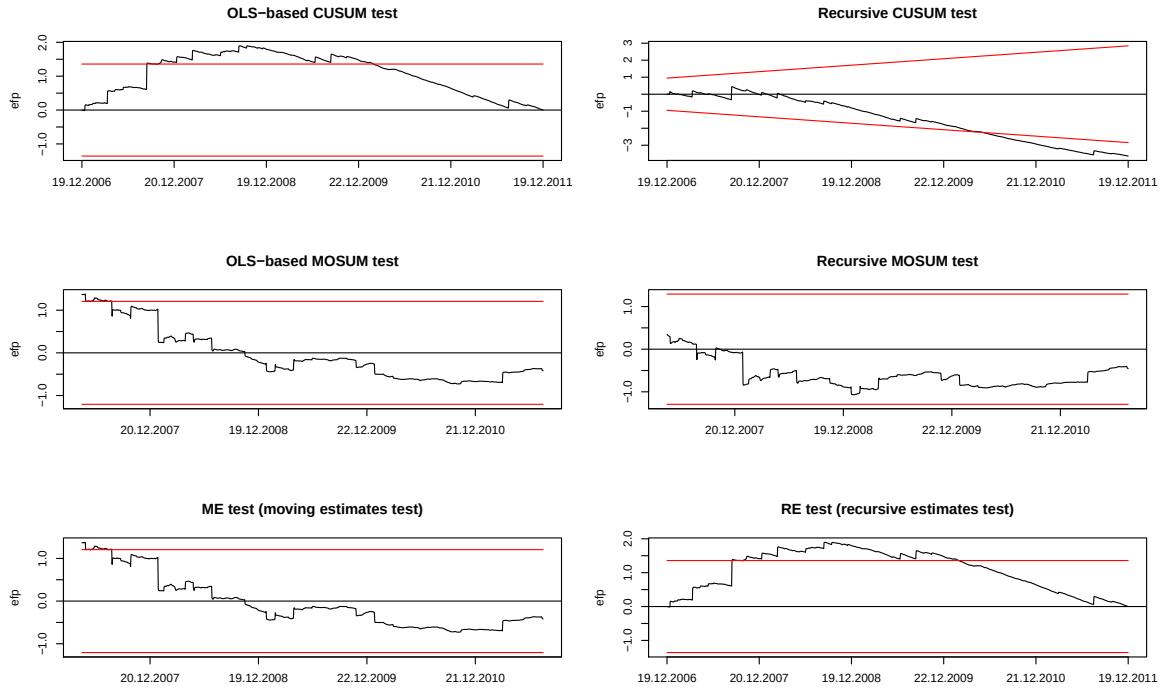


Figure 53: Empirical fluctuation processes for ELGP2MON.Index

	statistic	p-value
OLS-CUSUM	1.89556	0.00151
Rec-CUSUM	1.25001	0.00363
OLS-MOSUM	1.37729	0.01000
Rec-MOSUM	1.06841	0.21988
ME	1.37729	0.01000
RE	1.89556	0.00151

Table 21: General Test for all models for ELGP2MON.Index

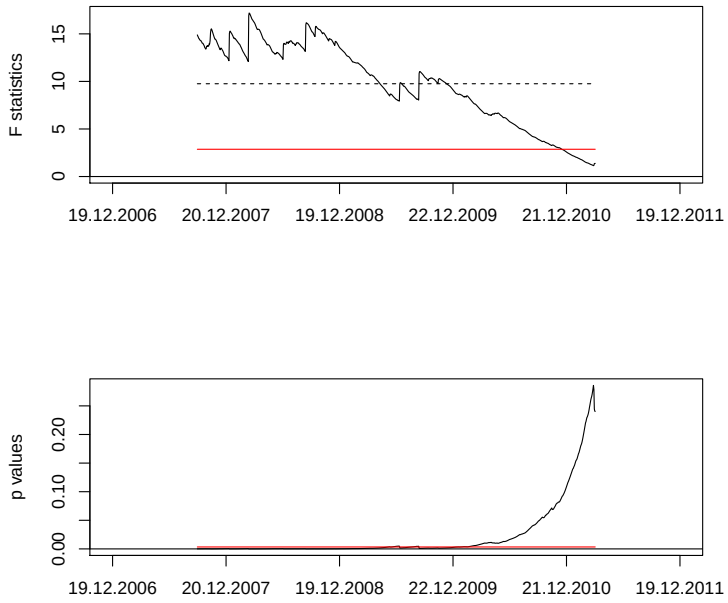


Figure 54: F-test statistic and p-values for ELGP2MON.Index

	statistic	p-value
sup	17.19258	0.00087
ave	9.75688	0.00000
exp	6.36675	0.00001

Table 22: F-Test with different functionals for ELGP2MON.Index

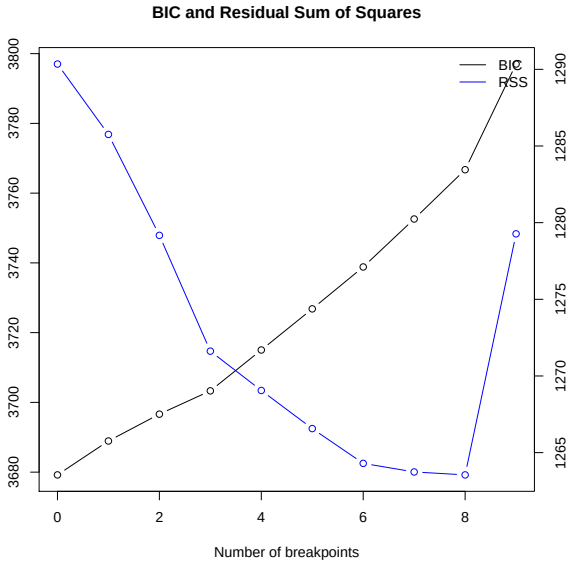


Figure 55: RSS and BIC with respect to number of possible breakpoints for ELGP2MON.Index

```
 Optimal (m+1)-segment partition:

Call:
breakpoints.formula(formula = x ~ 1, h = 0.1)

Breakpoints at observation number:

m = 1   152
m = 2               454 586
m = 3   152         454 586
m = 4   152 322     454 586
m = 5   152 322     454 586 717
m = 6   152 322     454 586 717 846
m = 7   152 322     454 586 717 846 978
m = 8   152 322     454 586 717 846 978  1119
m = 9   130 259 388 517 646 775 904 1034 1163

Corresponding to breakdates:

m = 1   0.117647058823529
m = 2                                                       0.351393188854489
m = 3   0.117647058823529                                   0.351393188854489
m = 4   0.117647058823529 0.24922600619195                  0.351393188854489
m = 5   0.117647058823529 0.24922600619195                  0.351393188854489
m = 6   0.117647058823529 0.24922600619195                  0.351393188854489
m = 7   0.117647058823529 0.24922600619195                  0.351393188854489
m = 8   0.117647058823529 0.24922600619195                  0.351393188854489
m = 9   0.10061919504644  0.20046439628483 0.30030959752322 0.40015479876161

m = 1
m = 2   0.453560371517028
m = 3   0.453560371517028
m = 4   0.453560371517028
m = 5   0.453560371517028 0.554953560371517
m = 6   0.453560371517028 0.554953560371517 0.654798761609907
m = 7   0.453560371517028 0.554953560371517 0.654798761609907 0.756965944272446
m = 8   0.453560371517028 0.554953560371517 0.654798761609907 0.756965944272446
m = 9   0.5               0.59984520123839  0.69969040247678  0.80030959752322
```

```
m = 1
m = 2
m = 3
m = 4
m = 5
m = 6
m = 7
m = 8   0.86609907120743
m = 9   0.90015479876161

Fit:

m   0        1        2        3        4        5        6        7
RSS 1290.347 1285.754 1279.172 1271.619 1269.054 1266.569 1264.300 1263.740
BIC 3679.211 3688.932 3696.629 3703.306 3715.025 3726.820 3738.831 3752.587

m   8        9
RSS 1263.548 1279.271
BIC 3766.719 3797.024
```